Microsoft

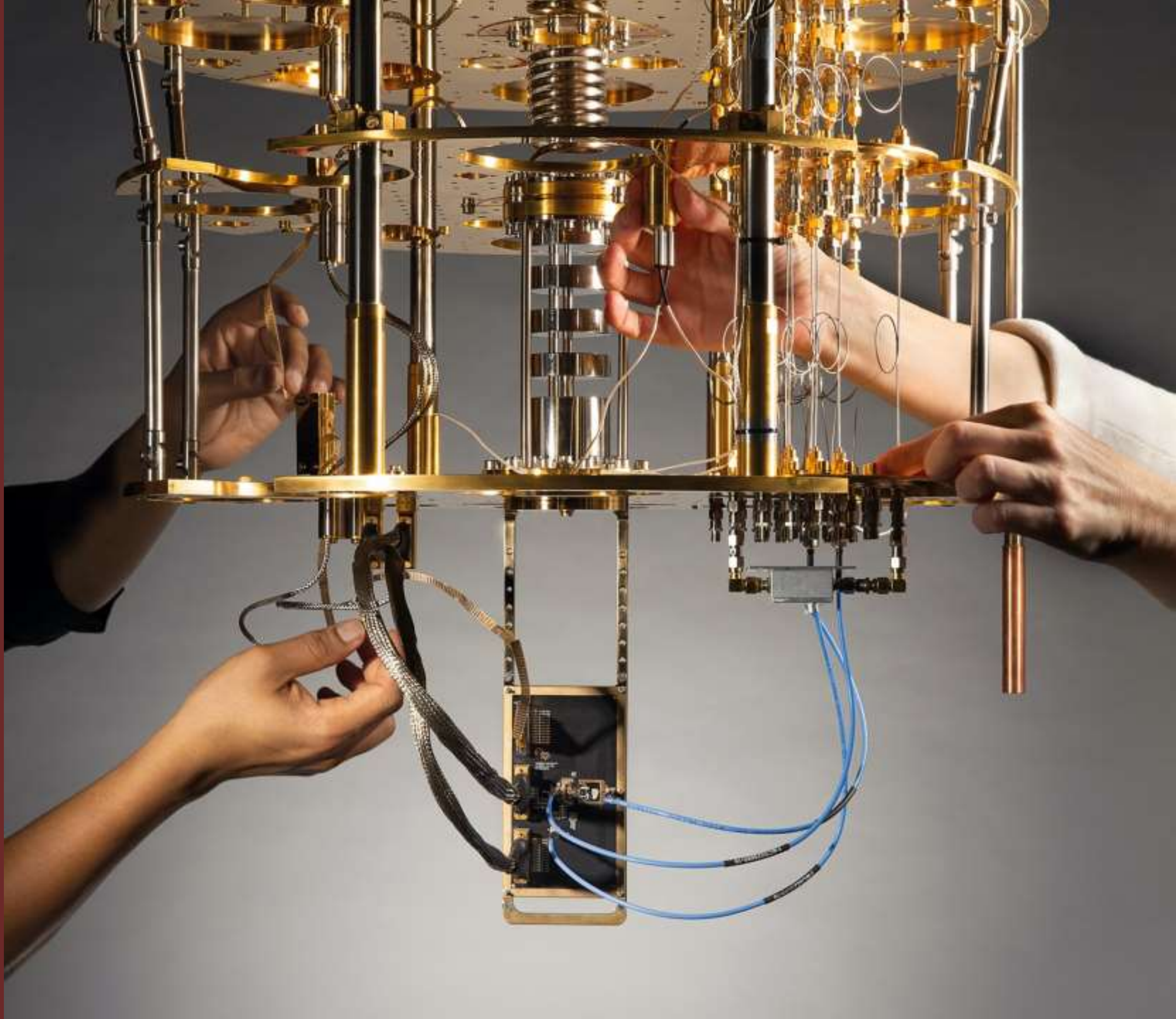
Two-dimensional implementations of quantum LDPC codes.

Nicolas Delfosse - Microsoft Quantum

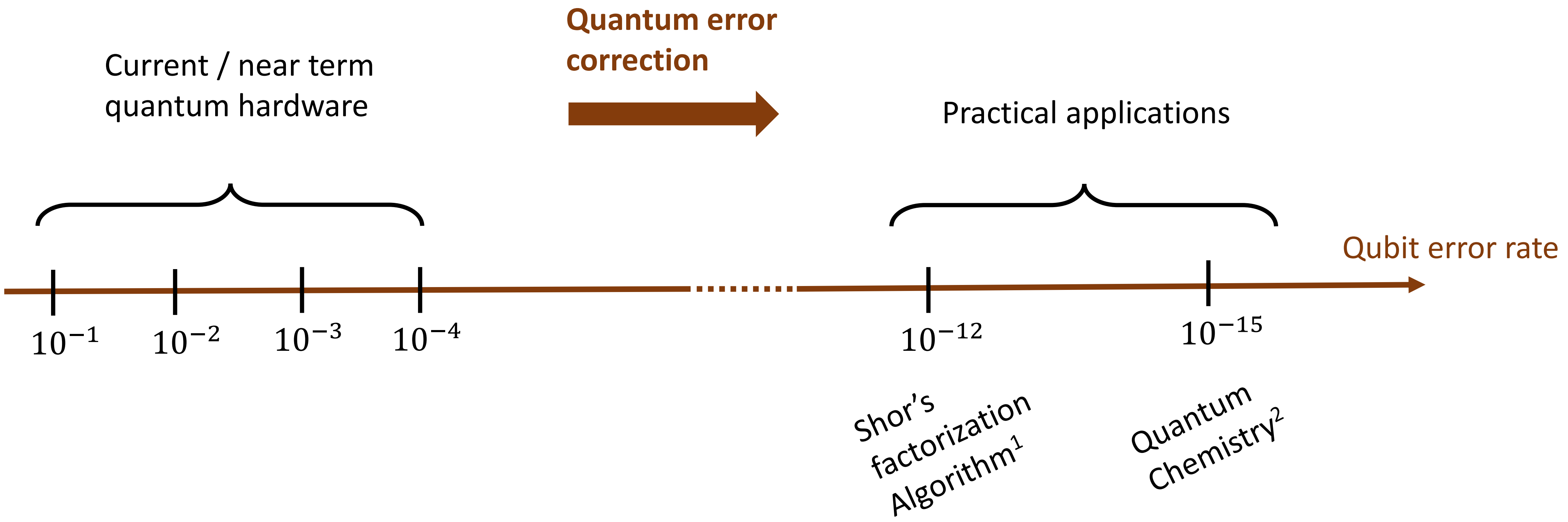
with Maxime Tremblay and Michael Beverland

