

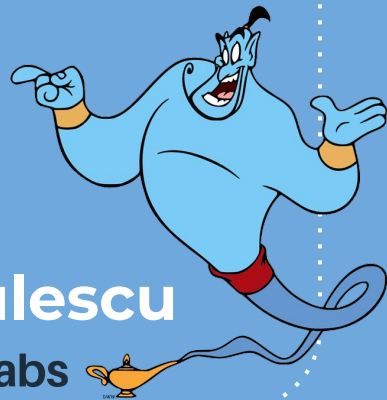


Vector Commitments

and practical applications

ia.cr/2022/705

with Matteo Campanelli, Carla Ràfols,
Alexandros Zacharakis, Arantxa Zapico



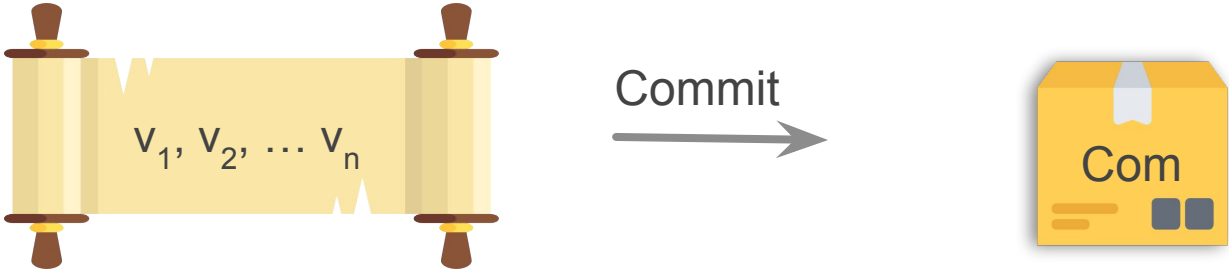
Anca Nitulescu



Protocol Labs

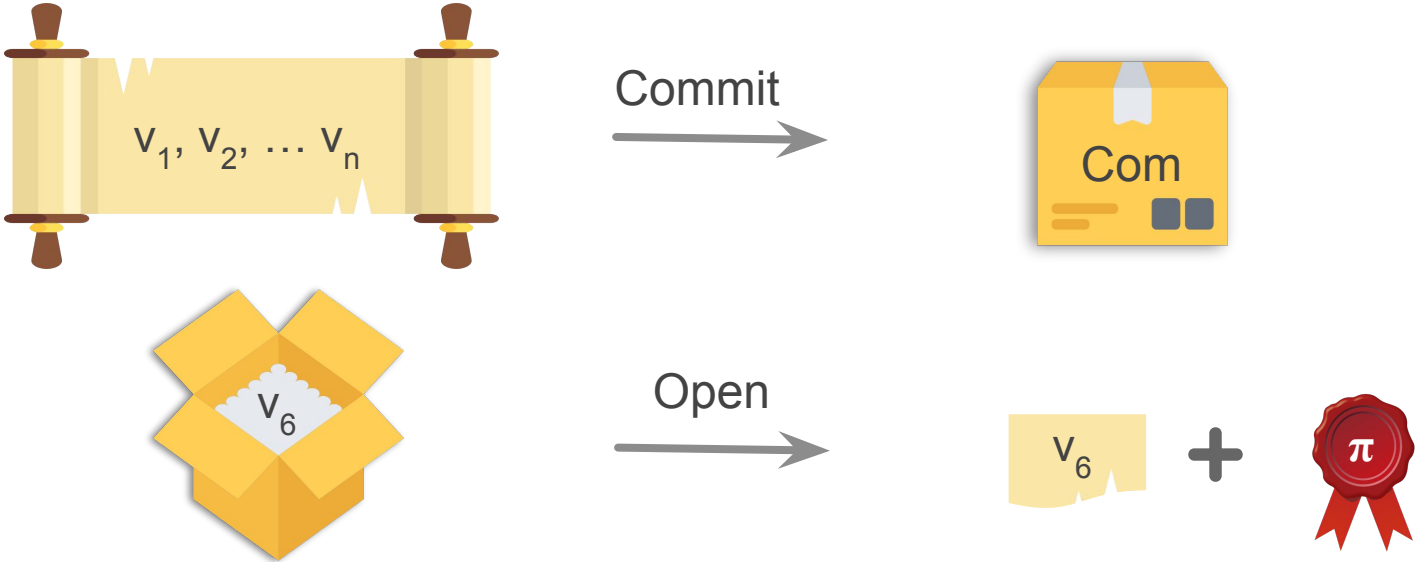


Vector Commitments

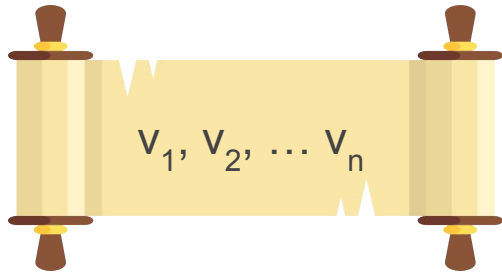




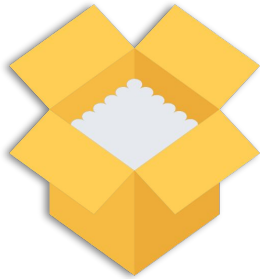
Vector Commitments



SubVector Commit



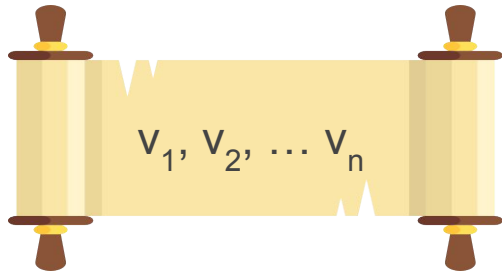
Commit
→



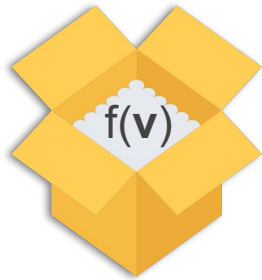
Open
→
subvector



Functional VC



Commit
→

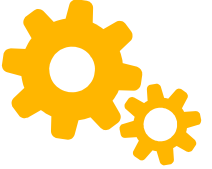


Open
→

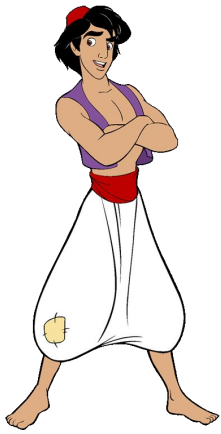


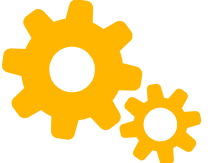


Motivation

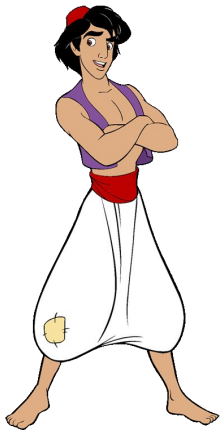


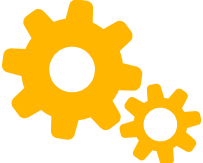
Storage Delegation



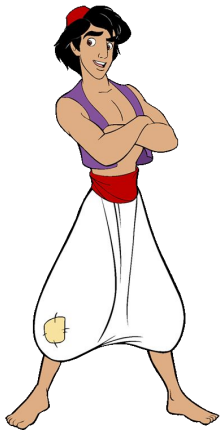


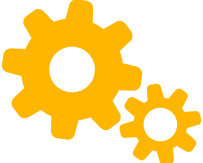
Storage Delegation





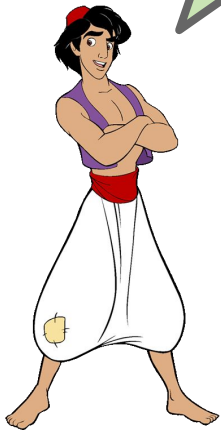
Storage Delegation





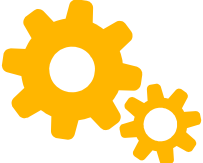
Storage Delegation

Query 3



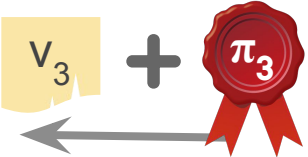
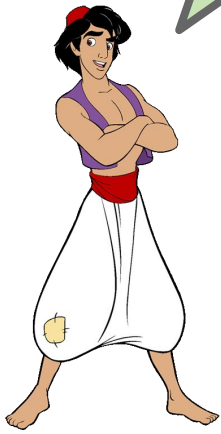
- V₁
- V₂
- V₃
- V₄
- V₅





Storage Delegation

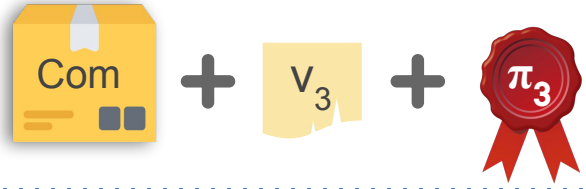
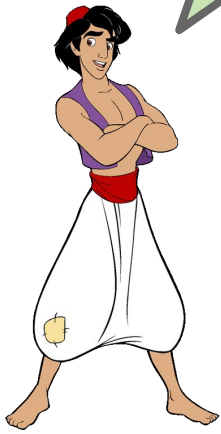
Query 3

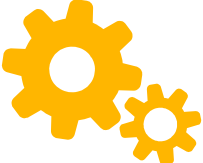




Storage Delegation

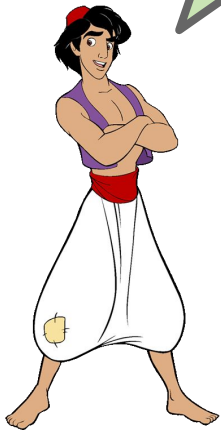
Verify v_3





Storage Delegation

Verify v_3



+



+

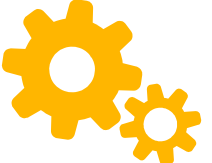


verify →



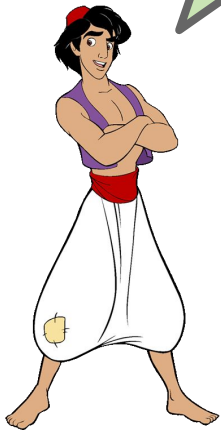
/

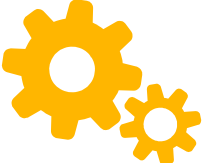




Storage Delegation

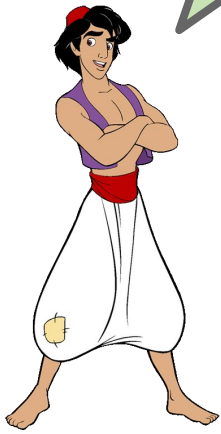
Query $f()$
on vector v





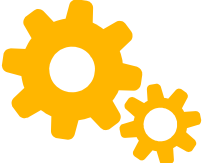
Storage Delegation

Query $f()$
on vector \mathbf{v}



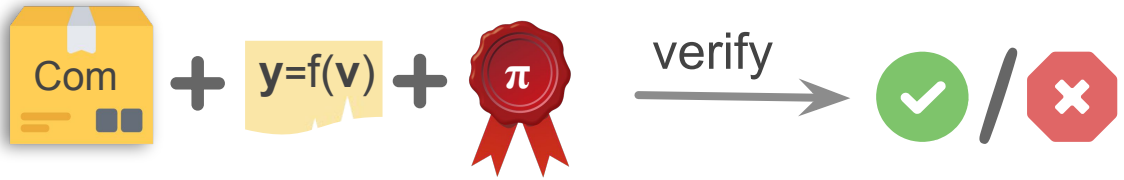
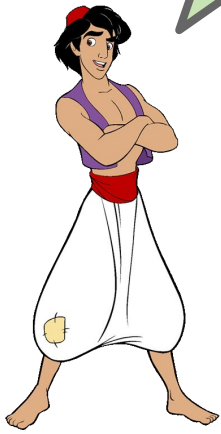
$y=f(\mathbf{v}) + \pi$





