An isogeny-based adaptor signature using SQISign

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University of Waterloo

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Payment Channel Networks

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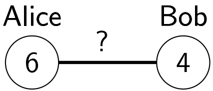

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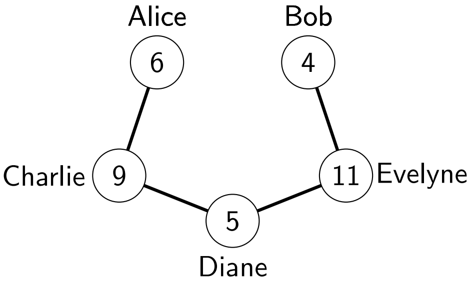

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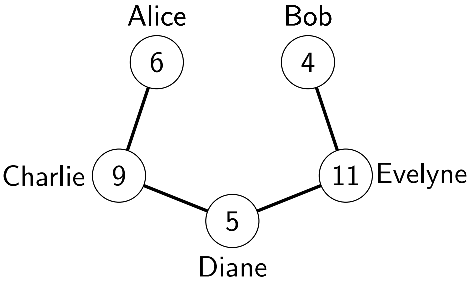

How can Alice be assured her money will arrive to Bob?

Anonymous Multi-Hop Locks (AMHL)

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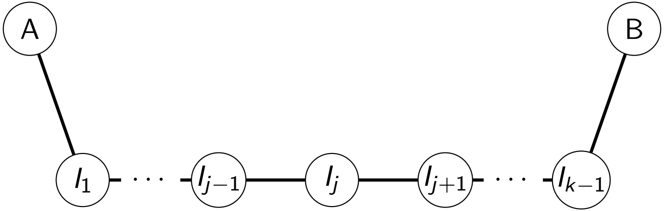

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$$f : \mathcal{L}_{witness} \rightarrow \mathcal{L}_{statement}$$

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She will then compute the following for each $j \in [1, \cdots k-1]$:

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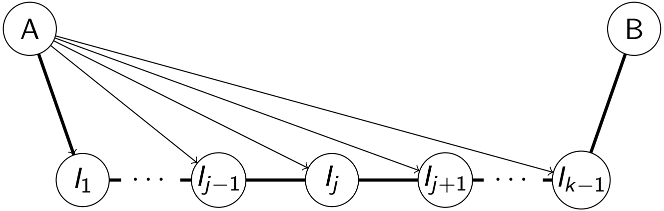

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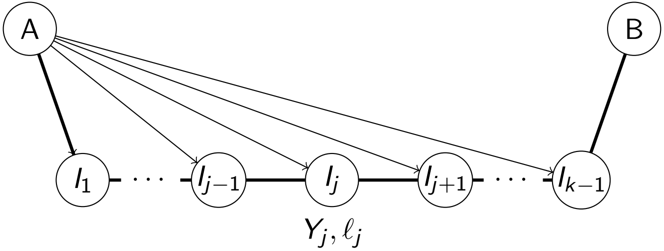

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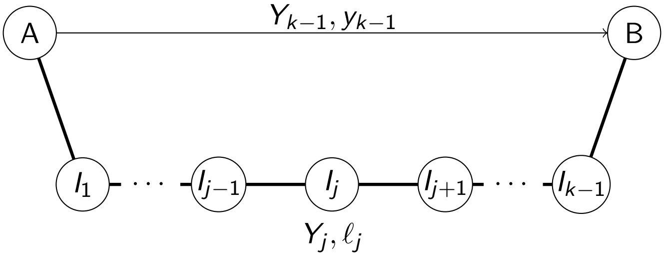

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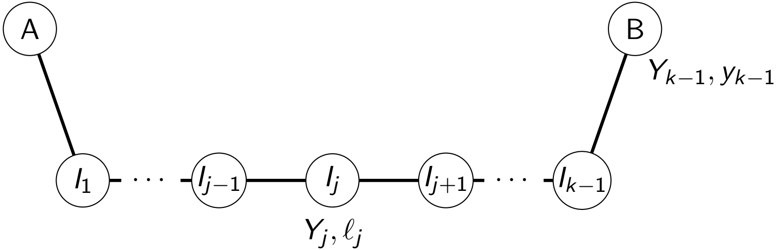

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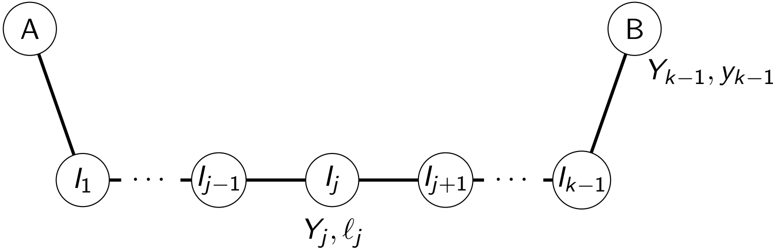

Intermediary I_j will sign a contract agreeing to release funds to I_{j+1} on the condition that I_{j+1} can provide y_j.