arxiv:2109.14599 and arxiv:2109.14609

Fault-tolerant quantum computing with the surface code

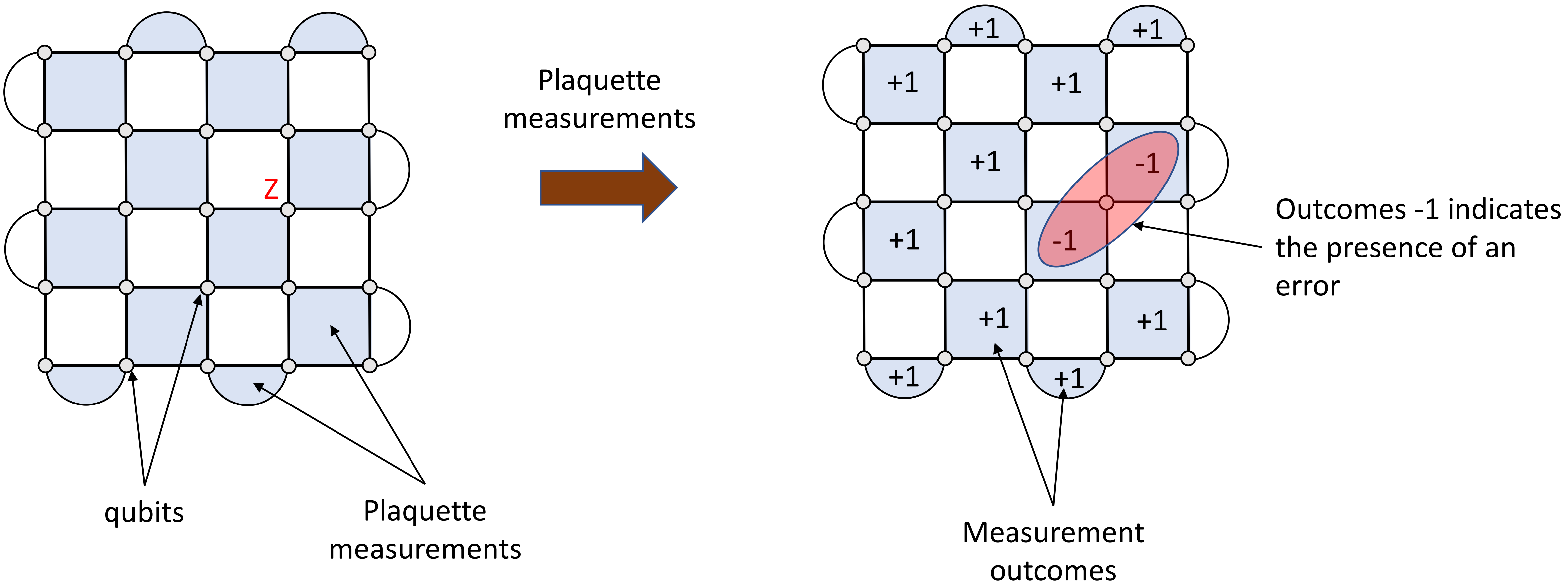


The need for quantum error correction



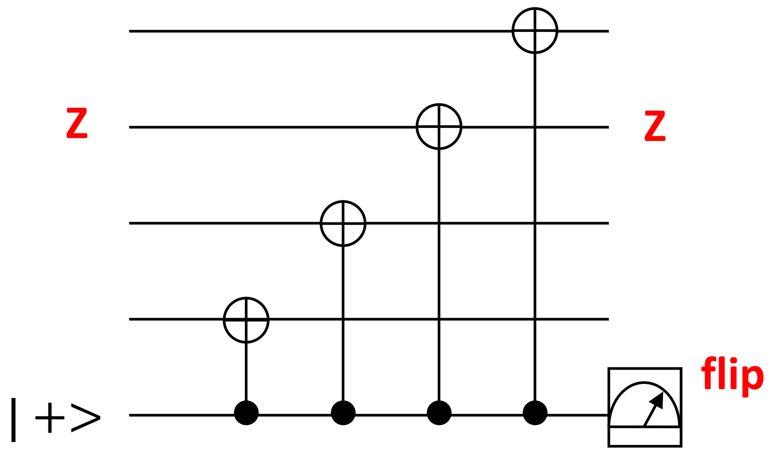
1. Fowler, Mariantoni, Martinis, Cleland (2012) arxiv:1208.0928
2. Reiher, Wiebe, Svore, Wecker, Troyer (2016) arxiv:1605.03590

Distance-three surface code[1]

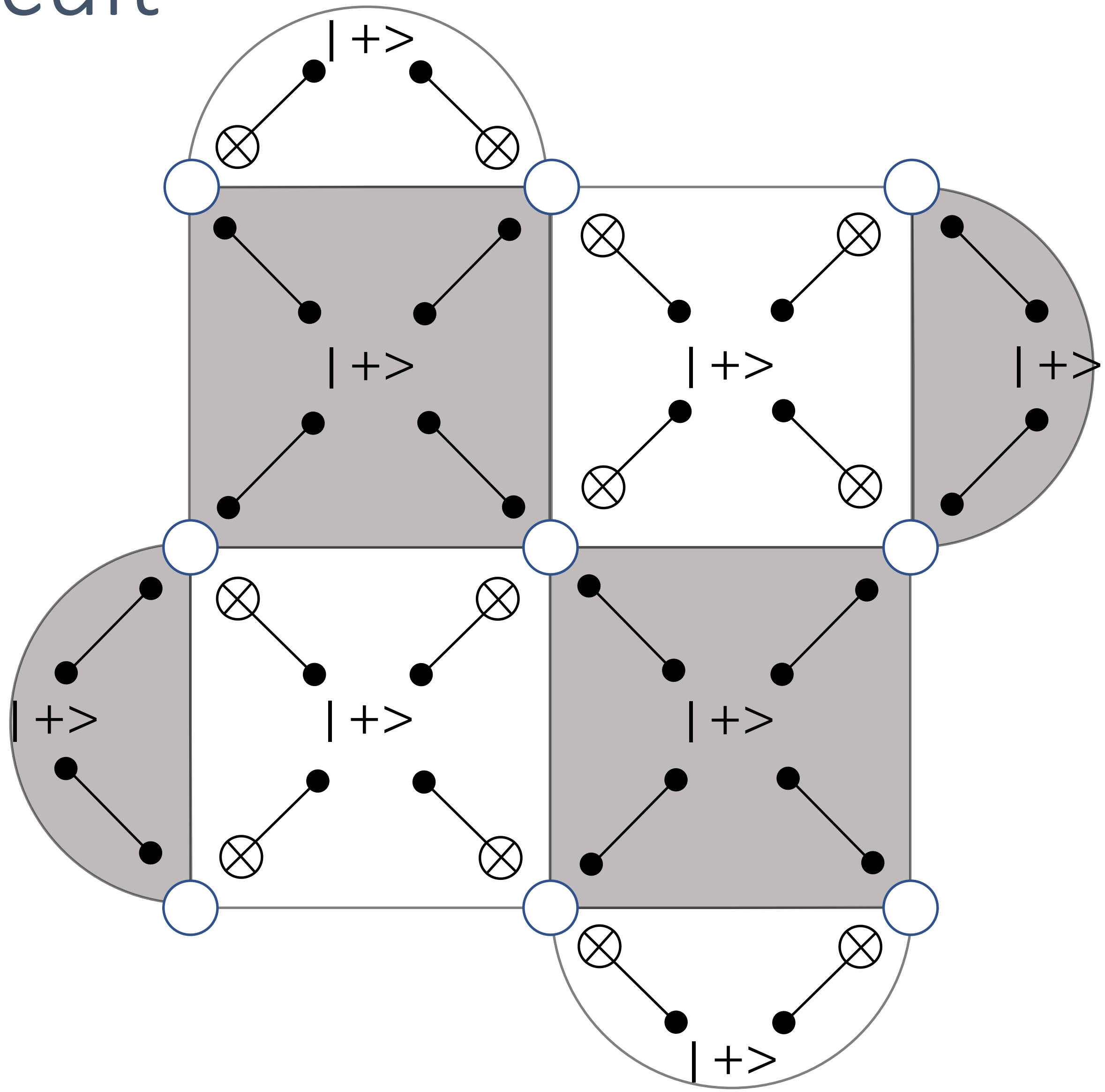


1. Kitaev (1997) [arxiv:9707021]

Syndrome extraction circuit



X plaquette circuit

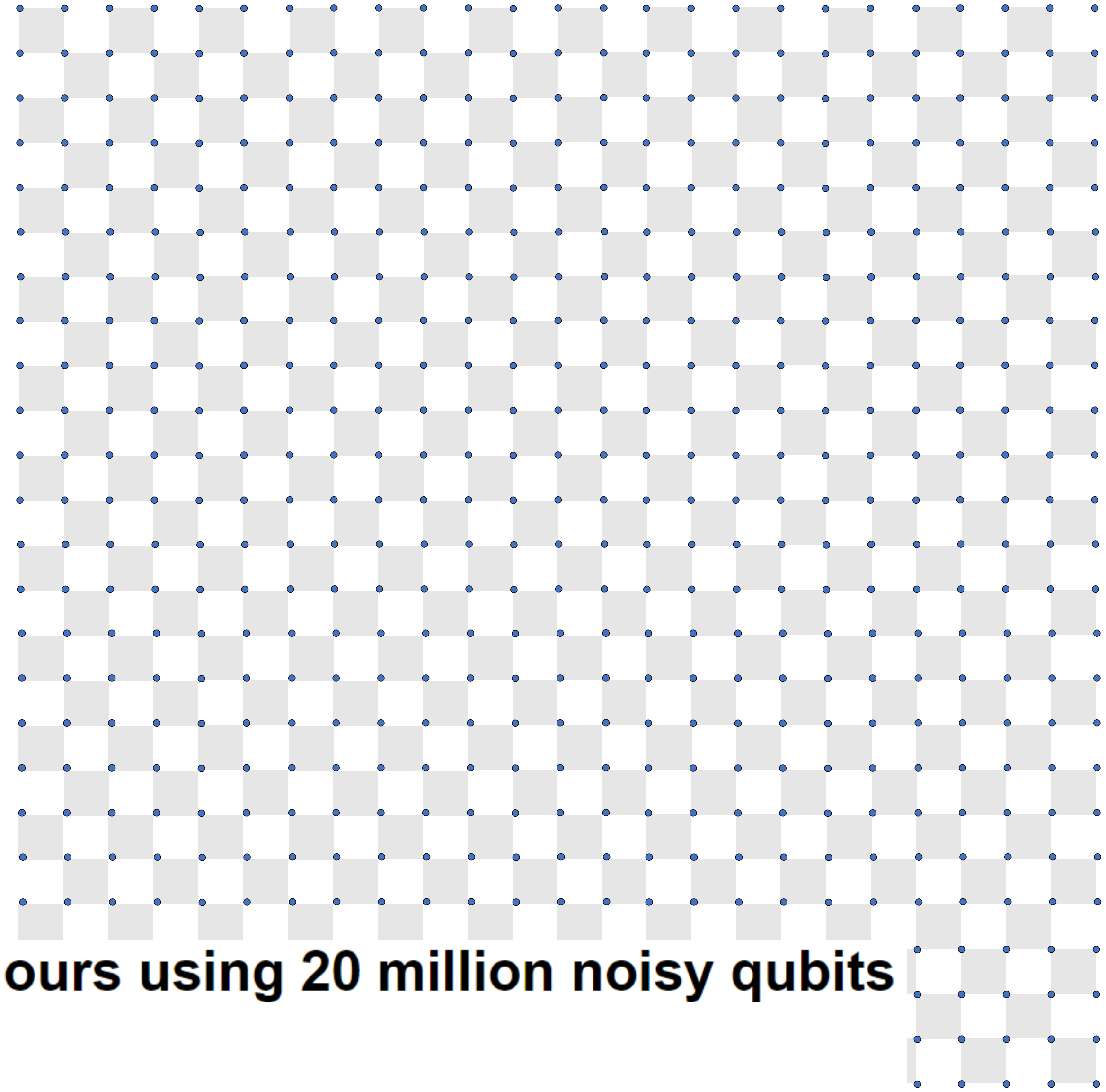


Surface codes

Why surface codes?
- High noise threshold (about 1%).
- Can be implement with 2D local gates.