Storage Delegation

Query $f()$
on vector \mathbf{v}





More Properties



Updates



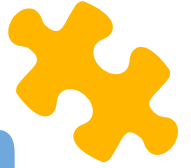
Aggregation



Updates



Aggregation



Updates



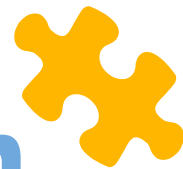
Aggregation



Updates



Aggregation



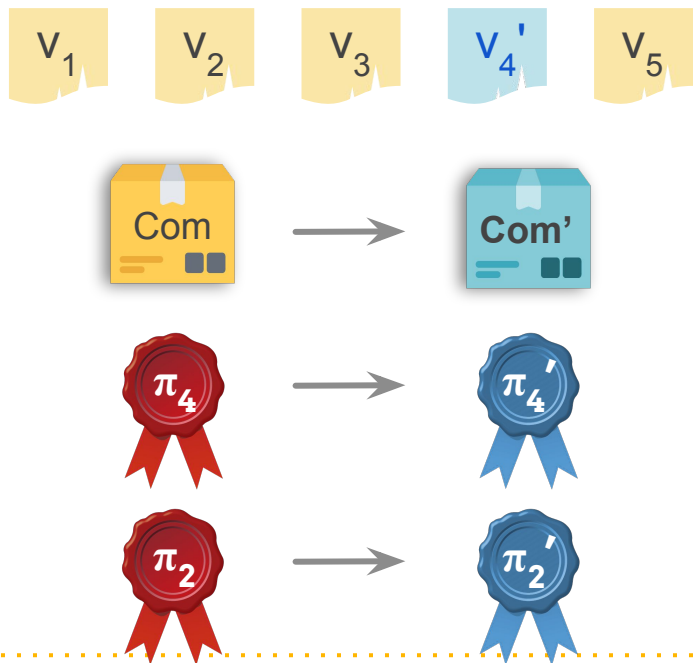
Updates



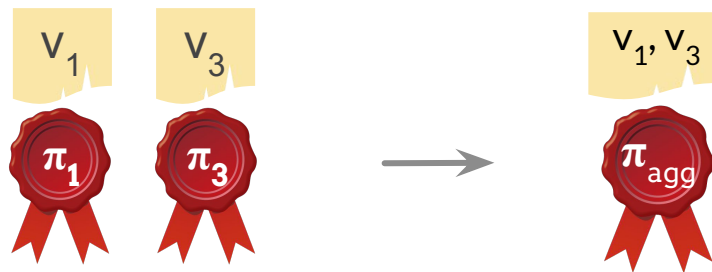
Aggregation



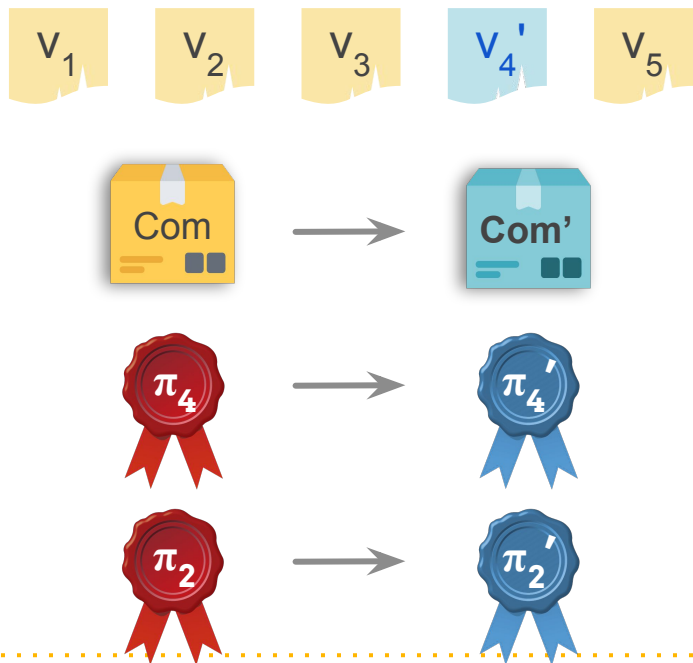
Updates



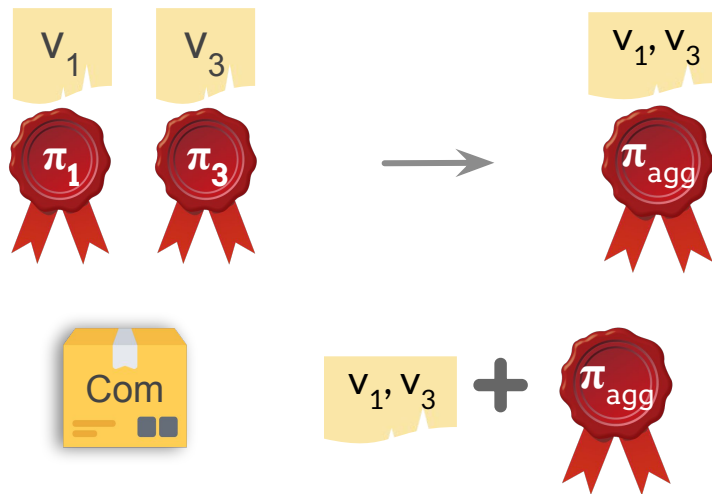
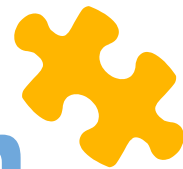
Aggregation



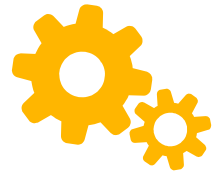
Updates



Aggregation

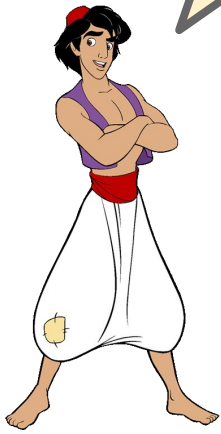


Does not need the entire vector v



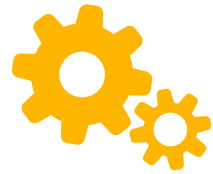
Updates for the vector

Update V_4'



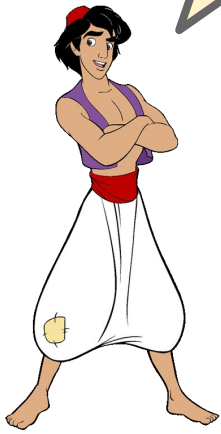
- V_1
- V_2
- V_3
- V_4
- V_5

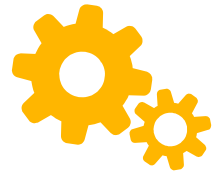




Updates for the vector

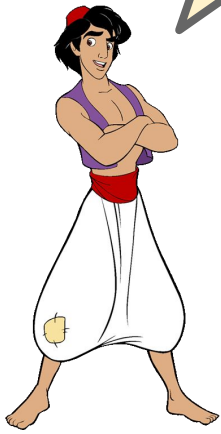
Update V_4'

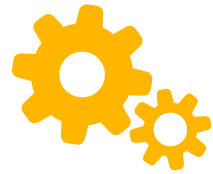




Updates for the vector

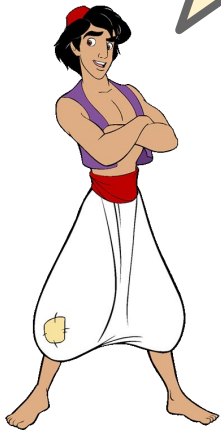
Update V_4'

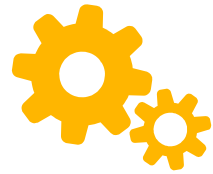




Updates for the vector

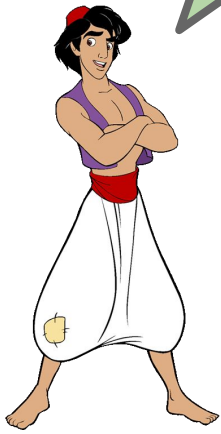
Update v_4'





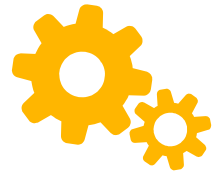
Proof Aggregation

Query 1 & 3



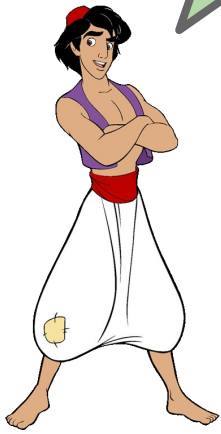
- V_1
- V_2
- V_3
- V_4
- V_5



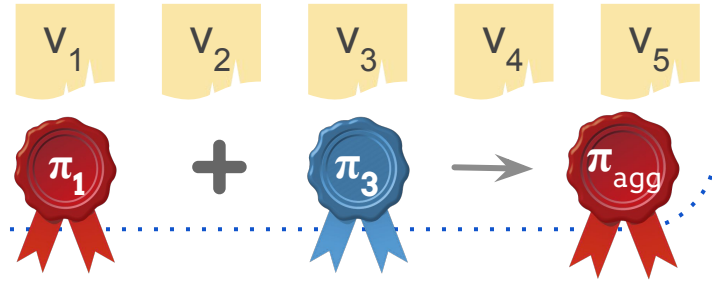


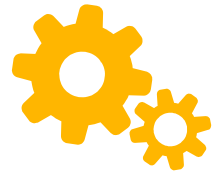
Proof Aggregation

Query 1 & 3



$$v_1, v_3 + \pi_{agg}$$





Drawbacks

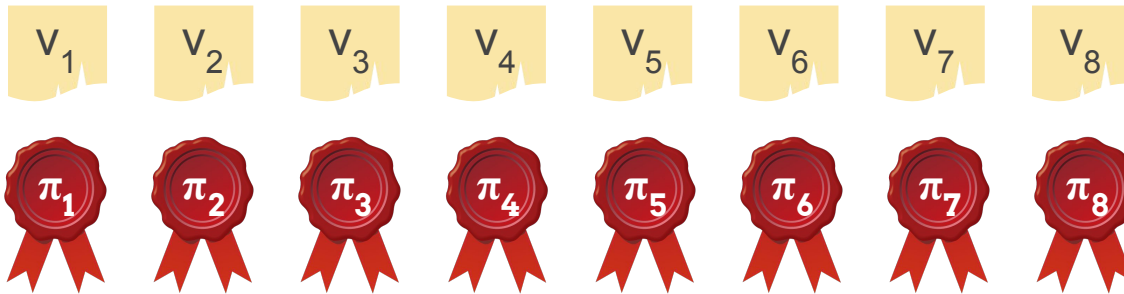


Properties:

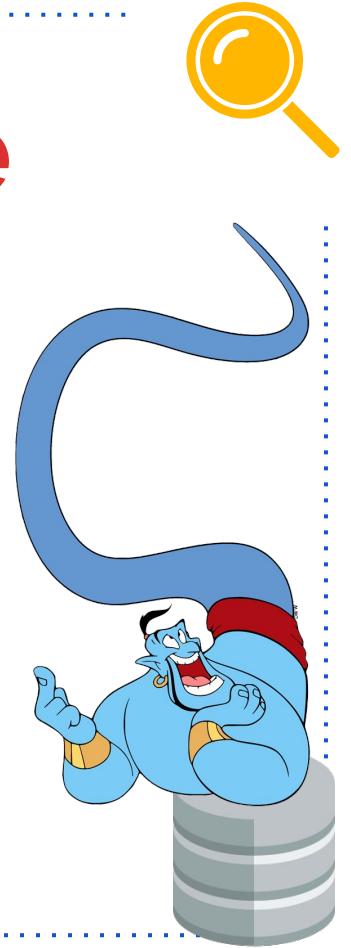
- Short proofs
- Fast verification
- Updates & Aggregation
- **Slow Prover**