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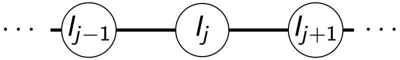

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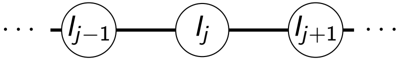

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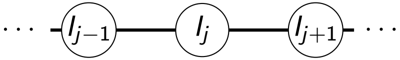

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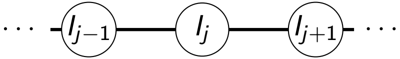

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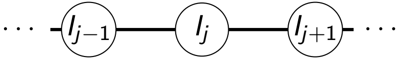

$$y_j \leftarrow I_{j+1}$$

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$$I_j \leftarrow y_{j-1}$$

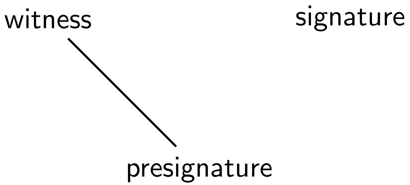
...how can we make this post-quantum?

Adaptor Signatures

witness signature

presignature

Adaptor Signatures



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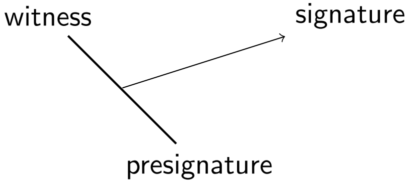

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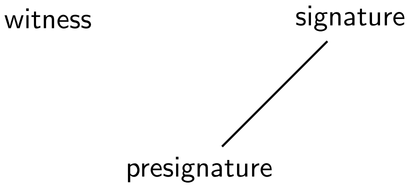

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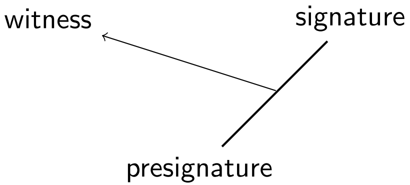

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Then an adaptor signature scheme with respect to R and Σ consists of four algorithms:

$$\mathsf{PreSig}(\mathsf{sk}, m, Y) \rightarrow \widetilde{\sigma}$$
$$\mathsf{PreVer}(\mathsf{pk}, m, Y, \widetilde{\sigma}) \rightarrow b \in \{0, 1\}$$
$$\mathsf{Adapt}(\widetilde{\sigma}, y) \rightarrow \sigma$$
$$\mathsf{Extract}(\sigma, \widetilde{\sigma}, Y) \rightarrow y$$

Example

Schnorr Signature

Alice chooses a cyclic group $\mathbb{G} = \langle g \rangle$ of prime order q, and a cryptographic hash function $\mathcal{H} : \{0,1\}^* \to \mathbb{Z}_q$.

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Commit:

Each I_j will create a pre-signature $\hat{\sigma}_i = \mathrm{PreSig}(\mathrm{sk_i}, \mathrm{tx_i}, \mathrm{Y_i})$ where $\mathrm{tx_i}$ is the conditional contract stating that I_j will release funds to I_{j+1} once I_j is provided their full signature.

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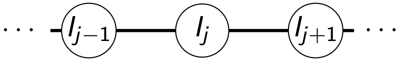

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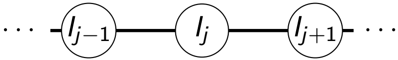

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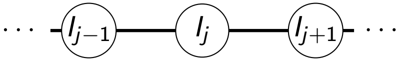

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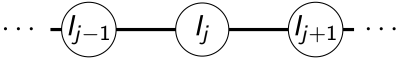

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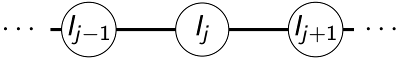

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SQISign Adaptor Signature (SAS)

Currently there are two post-quantum adaptor signatures schemes:

- Lattice Adaptor Signature (LAS) using Dilithium (Esgin, Ersoy, Erkin, 2020).
- Isogeny Adaptor Signature (IAS) using CSI-FiSh (Tairi, Moreno-Sanchez, Maffei, 2021).
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A generic construction was also published, but does not include most post-quantum signatures, such as SQISign.