Main issue:
- Encode **only 1 logical qubit.**
- Cost **thousands of physical qubits**.

(Here 1250 physical qubits)

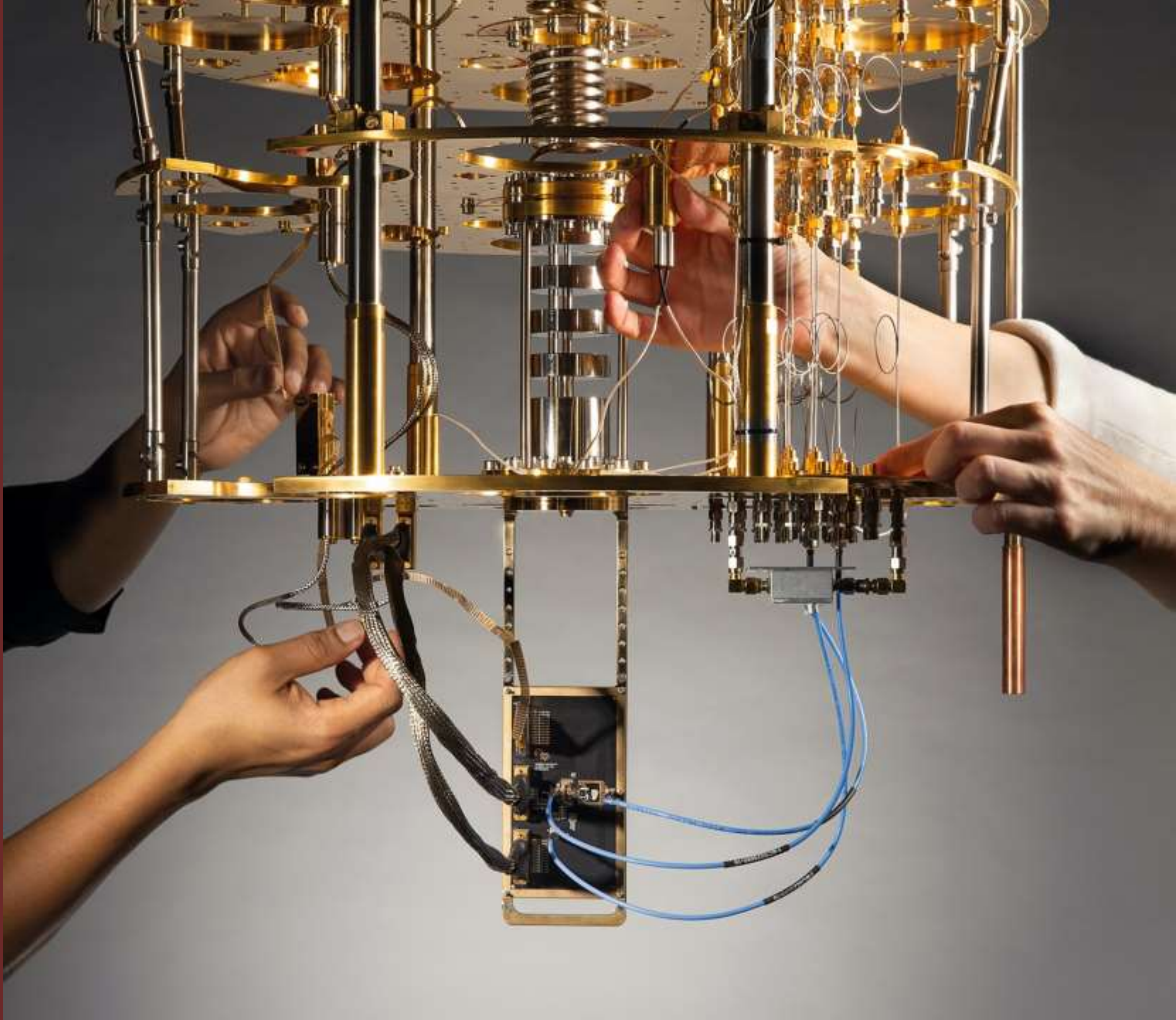


How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney, Martin Ekerå

25x25 surface code:
Physical error rate $= 10^{-3} \Rightarrow$ Logical error rate $= 10^{-12}$

The promise of quantum LDPC codes



Low Density Parity Check (LDPC) codes

- 2004: QLDPC codes outperform surface codes for quantum communication

Sparse Graph Codes for Quantum Error-Correction

David J.C. MacKay, Graeme Mitchison, Paul L. McFadden

- 2009: First satisfying QLDPC codes.

Quantum LDPC codes with positive rate and minimum distance proportional to n^{1/2}

Jean-Pierre Tillich, Gilles Zemor

- 2013: QLDPC codes asymptotically reduce the cost of fault-tolerant quantum computing.

Fault-Tolerant Quantum Computation with Constant Overhead

Daniel Gottesman

- 2020: High threshold for HGP codes (about 3% for phenomenological noise)

Combining hard and soft decoders for hypergraph product codes

Antoine Grospellier, Lucien Grouès, Anirudh Krishna, Anthony Leverrier

- 2021: Good QLDPC codes exist!

Asymptotically Good Quantum and Locally Testable Classical LDPC Codes

Pavel Panteleev, Gleb Kalachev

Main results:

Question: They seem promising, but can we implement them with a 2D grid of qubits?

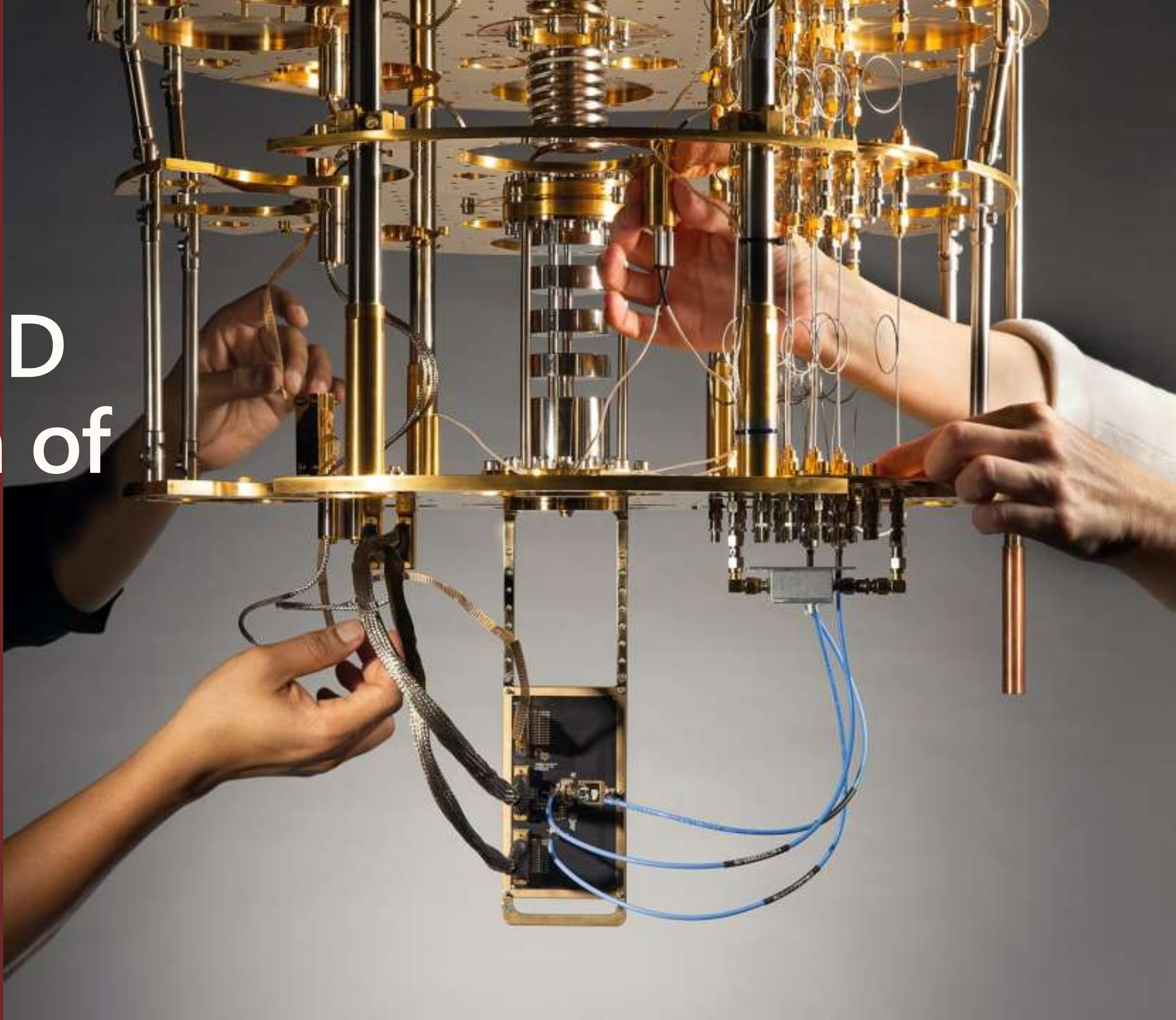
- **No,** if we use only 2D local gates.