Trade **Space** for **Time**

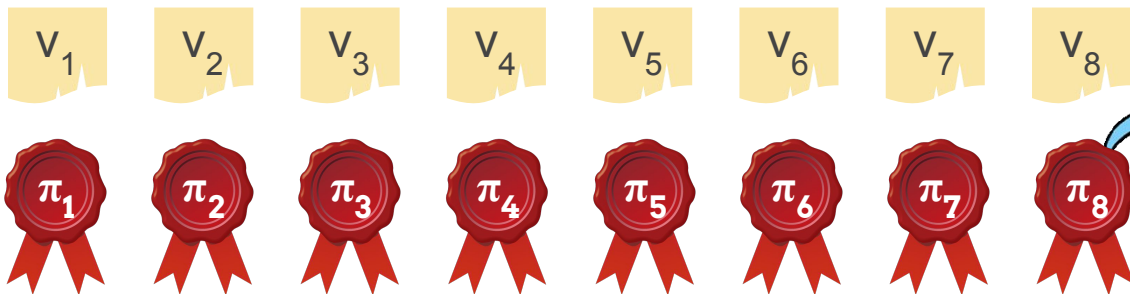


Offline: Compute all proofs





Trade Space

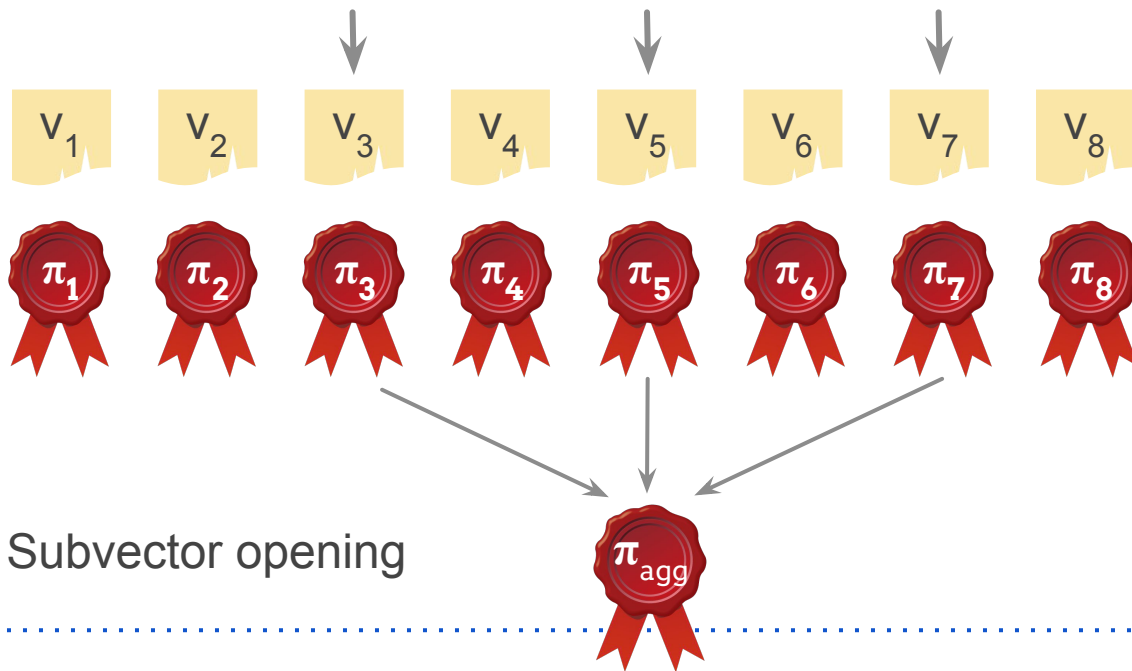


Offline: Compute all proofs

Online: Open by sending the stored proof

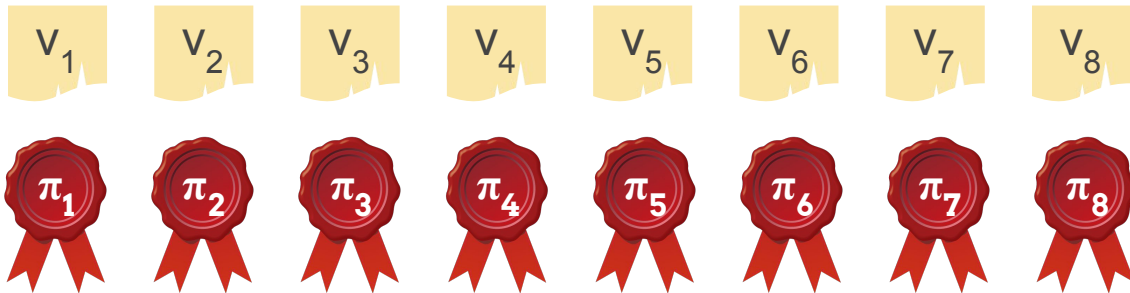


Trade Space

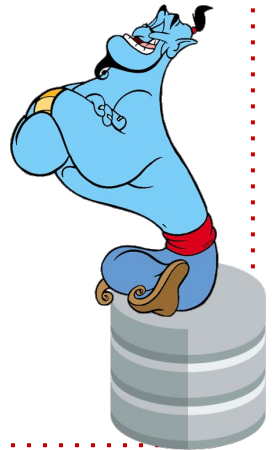




Space = expensive

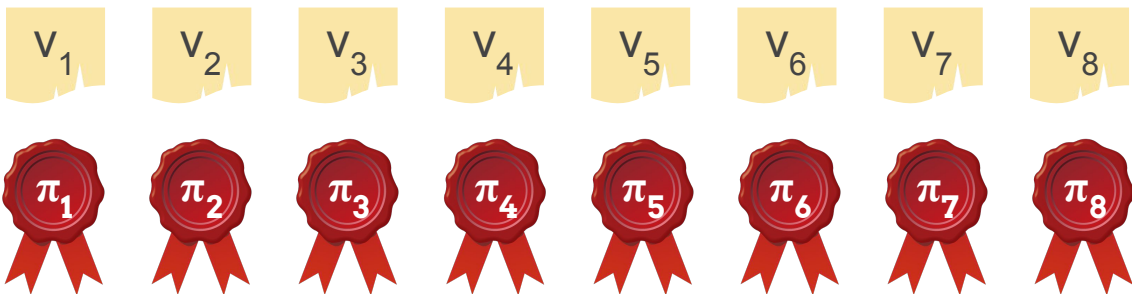


Computation: preprocessing with *small* amortized cost





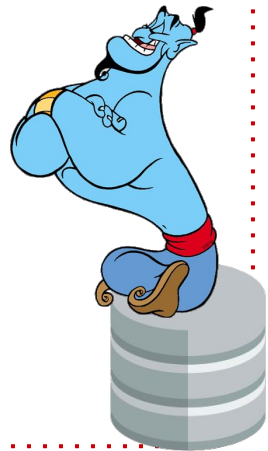
Space = expensive



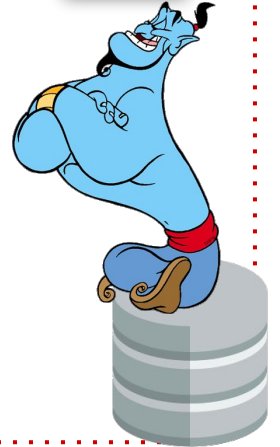
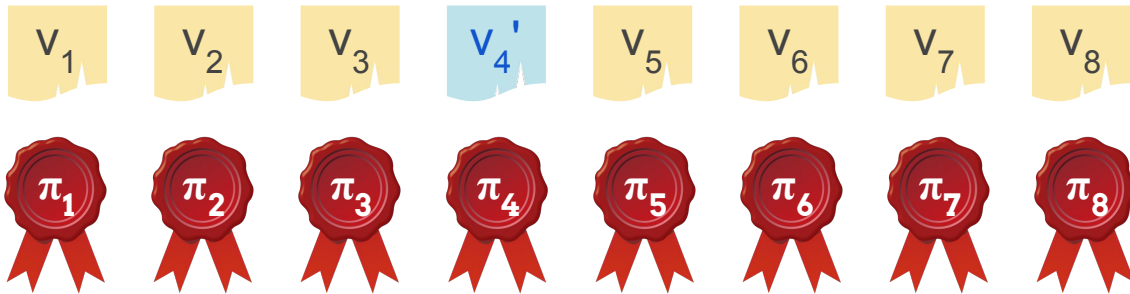
Computation: preprocessing with *small* amortized cost

Storage: doubles

Updates: recompute all



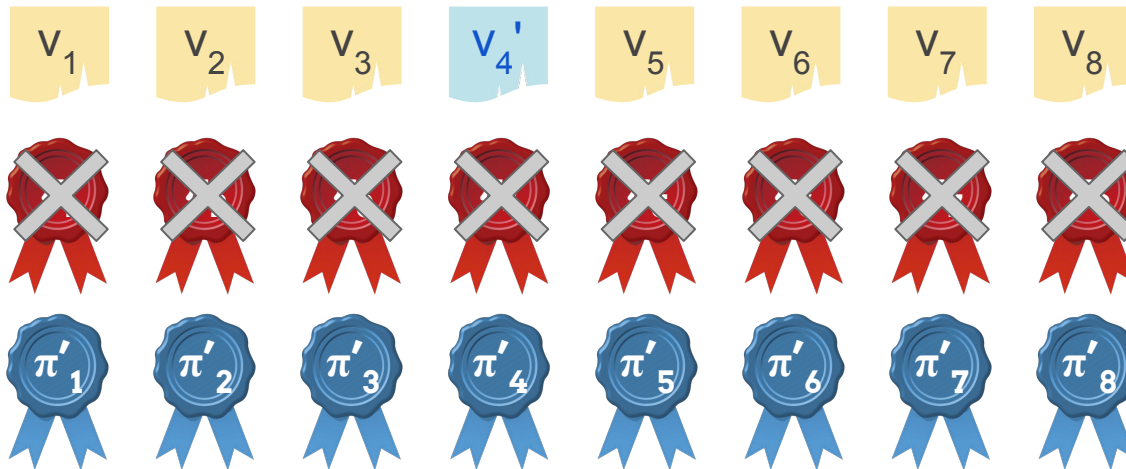
Updates = expensive



Updates = expensive



Updates = expensive





Maintainability



Update all: cost significantly smaller than recomputing all proofs





Space vs. Time



Space

- Sublinear, fast updates
- Ideally: flexibility - tune it



Time

- Sublinear(size of vector)
- Ideally: Sublinear(opening size)



Different solutions

Pairing-Based schemes

[KZG10], [CF13], [LM19],[GRWZ20], [TAB+20]

Advantages:

- constant-size openings
- constant-time verification
- **aggregatable**

Downsides:

- linear prover: no tradeoffs
- trusted setup
- linear parameters = bound on vector size
- updates: hard to maintain proofs



Tree-based schemes

[Merkle Trees, Hyperproofs, Verkle Trees]

Downsides:

- log-size openings
- log-time verification
- non-aggregatable

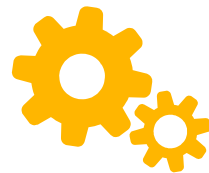
Advantages:

- linear prover: **tradeoffs**
- transparent setup
- independent parameters
- maintainability for proofs





Our Constructions



Contributions

Linear-Map Vector Commitments

- New simple constructions for Inner-Product Vector Commitments
- Unbounded Aggregation & Updatability with static keys
- Special Subset Openings and Subset opening with Efficient Verifier

Maintainable Constructions

- Maintainable Tree-based construction from any LVC
- Trade-offs for the prover with flexible Space and Time savings
- All this with reusable Setup “powers of tau” from SNARKs



LVC Algorithms

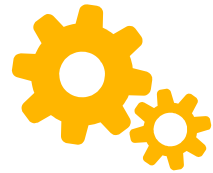


$\text{KeyGen}(\lambda, n) \rightarrow (\text{ck}, \text{vk})$

$\text{Commit}(\text{ck}, \mathbf{v}) \rightarrow (\text{C}, \text{aux})$

$\text{Open}(\text{ck}, \text{aux}, f, \mathbf{y}) \rightarrow \pi \quad \mathcal{R}: f(\mathbf{v}) = \mathbf{y}$