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- In particular, this means $E' \cong E/ker(\phi)$.

Isogeny Background

Problem (Computational Supersingular Isogeny (CSSI))

Consider two curves E and E' defined over $\mathbb{F}_{p^2}$.
Assuming it exists, find an isogeny $\phi : E \to E'$ of degree ℓ, for some prime power ℓ, with (cyclic) kernel.

Equivalently, find a generator of order ℓ for the kernel of such a map.

Isogeny Background

Let p be a prime of the form $p = \ell_A^{e_A} \ell_B^{e_B} f - 1$.
Let E and E' be two isogenous curves.

Problem (SIDH Relation)

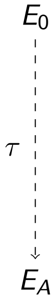
Suppose we have that (P_B, Q_B) is a basis of $E[\ell_B^{e_B}]$, and (P_A, Q_A) is a basis of $E[\ell_A^{e_A}]$.
Given $p, E, E', (P_B, Q_B), (P_A, Q_A), \varphi(P_B), \varphi(Q_B)$
find the isogeny $\varphi : E \to E'$ satisfying $\varphi(P_B), \varphi(Q_B)$.

SQISign

SQISign

E_0

SQISign



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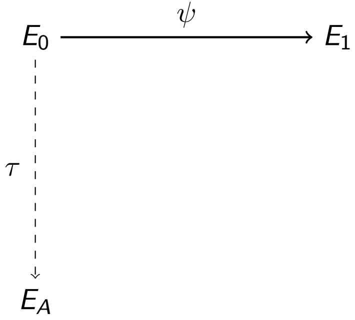

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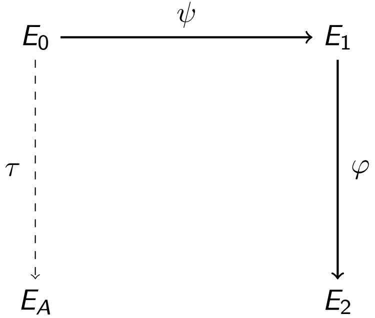

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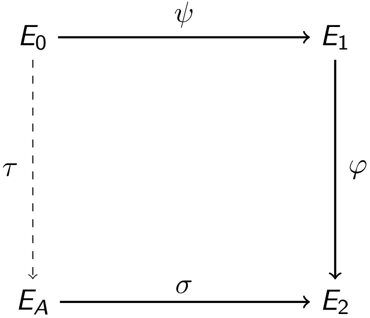

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Let (P_0, Q_0) be a basis for $E_0[\ell^e]$, for some small prime ℓ.
We choose our hard relation to be

$$\mathrm{R}_{SSI} := \{(y, E_Y) | y : E_0 \to E_Y \cong E_0/\langle P_0 + \alpha_y Q_0 \rangle\}$$

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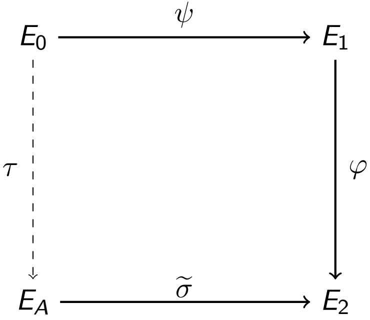

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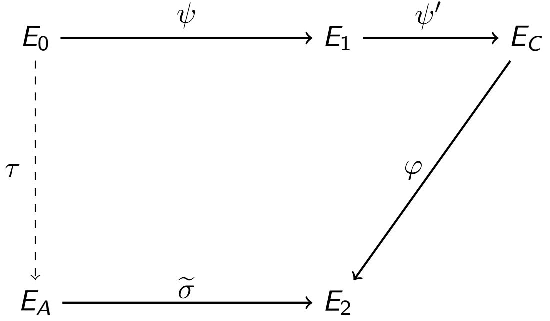

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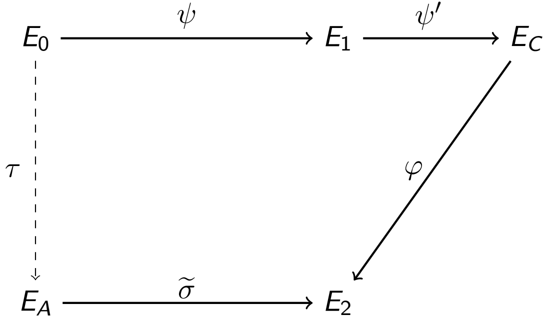
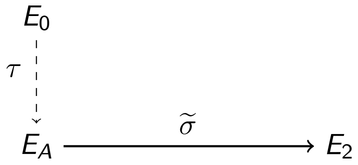