Because the syndrome extraction takes either too many gates or too many qubits which results in a degradation of the performance.

- **Yes**, if we allow for some long-range connection.

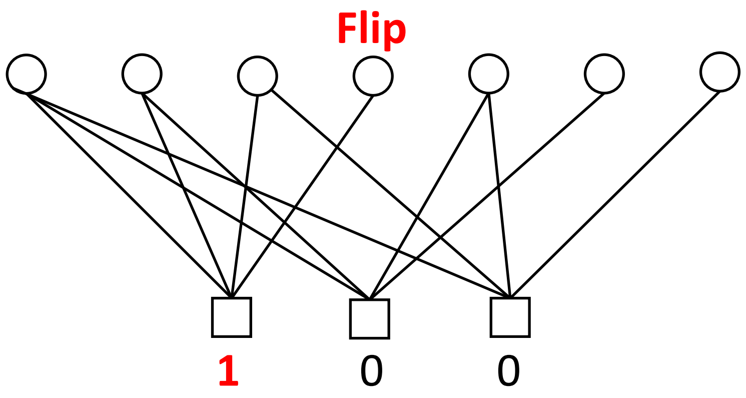
We propose a design based on a small number of layers of long-range connections.

Obstacles to a 2D implementation of quantum LDPC codes

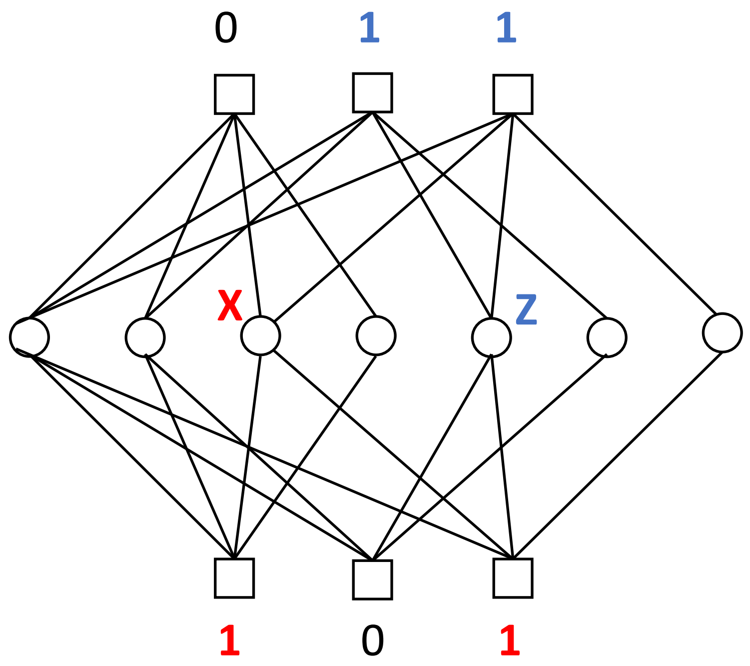


Quantum LDPC codes

Classical code detect bit flips:



Quantum codes detect X errors and Z errors:

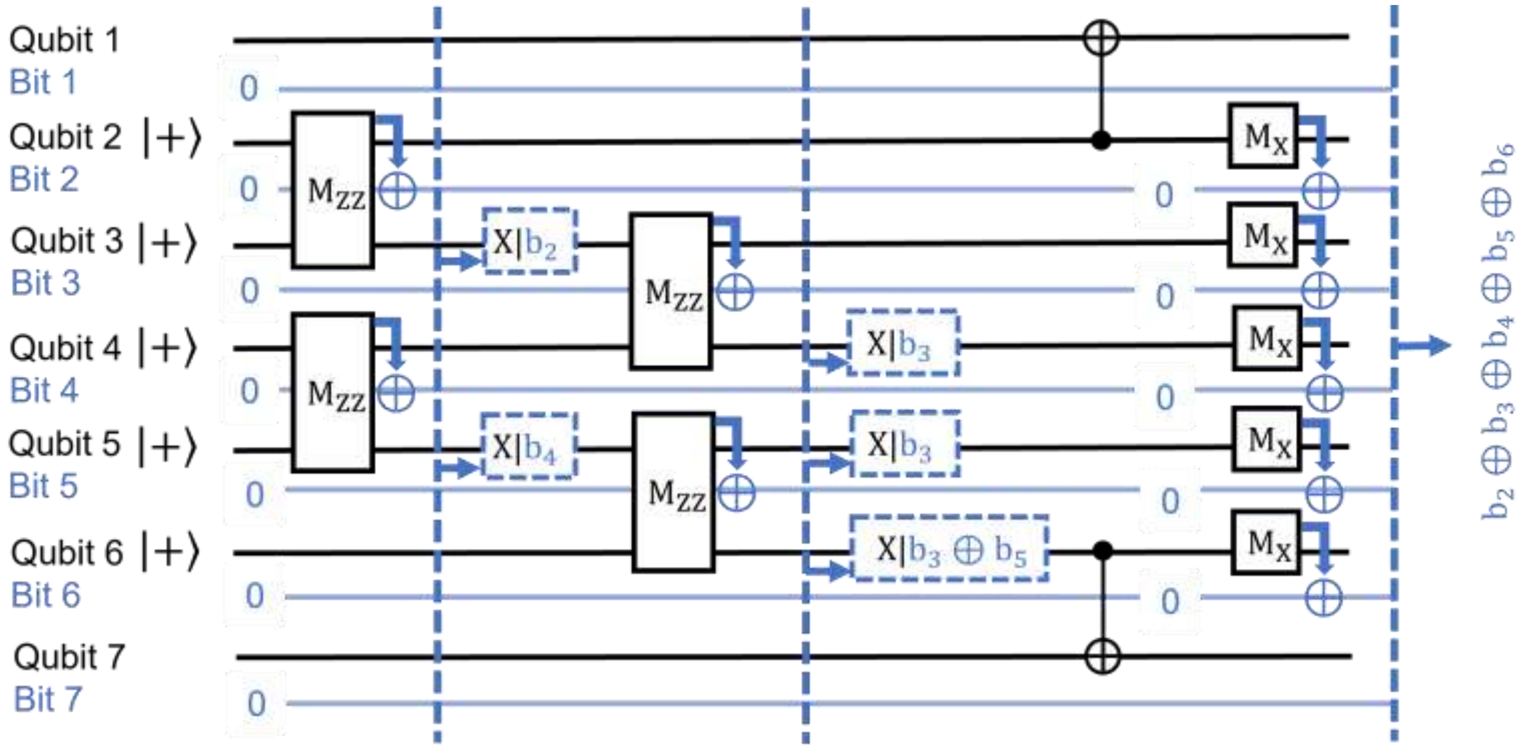


Quantum LDPC codes: Quantum codes defined by bounded degree graphs

Syndrome measurement circuits

Allowed operations:

- Preparations of $|0\rangle$ or $|+\rangle$.
- Single-qubit and two-qubit Pauli measurements.
- Single-qubit and two-qubit unitary Clifford gates.
- Classically-controlled Pauli operations, applied only if some subset of previous measurement outcomes has parity 1.
- Output a set of classical bits obtained by computing the parity of some subsets of measurement outcomes.