$\text{Verify}(\text{vk}, \text{C}, f, \mathbf{y}, \pi) \rightarrow 0/1$



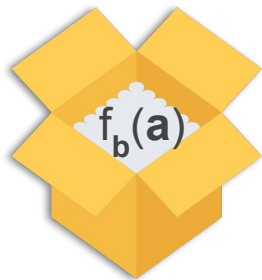
Inner-Product VC

Inner Product of a committed vector with a public vector



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$

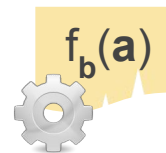
$$\mathbf{a}, \mathbf{b} \in \mathbb{F}^n$$



Open

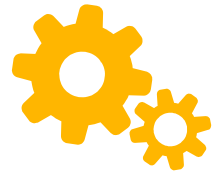


$$f : \mathbb{F}^n \rightarrow \mathbb{F}$$



+





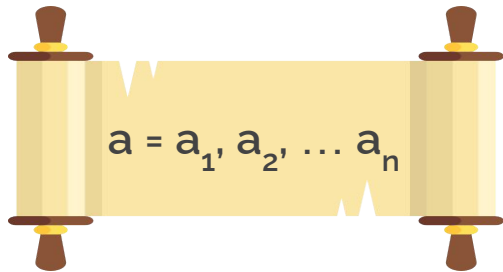
Inner-Product VC

Encode the committed vector \mathbf{a} as a polynomial



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$

$$\mathbf{a}, \mathbf{b} \in \mathbb{F}^n$$



$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$



Monomial basis

$$M_i(X) = X^i$$

$$a = a_1, a_2, \dots, a_n$$

$$A(X) = a_1 + a_2 X + a_3 X^2 + \dots + a_n X^{n-1}$$

Lagrange basis

$$L_i(X) = \prod_{j \neq i} \frac{X - j}{i - j}$$

$$a = a_1, a_2, \dots, a_n$$

$$A(X) = a_1 L_1(X) + a_2 L_2(X) + \dots + a_n L_n(X)$$

$$A(1) = a_1, A(2) = a_2, \dots, A(n) = a_n$$



Background



Bilinear Groups

$$[a]_1 \in \mathbf{G}_1, [b]_2 \in \mathbf{G}_2$$

$$e : \mathbf{G}_1 \times \mathbf{G}_2 \rightarrow \mathbf{G}_T$$

$$e([a]_1, [b]_2) = [ab]_T$$

Inner Product VC



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

Inner Product VC



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, \mathbf{a}) \rightarrow $C = [A(\tau)]_1$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$

$$A(X) B(X) =$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$

$$A(X) B(X) = \langle a, b \rangle X^{n-1}$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$

$$A(X) B(X) = \langle a, b \rangle X^{n-1} + H(X) X^n$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$

$$A(X) B(X) = \langle a, b \rangle X^{n-1} + H(X) X^n + R(X)$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$

$$B(X) = b_n + b_{n-1}X + b_{n-2}X^2 + \dots + b_1X^{n-1}$$

$$A(X) = a_1 + a_2X + a_3X^2 + \dots + a_nX^{n-1}$$

$$A(X) B(X) = \langle a, b \rangle X^{n-1} + H(X) X^n + R(X)$$

$$\check{R}(X) = XR(X)$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$



Verify(vk, C, b, y, π):

$$A(X) B(X) = y X^{n-1} + H(X) X^n + R(X)$$

$$\check{R}(X) = XR(X)$$



$$f_b(a) = \langle a, b \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, b, $y = \langle a, b \rangle$) \rightarrow π :

$$[H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$$



Verify(vk, C, b, y, π):

$$e([A(\tau)]_1, [B(\tau)]_2) = e([y \tau^{n-1}]_1, [1]_2) e([H(\tau)]_1, [\tau^n]_2) e([R(\tau)]_1, [1]_2)$$



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

vk: $[1]_2, [\tau]_2, \dots, [\tau^n]_2$



Commit(ck, \mathbf{a}) \rightarrow $C = [A(\tau)]_1$



Open(ck, $\mathbf{a}, \mathbf{b}, y = \langle \mathbf{a}, \mathbf{b} \rangle$) \rightarrow $\pi: [H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$



Verify(vk, C, \mathbf{b}, y, π): $e([A(\tau)]_1, [B(\tau)]_2) = e([y \tau^{n-1}]_1, [1]_2) e([H(\tau)]_1, [\tau^n]_2) e([R(\tau)]_1, [1]_2)$



$$f_b(\mathbf{a}) = \langle \mathbf{a}, \mathbf{b} \rangle$$

Inner Product VC



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

vk: $[1]_2, [\tau]_2, \dots, [\tau^n]_2$



Commit(ck, \mathbf{a}) \rightarrow $C = [A(\tau)]_1$



Open(ck, $\mathbf{a}, \mathbf{b}, y = \langle \mathbf{a}, \mathbf{b} \rangle$) \rightarrow $\pi: [H(\tau)]_1, [R(\tau)]_1, [\check{R}(\tau)]_1$



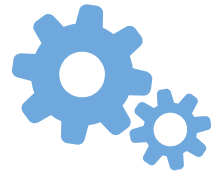
Verify(vk, C, \mathbf{b}, y, π):

$$e([\check{R}(\tau)]_1, [1]_2) = e([R(\tau)]_1, [\tau]_2)$$

$$\check{R}(X) = XR(X)$$

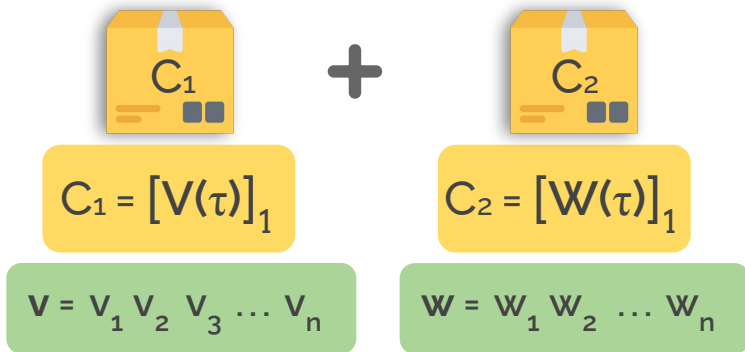


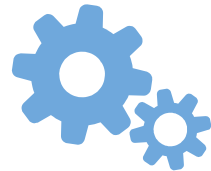
Properties of LVC



Homomorphism

Commitments

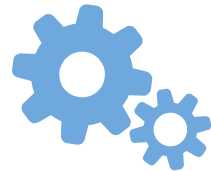




Homomorphism

Commitments

$$\begin{array}{ccc} \begin{array}{c} \text{C}_1 \\ \text{C}_1 = [V(\tau)]_1 \\ v = v_1 v_2 v_3 \dots v_n \end{array} & + & \begin{array}{c} \text{C}_2 \\ \text{C}_2 = [W(\tau)]_1 \\ w = w_1 w_2 \dots w_n \end{array} = \begin{array}{c} \text{C}_3 \\ \text{C}_3 = [(V+W)(\tau)]_1 \\ v + w = v_1 + w_1 \quad v_2 + w_2 \dots v_n + w_n \end{array} \end{array}$$



Homomorphism

Openings



$y_1 = \langle a, b_1 \rangle$

+

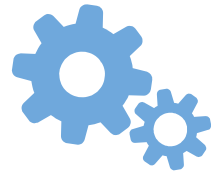


$y_2 = \langle a, b_2 \rangle$

=



$y_3 = \langle a, b_1 + b_2 \rangle$



Homomorphism

Openings

$$C_a \quad \pi_1 \quad + \quad \pi_2 \quad = \quad \pi_3$$
$$y_1 = \langle a, b_1 \rangle \quad y_2 = \langle a, b_2 \rangle \quad y_3 = \langle a, b_1 + b_2 \rangle$$

Proofs

$$C_{a_1} \quad \pi_1 \quad + \quad C_{a_2} \quad \pi_2 \quad = \quad C_{a_1 + a_2} \quad \pi_3$$
$$z_1 = \langle a_1, b \rangle \quad z_2 = \langle a_2, b \rangle \quad z_3 = \langle a_1 + a_2, b \rangle$$

Updatability for Com



$$a' = a_1 a_2 \dots a_i + \delta \dots a_n$$

$$\text{upk: } [1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

$$C' = [A(\tau)]_1 + [\delta \tau^i]_1$$



Updatability for Proof

$$\mathbf{a}' = a_1 a_2 \dots a_i + \delta \dots a_n$$

$$\text{upk: } [1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

$$C' = [A(\tau)]_1 + [\delta \tau^i]_1$$



=



+



$$y' = \langle \mathbf{a} + \delta \mathbf{e}_i, \mathbf{b} \rangle = y_1 = \langle \mathbf{a}, \mathbf{b} \rangle + \delta \langle \mathbf{e}_i, \mathbf{b} \rangle$$



Updatability for Proof

$$a' = a_1 a_2 \dots a_i + \delta \dots a_n$$

$$\text{upk: } [1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

$$C' = [A(\tau)]_1 + [\delta \tau^i]_1$$

$$A(X) B(X) = \langle a, b \rangle X^{n-1} + H(X) X^n + R(X)$$

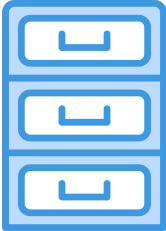
$$\begin{array}{c} \text{Red ribbon seal with } \pi' \\ \text{Red ribbon seal with } \pi \\ \text{Red ribbon seal with } \delta \pi_i \end{array} = \begin{array}{c} \text{Red ribbon seal with } \pi \\ \text{Red ribbon seal with } \delta \pi_i \end{array} + \begin{array}{c} \text{Red ribbon seal with } \delta \pi_i \end{array}$$
$$y' = \langle a + \delta e_i, b \rangle = y_1 = \langle a, b \rangle + \delta \langle e_i, b \rangle$$

$$\pi_{ij} : \langle e_i, e_j \rangle = 0$$

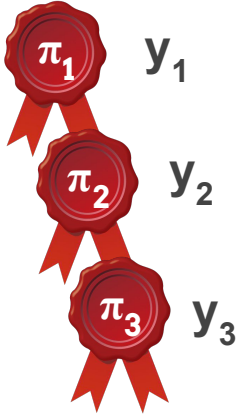
$$H(X)/R(X) = X^{i+n-j}$$

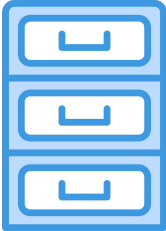
$$\pi_{ii} : \langle e_i, e_i \rangle = 1$$