Include $\tau(P_0), \tau(Q_0)$ in PreSig

SQISign Adaptor Signature (SAS)

Adapt : (y, E_Y) where $y : E_0 \to E_Y \cong E_0/\langle P_0 + \alpha_y Q_0 \rangle$



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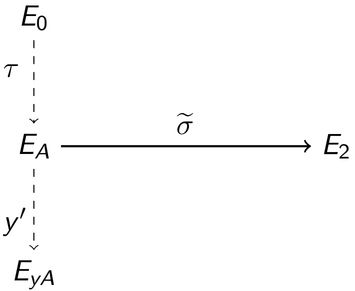
Adapt : (y, E_Y) where $y : E_0 \to E_Y \cong E_0/\langle P_0 + \alpha_y Q_0 \rangle$

$$y' : E_A \to E_{yA} = E_A/\langle \tau(P_0) + \alpha_y \tau(Q_0) \rangle$$



SQISign Adaptor Signature (SAS)

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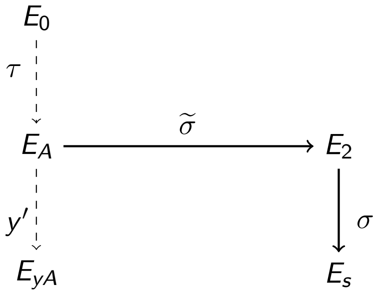
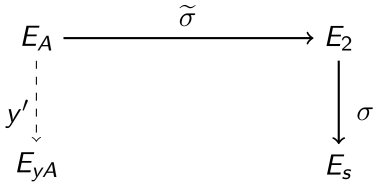
$$\sigma : E_2 \to E_s = E_2/\langle \widetilde{\sigma}(\tau(P_0) + \alpha_y \tau(Q_0)) \rangle$$



SQISign Adaptor Signature (SAS)

Extract :



Anonymous Multi-Hop Locks (AMHL) via SAS

Setup:

Anonymous Multi-Hop Locks (AMHL) via SAS

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$$\{\ell_1, \cdots \ell_{k-1}\} \subset \mathbb{Z}.$$

For each $j \in [1, \cdots k-1]$:

$$\alpha_j = \sum_{i=0}^{j} \ell_i, y_j : E_0 \to E_{Yj} \cong E_0/\langle P_0 + \alpha_j Q_0 \rangle$$

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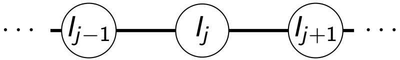
Commit:

Each I_j will create a pre-signature $\hat{\sigma}_i = \mathrm{PreSig}(\mathrm{sk_i}, \mathrm{tx_i}, \mathrm{E_{Yj}})$ where $\mathrm{tx_i}$ is the conditional contract stating that I_j will release funds to I_{j+1} once I_j is provided their full signature.

Anonymous Multi-Hop Locks (AMHL) via SAS

Release:



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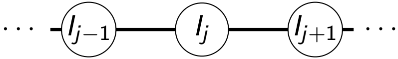

$$\sigma_j \leftarrow l_{j+1}$$

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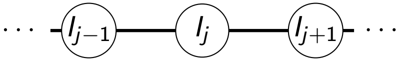

$$\sigma_j \leftarrow I_{j+1}$$

$$y_j \leftarrow \mathsf{Extract}(\sigma_j, \widetilde{\sigma}_j, E_{Yj})$$

$$\alpha_j \leftarrow y_j$$

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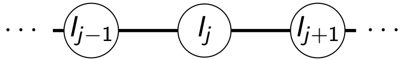

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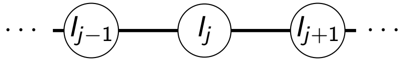

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$$\sigma_{j-1} \leftarrow \mathrm{Adapt}(\widetilde{\sigma}_{j-1}, y_{j-1})$$

Size Comparison in Bytes for 128-bit Security

	LAS	IAS	SAS
public key (bytes)	1472	128 - 2097152	64
presig (bytes)	2701	18327	226
sig (bytes)	3210	263 - 1880	15704

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The smaller presignature sizes in SAS make it better suited for *long* payment channel networks

- longer networks mean a longer set-up phase
- more will need to be transmitted to the participants