Our circuit bounds and saturating circuits

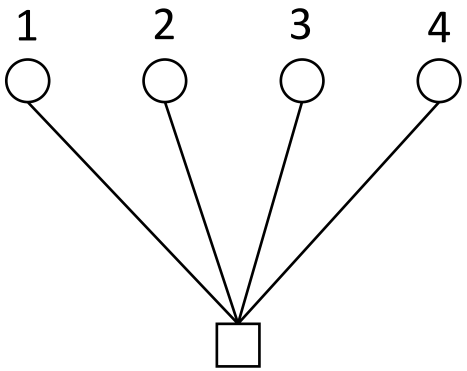
Theorem: Consider local-expander quantum LDPC codes with length n implemented with local gates on a grid of $\sqrt{N} \times \sqrt{N}$ qubits. The depth of the syndrome extraction circuit satisfies:

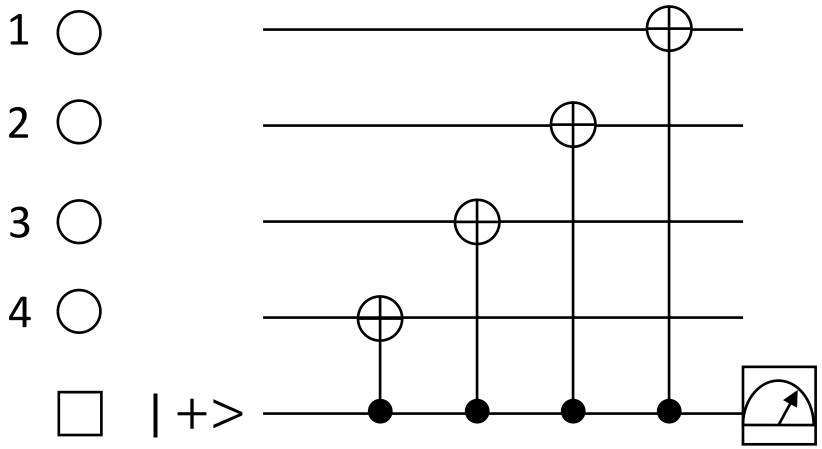
$$\mathbf{depth} \geq \boldsymbol{\Omega}\left(\frac{\boldsymbol{n}}{\sqrt{\boldsymbol{N}}}\right)$$

	Constant depth	**Constant overhead (# ancillas = O(n))**
Bound:	# ancilla ≥ $\boldsymbol{\Omega}(\boldsymbol{n}^2)$	Depth ≥ $\boldsymbol{\Omega}(\sqrt{\boldsymbol{n}})$
Saturating circuits:	Switch-based circuits (next slide)	HGP code circuits

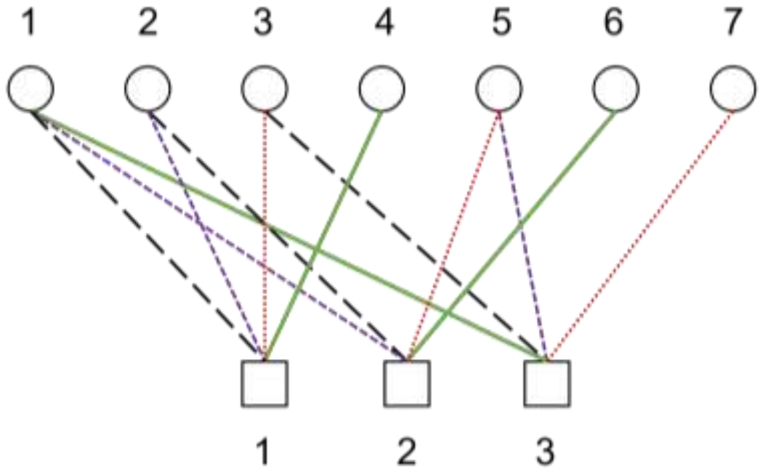
Color-based circuit for fully connected qubits

Measurement of a single X check:





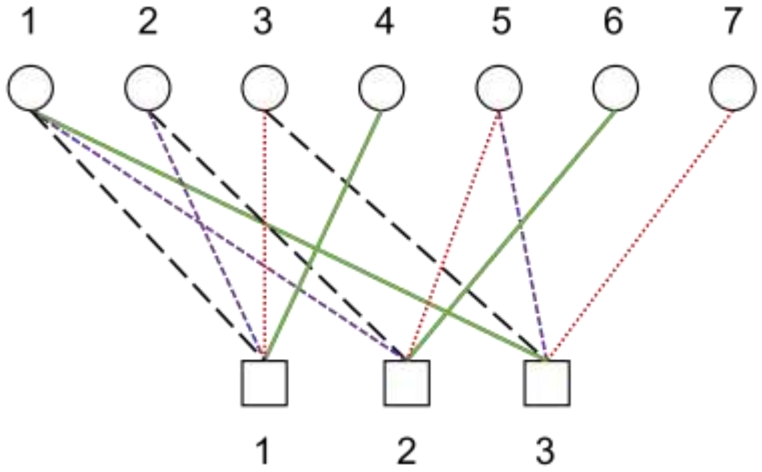
Simultaneous measurement of all X checks:

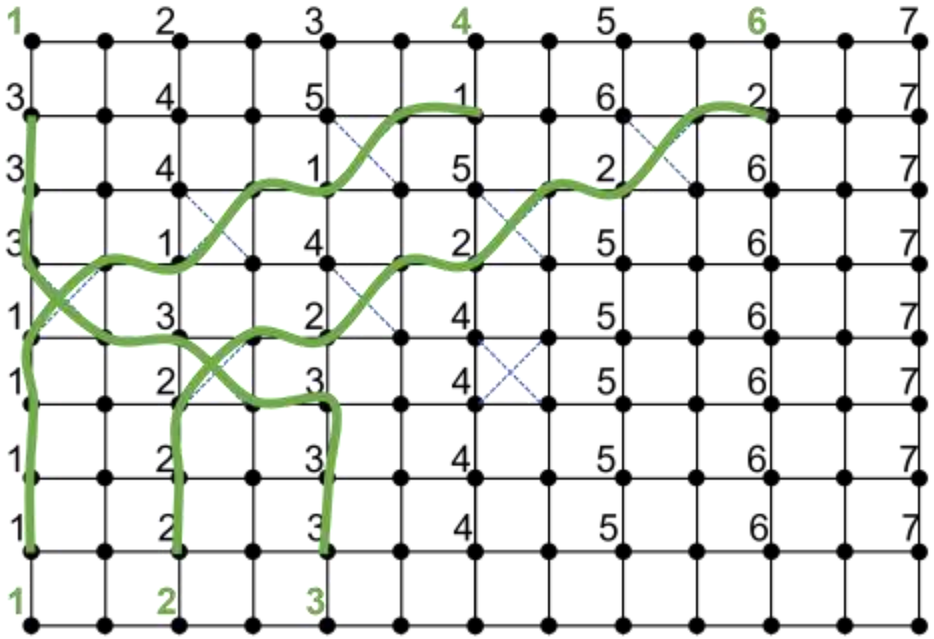


1. Prepare a readout qubits in $|+\rangle$ for each check.
2. For each color c do:
3. Apply a CNOT from each readout to its data qubit with color c
4. Measure each readout qubit in the X basis.