$$H(X)/R(X) = 0$$

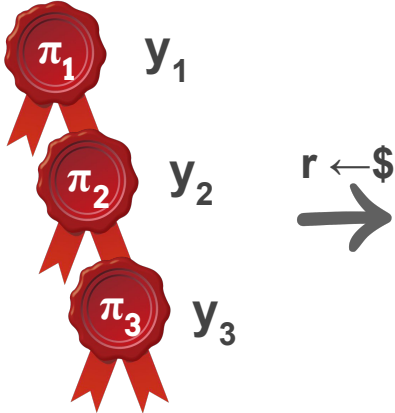


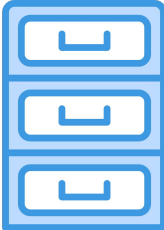
Aggregation



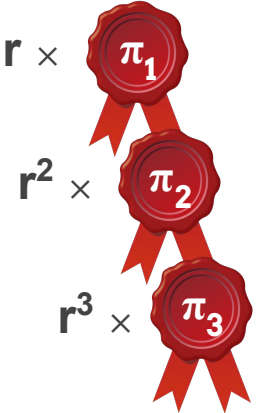


Aggregation

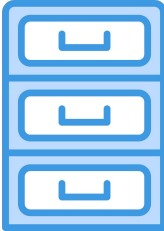




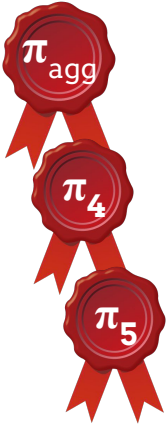
Aggregation

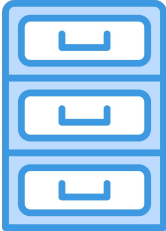


$$\pi_{agg} = \sum r^i \pi_i$$

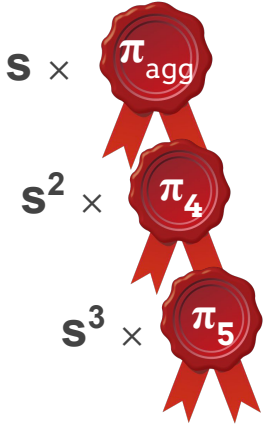


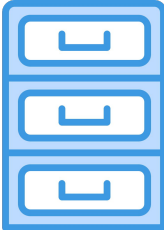
Aggregation



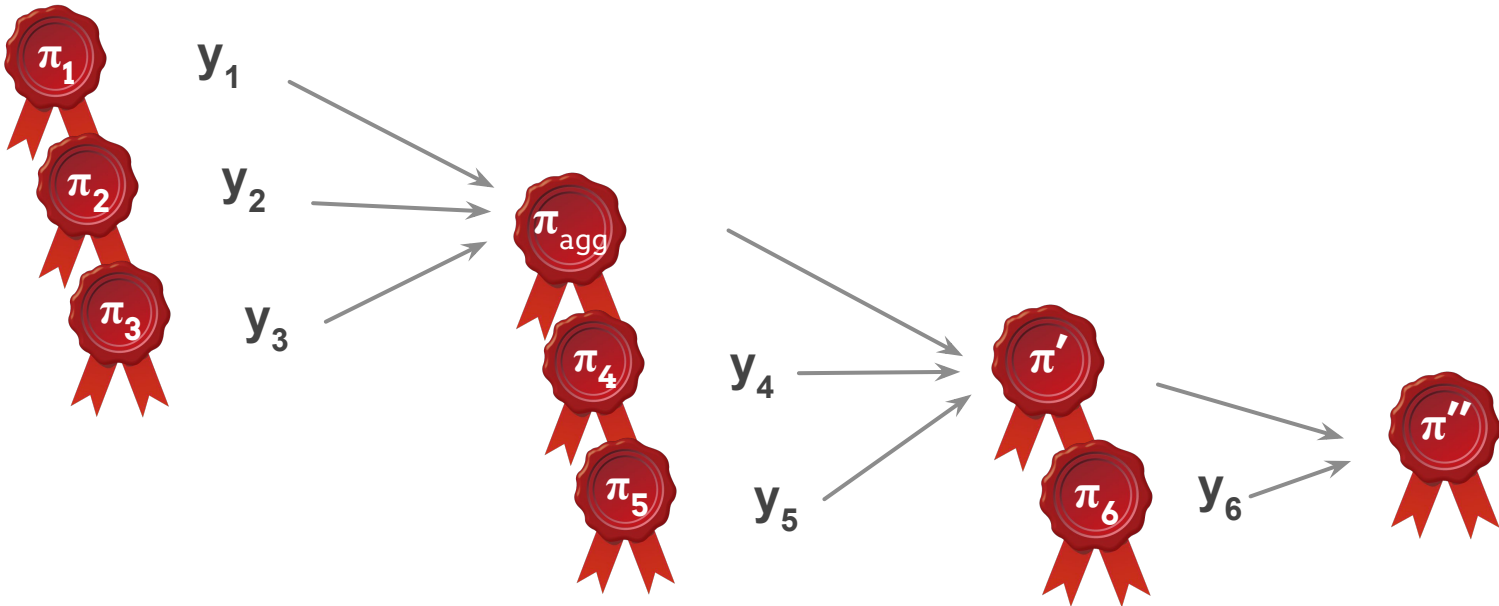


Aggregation





Aggregation



Linear-Map VC from IP



$$f : \mathbb{F}^n \rightarrow \mathbb{F}^\ell$$



KeyGen(λ, n) \rightarrow ck:

$$[1]_1, [\tau]_1, [\tau^2]_1, \dots, [\tau^{n-1}]_1$$

vk: $[1]_2, [\tau]_2, \dots, [\tau^n]_2$



Commit(ck, a) \rightarrow C = $[A(\tau)]_1$



Open(ck, a, f: $\mathbf{B}=(\mathbf{b}_i), \mathbf{y}$) \rightarrow $\pi_1, \pi_2, \dots, \pi_\ell \rightarrow \pi_{\text{agg}}$



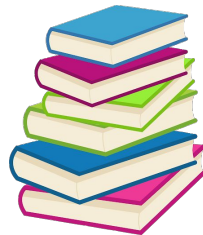
Verify(vk, C, f, $\mathbf{y}, \pi_{\text{agg}}$)

LVC comparison

$$f : \mathbb{F}^n \rightarrow \mathbb{F}^\ell$$



Schemes	ck	vk	upk	Updates	Aggregation	SVC	LVC
[LRY16]	n	n	—	×	×	×	✓
[LM19] SVC	n^2	n	n	key	×	✓	×
LVC	$n\ell$	n	n	key	×	✓	✓
Pointproofs	n	n	n	key	cross & one-hop	✓	×
[CFG+20]	1	1	1	hint	same & incremental	✓	×
Our Monomial	n	n	n	key	cross & unbounded	✓	✓
Our Lagrange	n	ℓ	n	key	cross & unbounded	✓	✓



References

[CF13] **Vector Commitments and their Applications**, by D. Catalano, D. Fiore, in PKC'13, <https://eprint.iacr.org/2011/495>

[CFG+20] **Vector Commitment Techniques and Applications to Verifiable Decentralized Storage**, by M. Campanelli, D. Fiore, N. Greco, D. Kolonelos, L. Nizzardo, in ASIACRYPT'20, <https://eprint.iacr.org/2020/149>

[GRWZ20] **Pointproofs: Aggregating Proofs for Multiple Vector Commitments**, by S. Gorbunov, L. Reyzin, H. Wee, Z. Zhang, in CCS'20, <https://eprint.iacr.org/2020/419>

[KZG10] **Constant-Size Commitments to Polynomials and Their Applications**, by A. Kate, G. Zaverucha, I. Goldberg, in ASIACRYPT'10,

[LRY10] **Functional commitment schemes: From polynomial commitments to pairing-based accumulators from simple assumptions**, by B. Libert, S. Ramanna and M. Yung, in ICALP'16

[LM19] **Subvector Commitments with Application to Succinct Arguments**, by R. W. F. Lai, G. Malavolta, in CRYPTO'19, <https://eprint.iacr.org/2018/705>



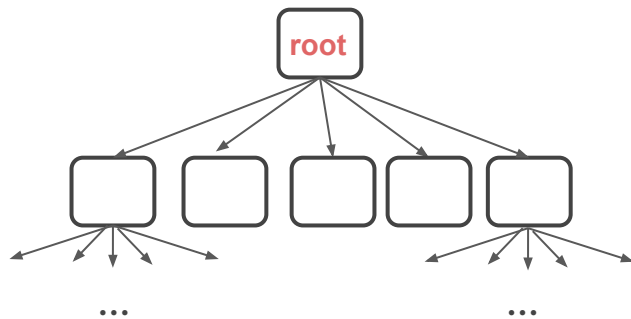
Maintainable Trees



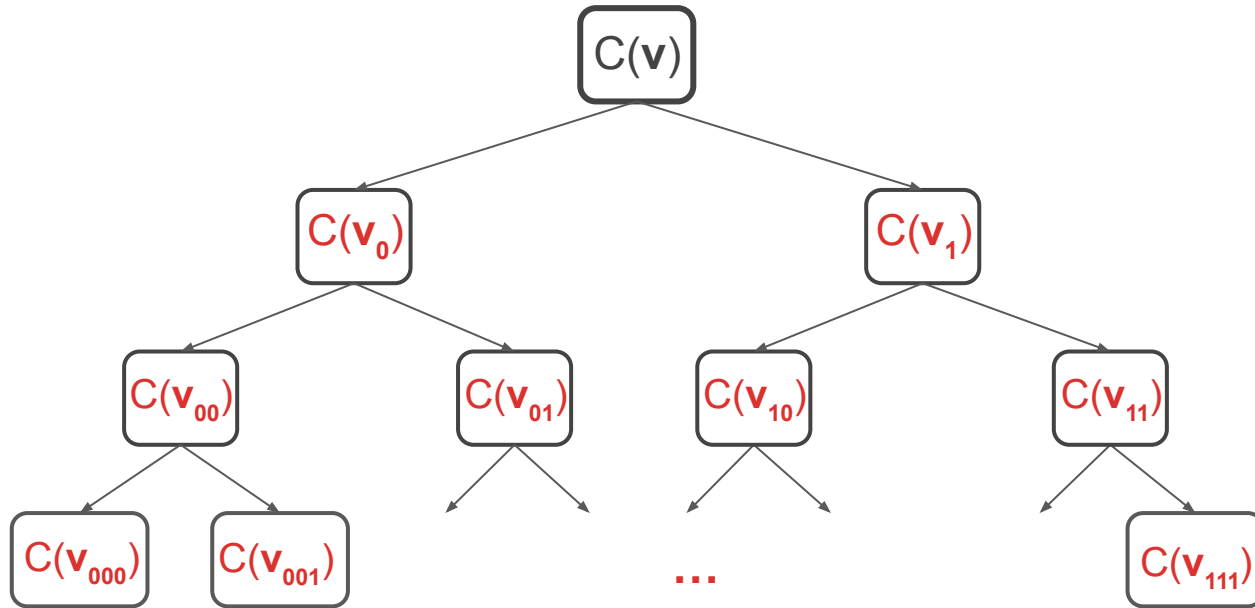
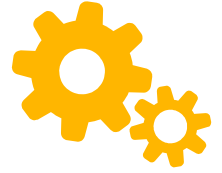
Tree-based Techniques

Ideas for tree-based VC:

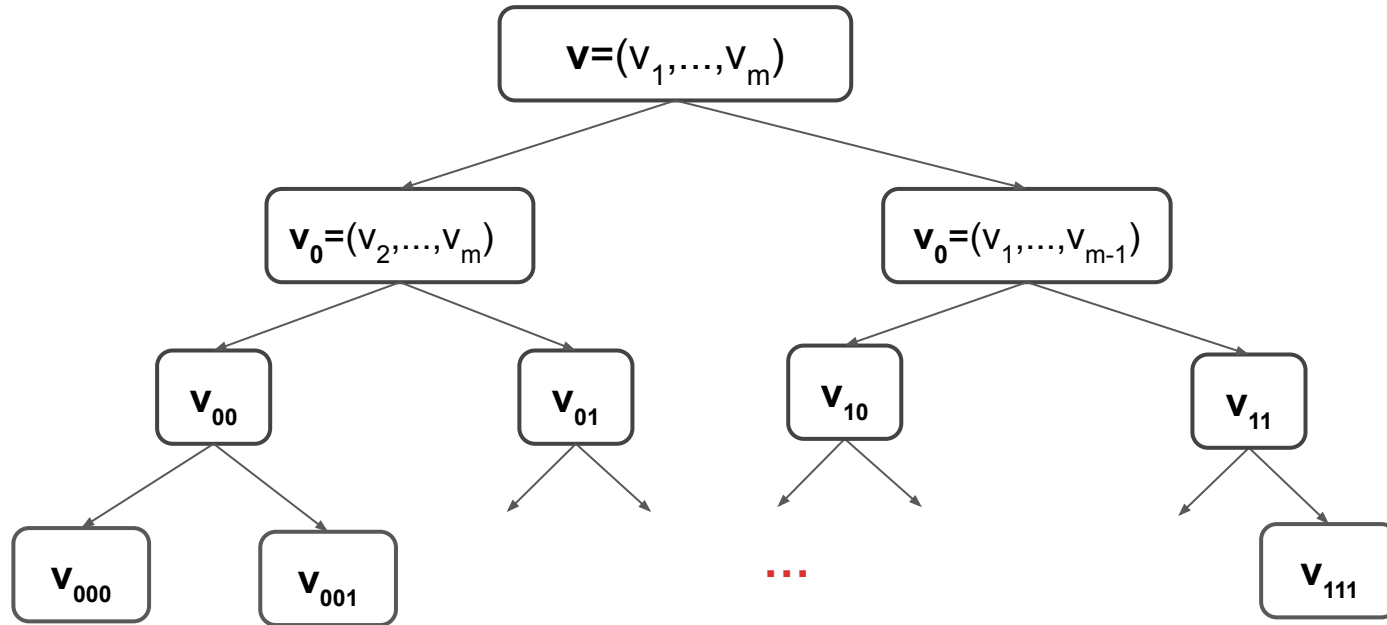
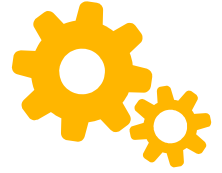
- maintainability for proofs
- generic for any homomorphic VC scheme
- any arity k vs. binary
- tune the tradeoff
- from multivariate polynomial to univariate
- **practical**: reusable trusted setup (Groth16)