Switch-based circuit

Tanner graph of Hamming code

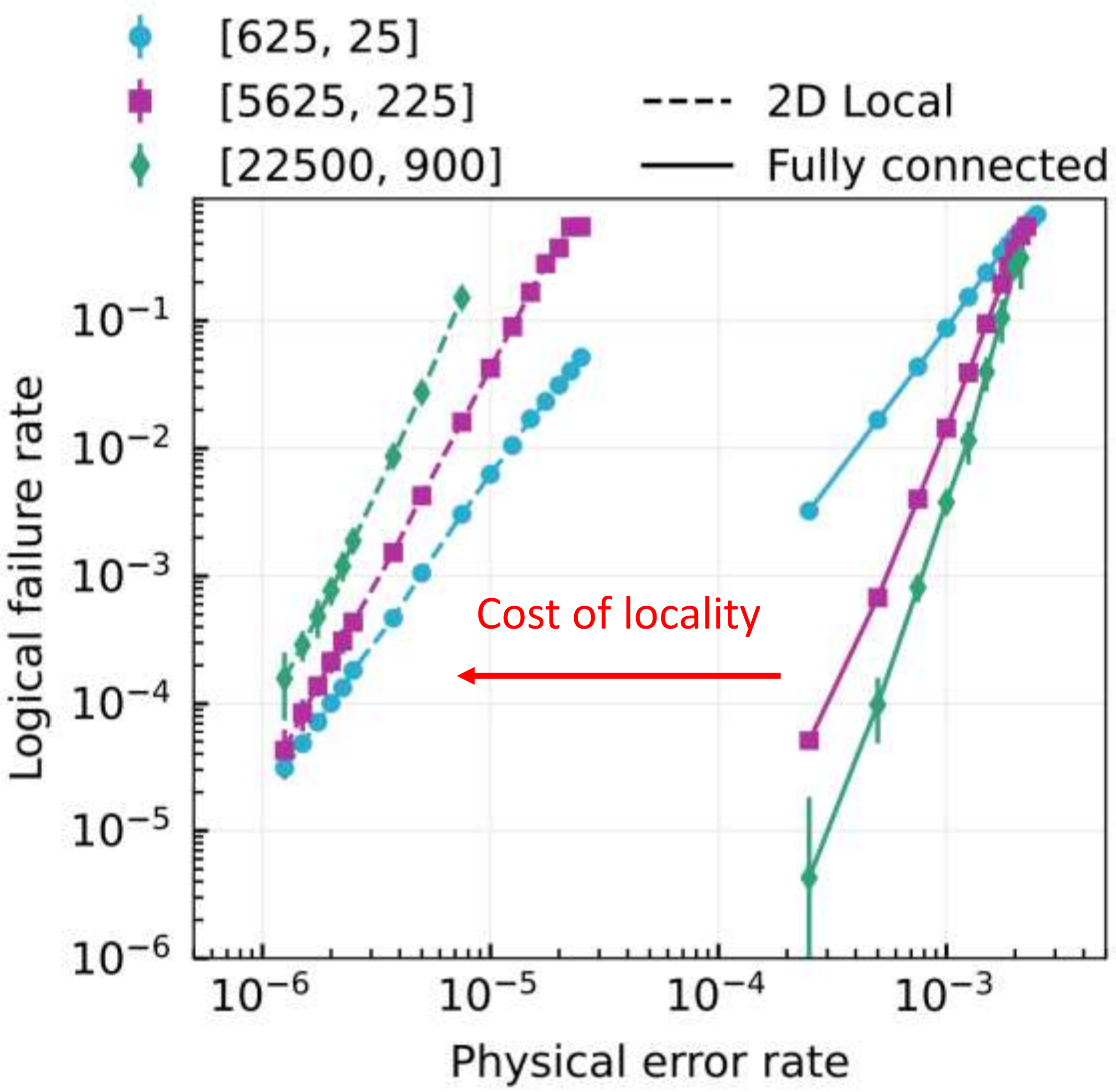




Uses a sorting network

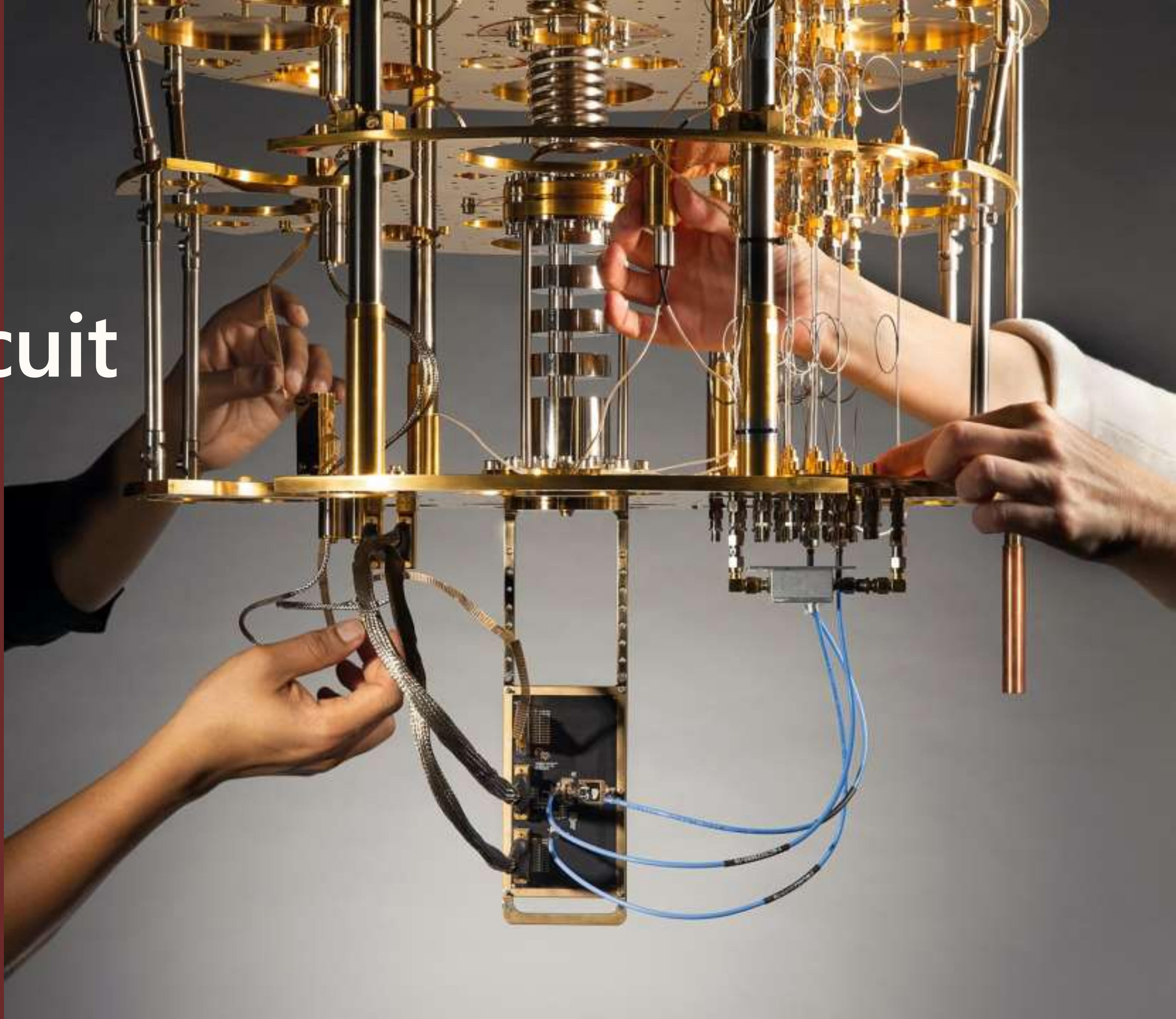
1. Place data qubits $q_1, \ldots, q_n$ on top of the grid
2. Place readout qubits $s_1, \ldots, s_n$ on the bottom of the grid
3. Prepare readout qubits in $|+\rangle$.
4. For each color c do:
5. Build **paths connecting each readout qubit with its data qubit**.
6. Prepare a Bell state on each diagonal edge of a path.
7. Apply a long distance CNOT from each readout to its data qubit
8. Measure each readout qubit in the X basis.

2D local vs fully connected implementation

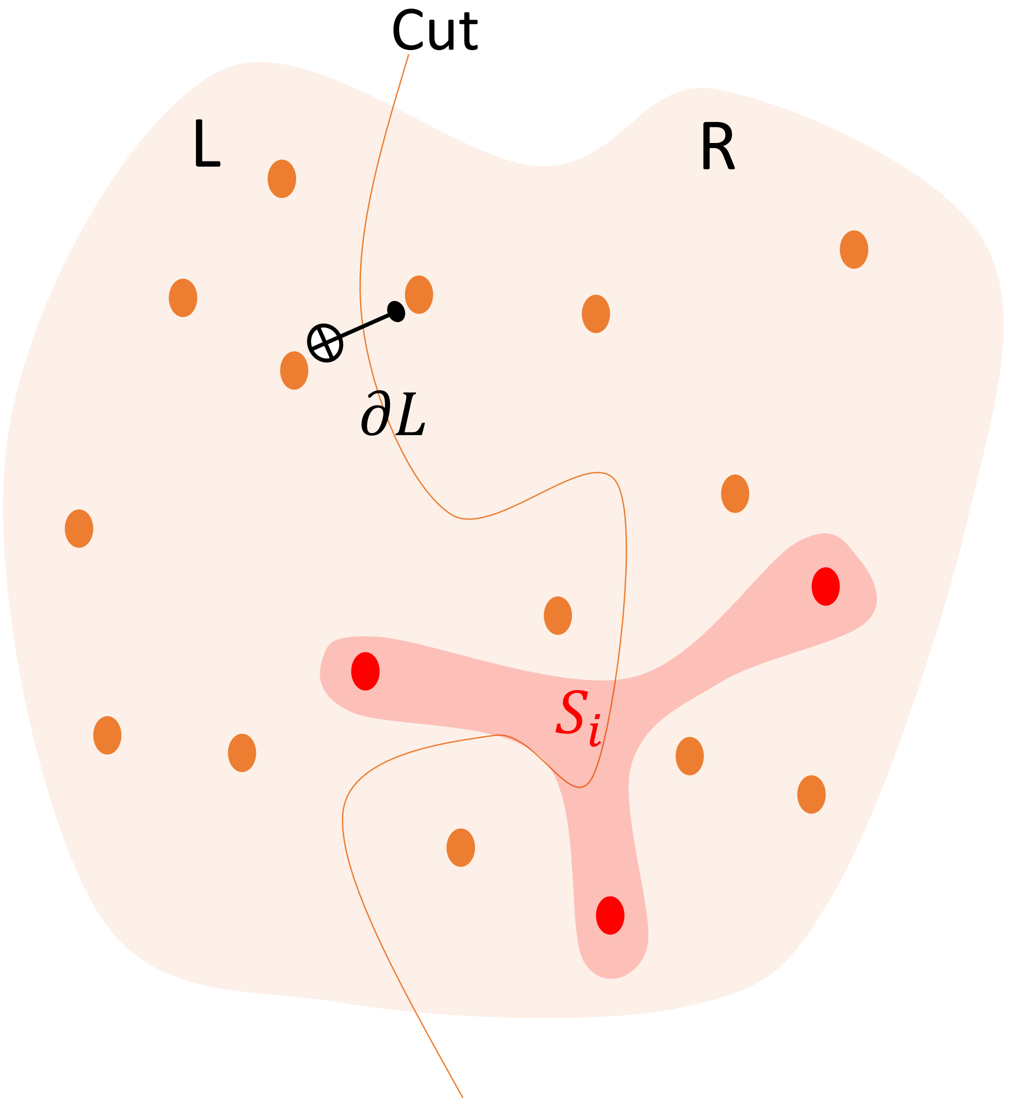


- 2D local syndrome extraction circuit with:
  - **n ancilla qubits**
  - **optimal depth $\Omega(\sqrt{n})$.**
- Simulation with circuit-level noise