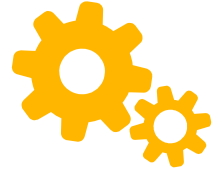
VC Tree



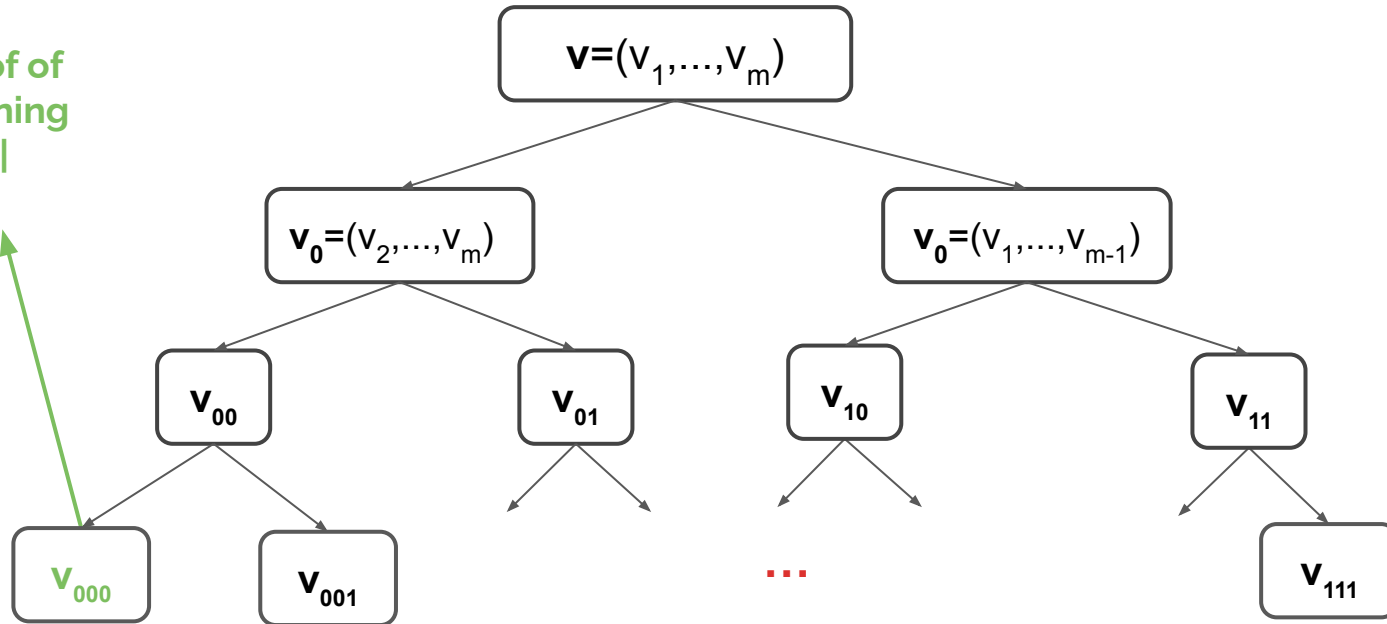
VC Tree



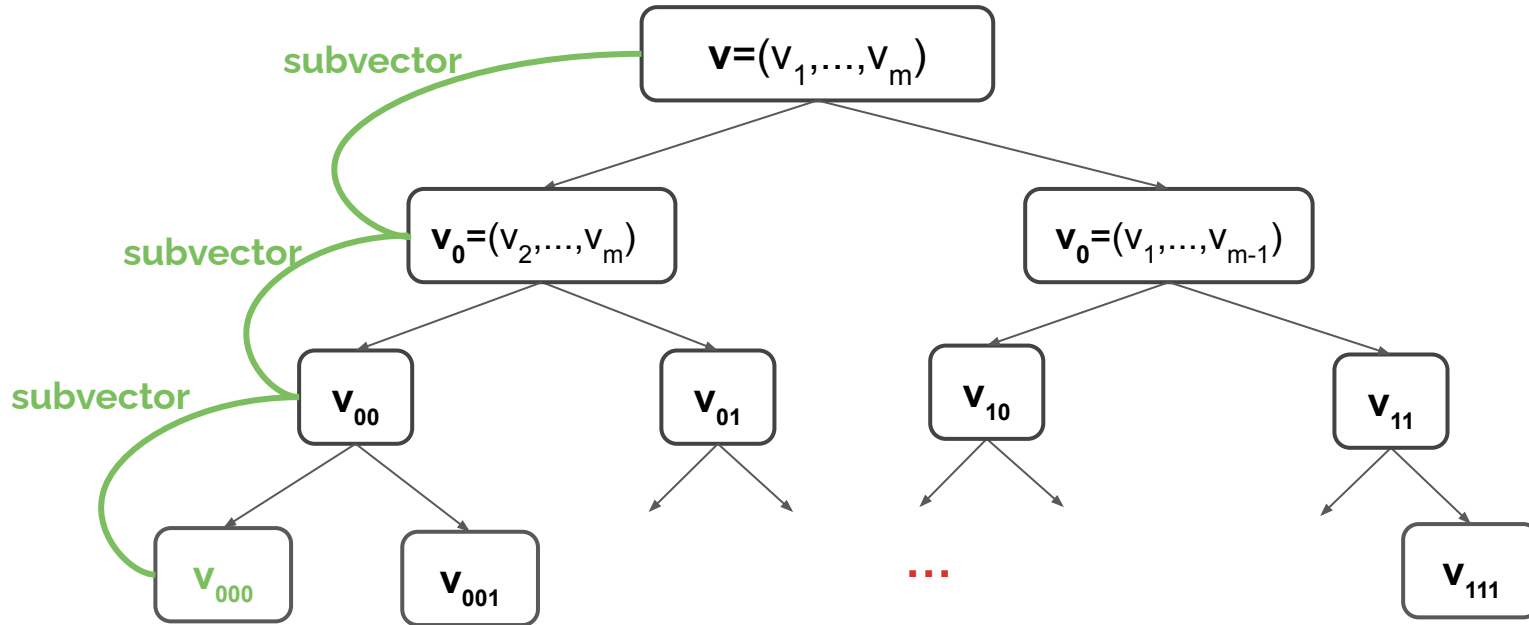
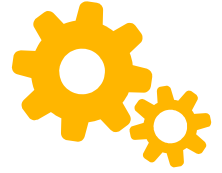
VC Tree - opening proof



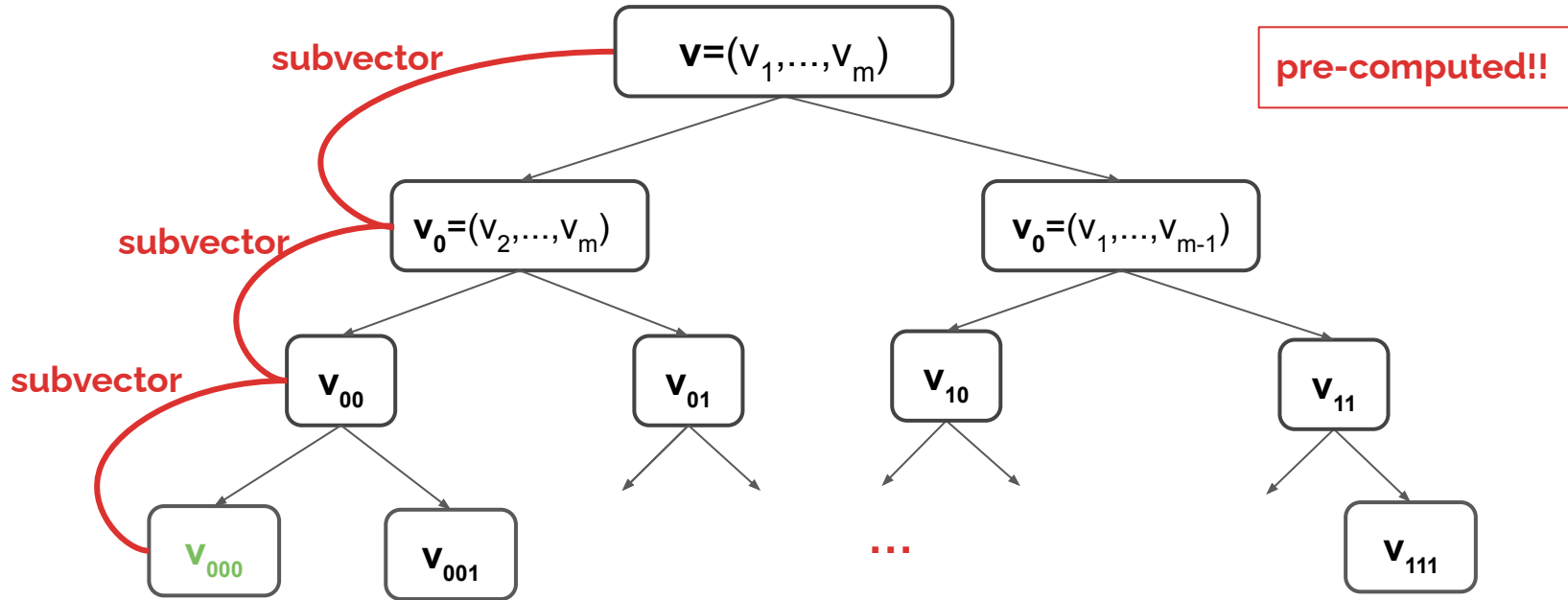
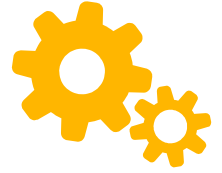
Proof of opening
 $|v_{000}|$