Proof of our circuit bound



Main technical result



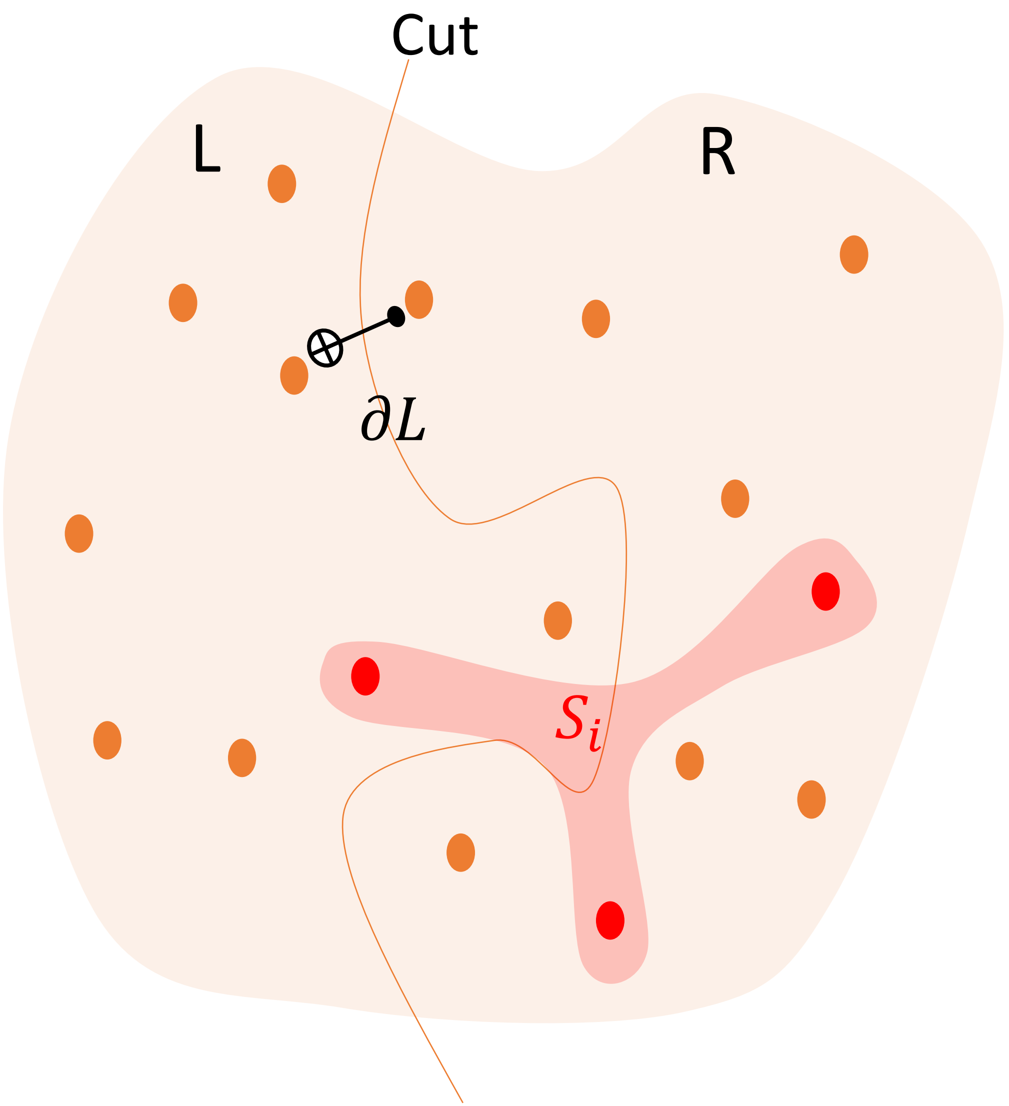
Theorem 1. *Let C be a Clifford circuit measuring commuting Pauli operators $S_1, \ldots, S_r$. Then, for any subset of qubits L, we have*

$$\mathrm{depth}(C) \geq \frac{n_{\mathrm{cut}}}{64|\partial L|},$$

- n_{cut} = number of independent Si with support on L and R.
- $|\partial L|$ = maximum number of gates acting on L and R in a single round.

The previous bounds are corollaries of this result.

Proof strategy



Consider the correlations between L and R

Lemma 1 (Informal version)

$$\#\text{ bits of correlations} \leq 32\,|\partial L|\text{depth}$$

Lemma 2 (Informal version)

$$\#\text{ bits of correlations} \geq \frac{n_{\text{cut}}}{2}$$

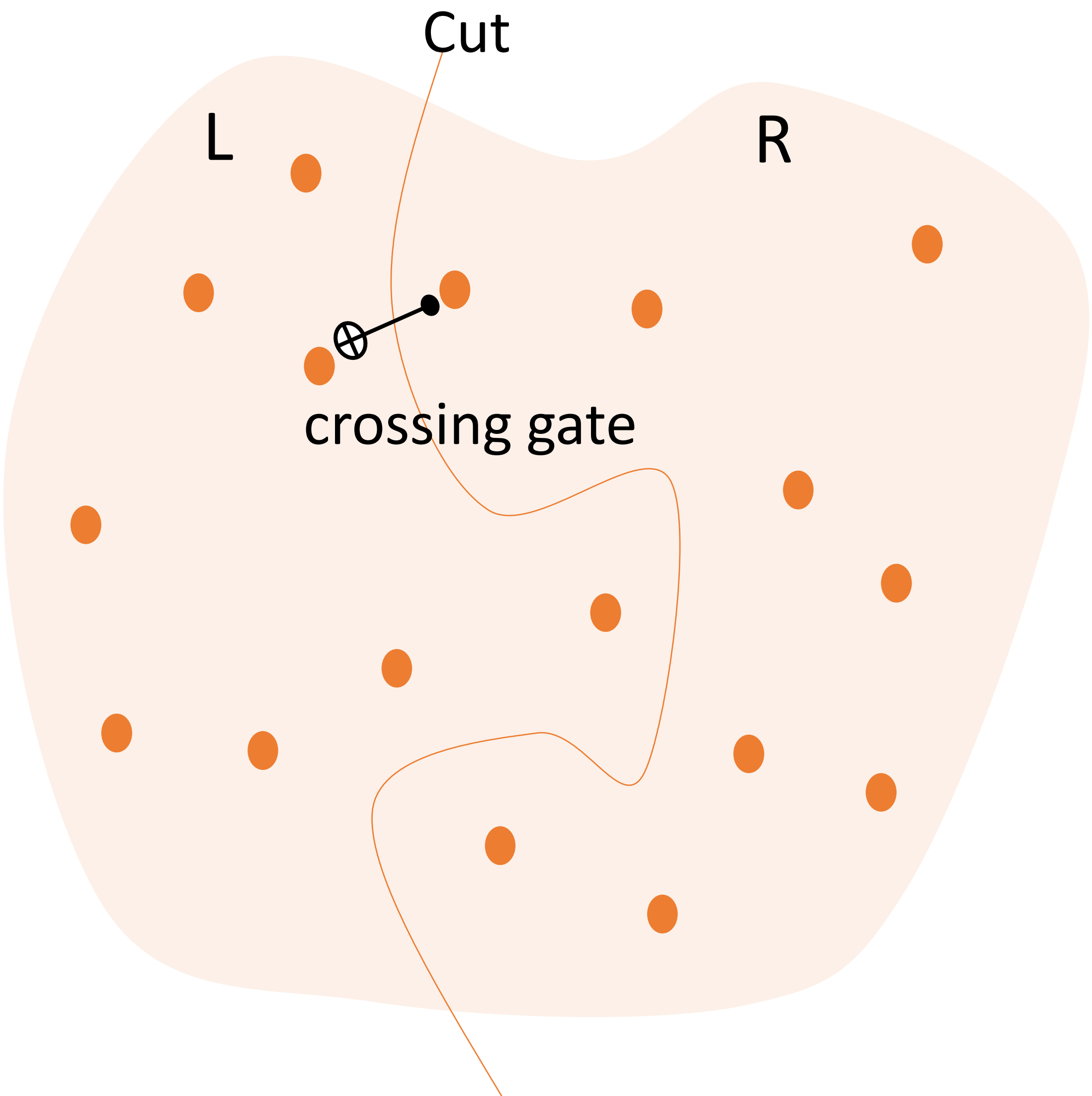
Application: $\text{depth} \geq \frac{n_{\text{cut}}}{64|\partial L|}$