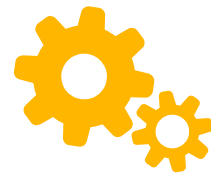
VC Tree - opening proof



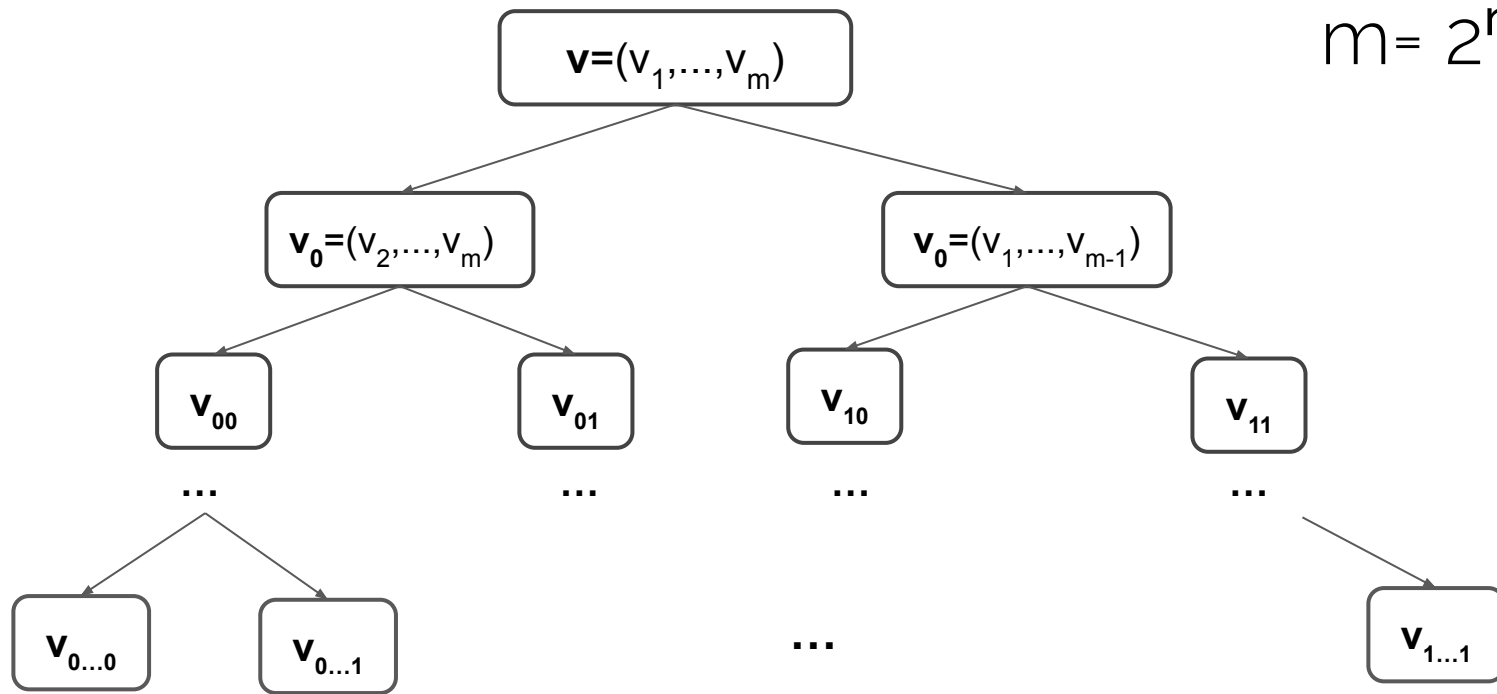
VC Tree - opening proof



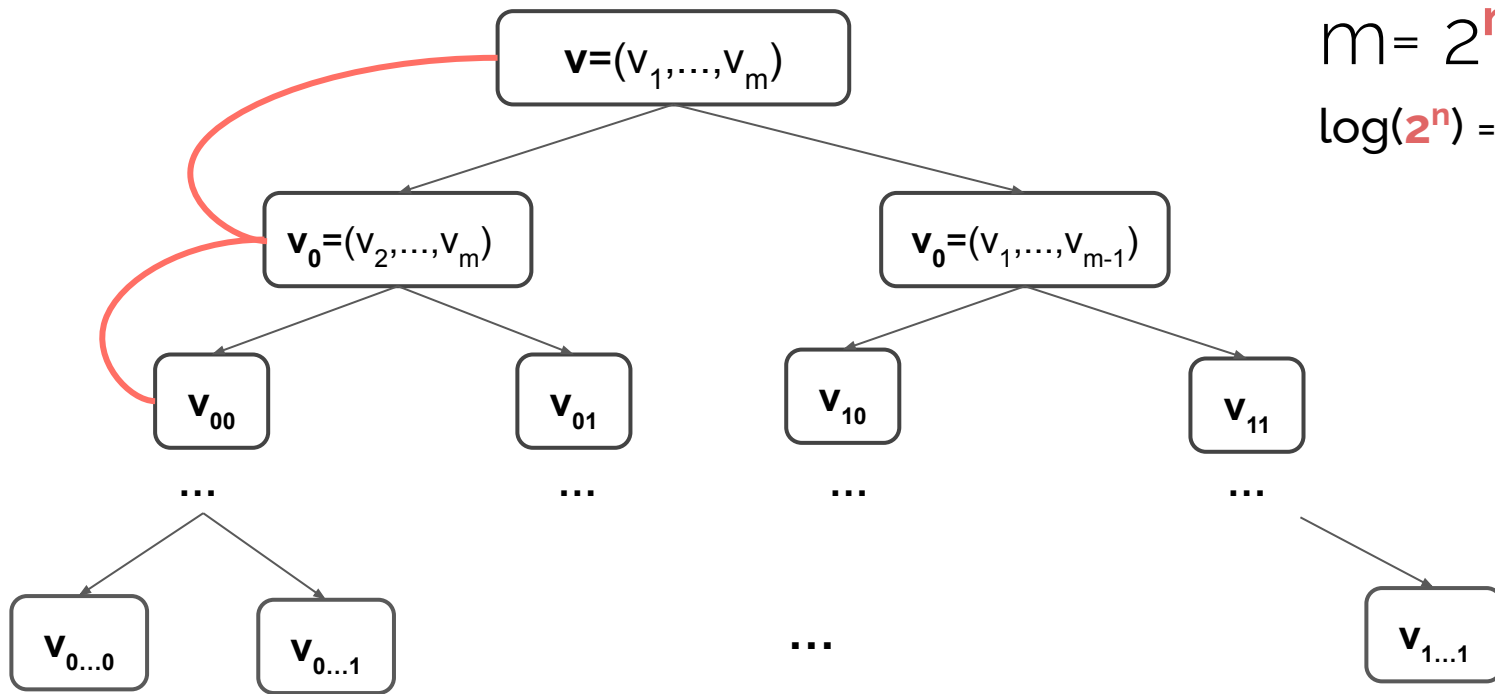
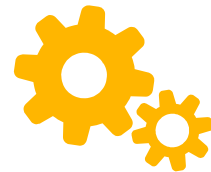
VC Tree



$$m = 2^{n+k}$$



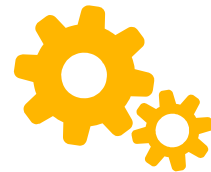
VC Tree



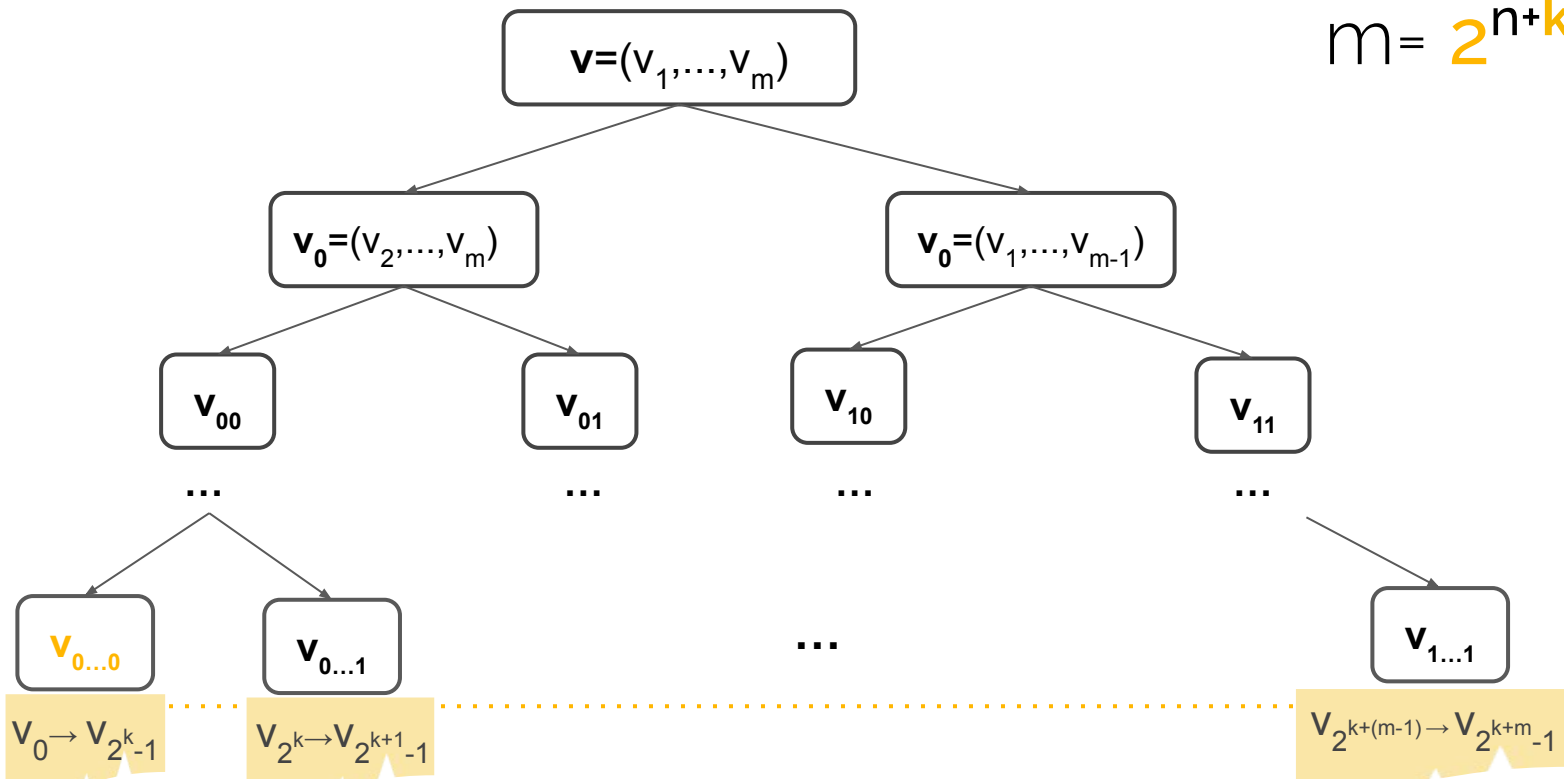
$$m = 2^{n+k}$$

$$\log(2^n) = n$$

VC Tree

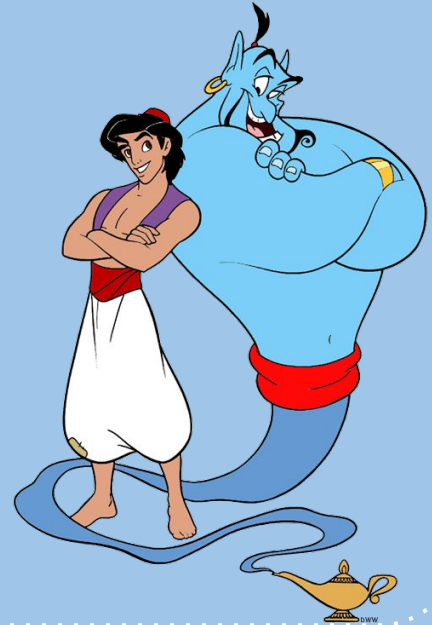


$$m = 2^{n+k}$$



Stateless Cryptocurrency

application



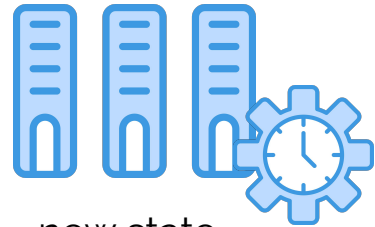


Distributed Ledger

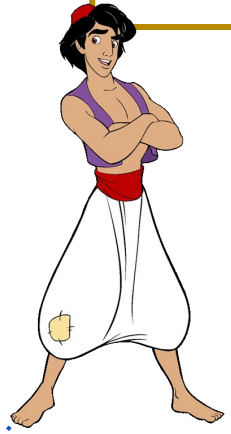
New Block
A → B, 5 Coins
C → D, 10 Coins
A → C, 5 Coins




previous state




new state



A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...



A: 10 Coins
B: 60 Coins
C: 25 Coins
D: 10 Coins
E: 90 Coins
F: 10 Coins
...



Grows over time
High storage & download cost





Distributed Ledger

New Block

A → B, 5 Co
C → D, 10 c
A → C, 5 Co



Solution:



VC

Grows over time
High storage & download cost



1. 10 Coins
...



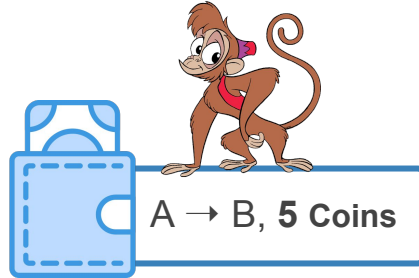
1. 10 Coins
...



Transaction validation



current state



A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins

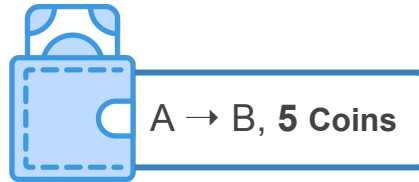
...



Transaction validation



current state



Validate
Transaction

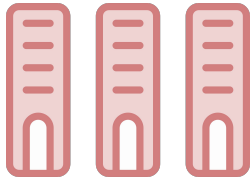
A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...



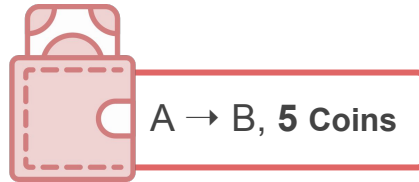
Check that
A has > 5 Coins



Stateful validation



current state



Validate
Transaction

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...



Stateless validation



State Commitment
(vector of balances)



A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins

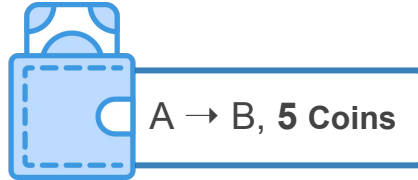
...

Stateless validation

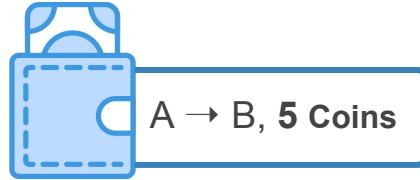


State Commitment
(vector of balances)

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...



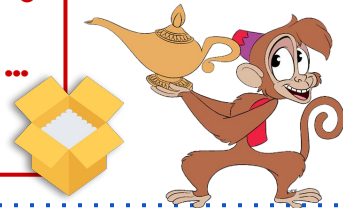
Stateless validation



State Commitment
(vector of balances)

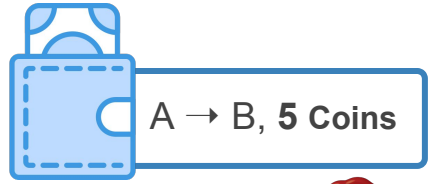
A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π_A
 π_B
 π_C
...





Stateless validation



State Commitment
(vector of balances)

A: 20

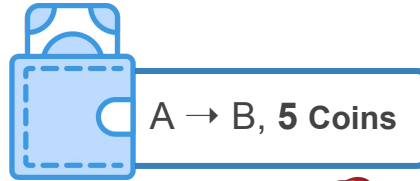


- A: 20 Coins
- B: 55 Coins
- C: 30 Coins
- D: 0 Coins
- E: 90 Coins
- F: 10 Coins
- ...

- π_A
- π_B
- π_C
- ...



Stateless validation

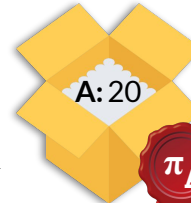


State Commitment
(vector of balances)

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π_A
 π_B
 π_C
...

A: 20



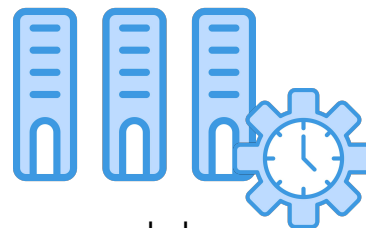
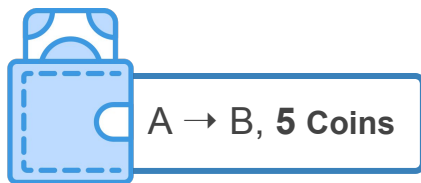
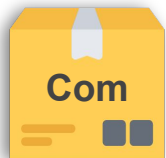
Check that

π_A is a valid opening
for Com





More Requirements



State Commitment
(vector of balances)

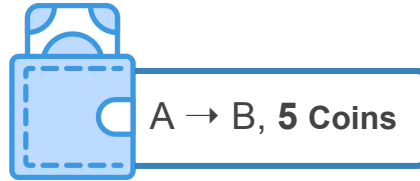
new state

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π_A
 π_B
 π_C
...

A: 15 Coins
B: 60 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

Updatability for Com



State Commitment
(vector of balances)

Update

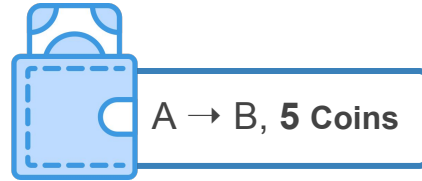
A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π_A
 π_B
 π_C
...



A: 15 Coins
B: 60 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

Updatability for Proofs



State Commitment
(vector of balances)

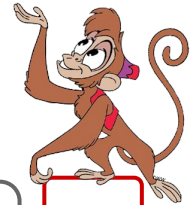
Update all

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

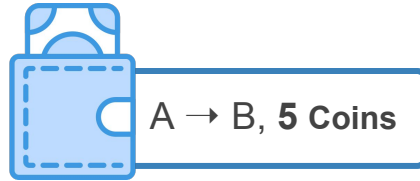
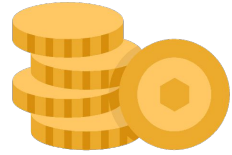
π_A
 π_B
 π_C
...

A: 15 Coins
B: 60 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π'_A
 π'_B
 π'_C
...



Updatability for Proofs



State Commitment
(vector of balances)

A: 20 Coins
B: 55 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

π_A
 π_B
 π_C
...

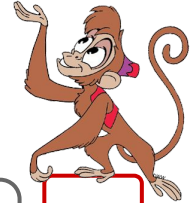
Updates types

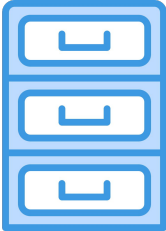
- hint: needs opening
- key: needs static key
- + maintainability

Update all

A: 15 Coins
B: 60 Coins
C: 30 Coins
D: 0 Coins
E: 90 Coins
F: 10 Coins
...

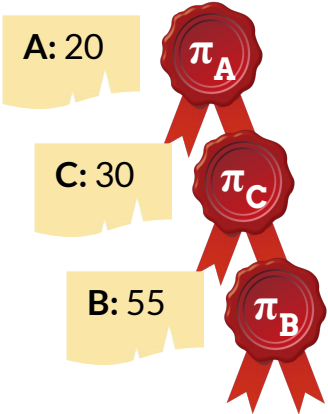
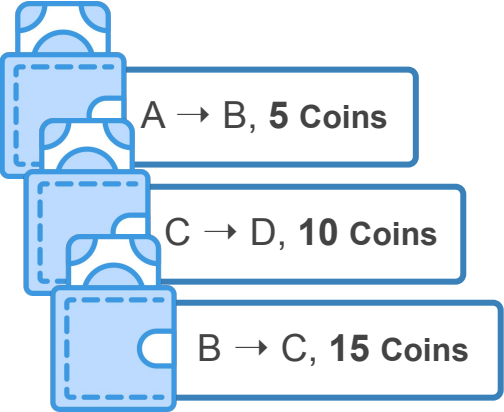
π'_A
 π'_B
 π'_C
...

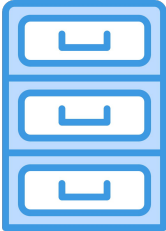




Aggregation

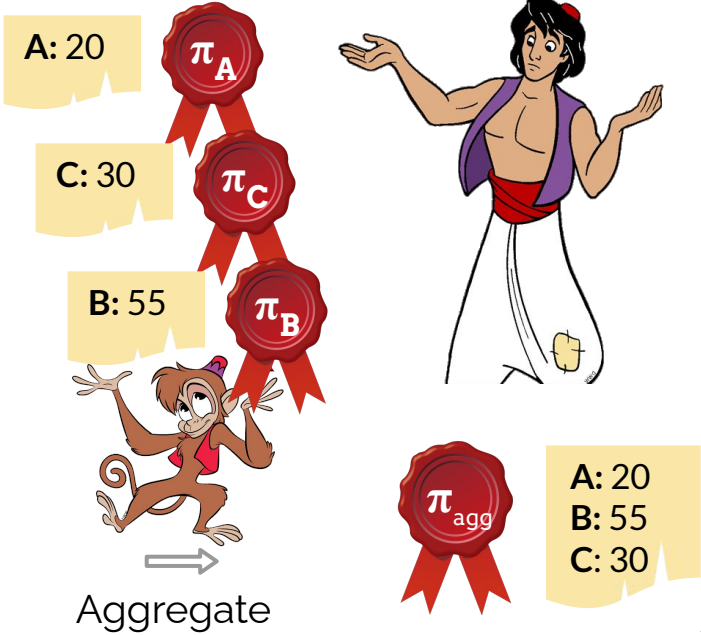
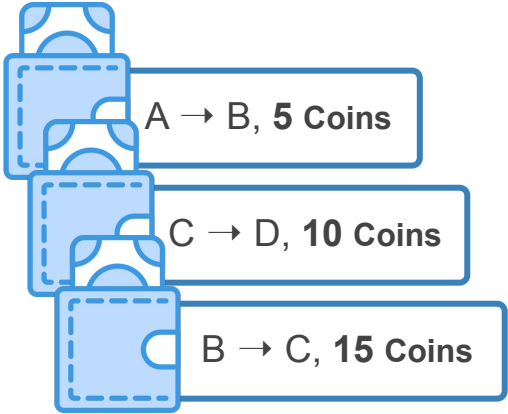
New Block
A → B, 5 Coins
C → D, 10 Coins
B → C, 15 Coins

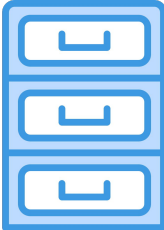




Aggregation

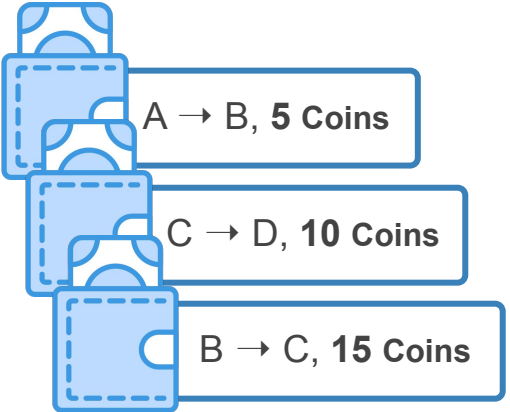
New Block
A → B, 5 Coins
C → D, 10 Coins
B → C, 15 Coins



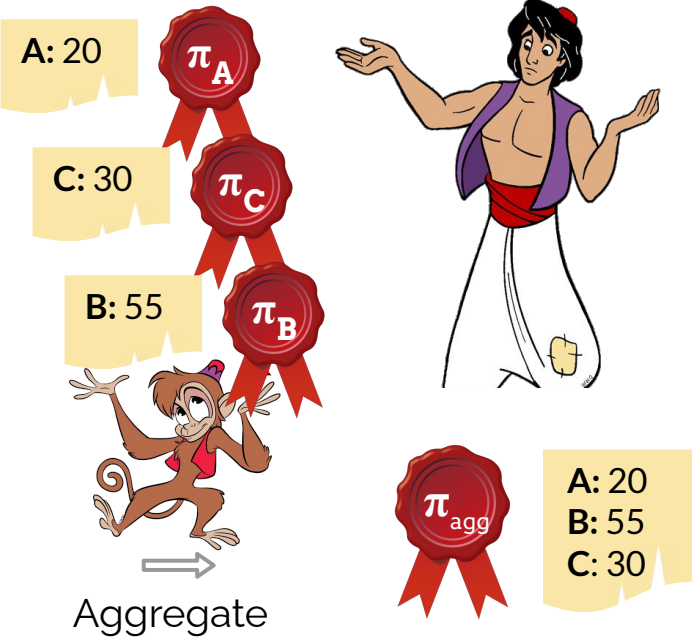


Aggregation

New Block
A → B, 5 Coins
C → D, 10 Coins
B → C, 15 Coins



- same-commitment/**cross-commitment**
- one-hop/**unbounded**
- **incremental**: can disaggregate





Future Directions



What's next?

- **Transparent Setup Vector Commitments**
- **Functional Vector Commitments for larger classes of functions**
- **Post-Quantum Vector Commitments with same properties**
 - **Code-based or lattice-based?**
- **More efficient Aggregation for Tree-based VC (relying on IPA)**

Thanks!

ia.cr/2022/705





Credits

Special thanks to all those who made and released these resources for free:

- Presentation template by [SlidesCarnival](#)
- Clip arts by [Iconfinder](#)
- Illustrations by [Disneyclips](#)