Main difficulty: Find the right notion of correlation[1]:

$$I(O_L^{(2)}, E_L; O_R^{(2)}, E_R | O^{(1)})$$

1. See Section 4.1 of arxiv:2109.14599 for the complete proof strategy.

First Lemma: Upper bound on correlations



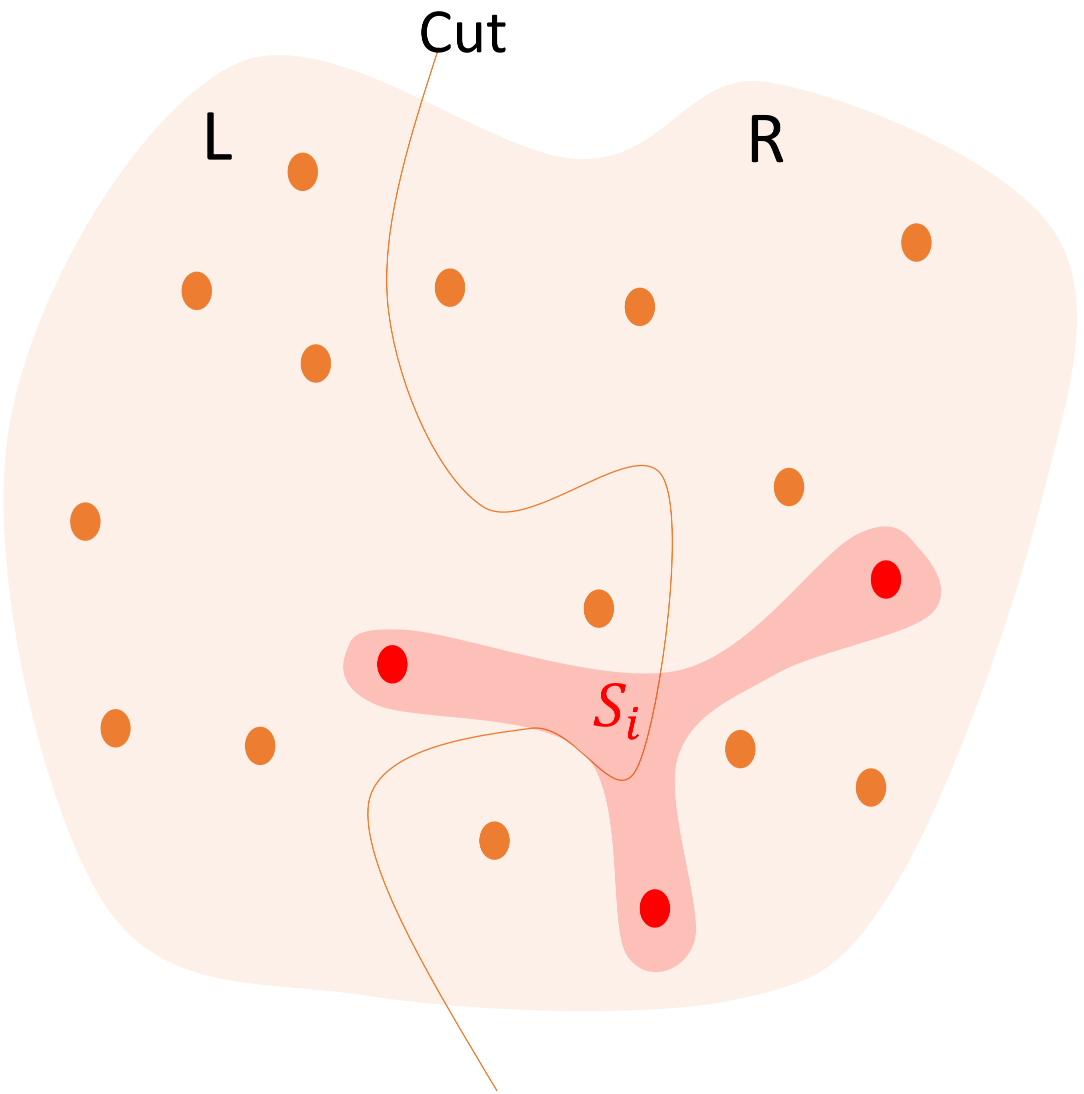
Lemma 1 (Informal version)

bits of correlations $\leq 32\ |\partial L|$ depth

Sketch of proof:

- Only crossing gates (gates supported on L and R) can introduce correlations.
- The circuit contains $|\partial L|$ depth crossing gates.
- Each crossing gate introduces a bounded amount of correlation.

Second Lemma: Lower bound on correlations



Lemma 2 (Informal version)

$$\#\text{ bits of correlations} \geq \frac{n_{\text{cut}}}{2}$$

Sketch of proof:

- We repeat the measurement of all the S_i twice so that the second round of measurement gives the same outcome as the first one.
- The two outcomes of the measurement of S_i are

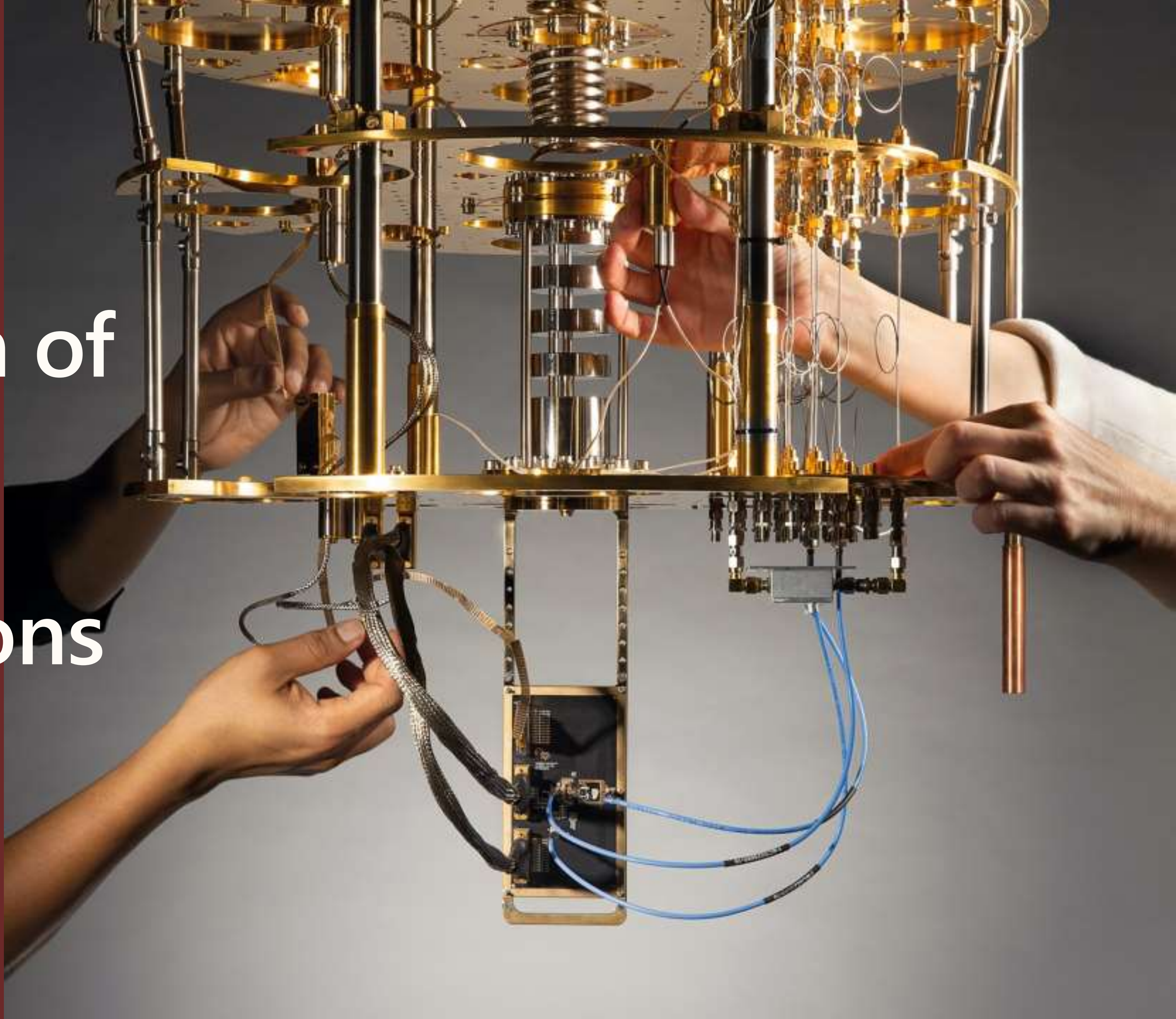
$$\alpha_1 + \cdots + \alpha_{a_i} + \beta_1 + \cdots + \beta_{b_i} \qquad \text{(first round)}$$

$$\alpha_1{}' + \cdots + \alpha_{a_i}{}' + \beta_1{}' + \cdots + \beta_{b_i}{}' \qquad \text{(second round)}$$

where the α_i are measured in L and the β_i in R.

- Then, we have $\sum \alpha_i + \sum \alpha_i' = \sum \beta_i + \sum \beta_i'$.
- If $a_i > 0$ and $b_i > 0$, this induces one bit of correlation between L and R.

Implementation of quantum LDPC codes with long range connections



Naïve layout with long-range connectivity

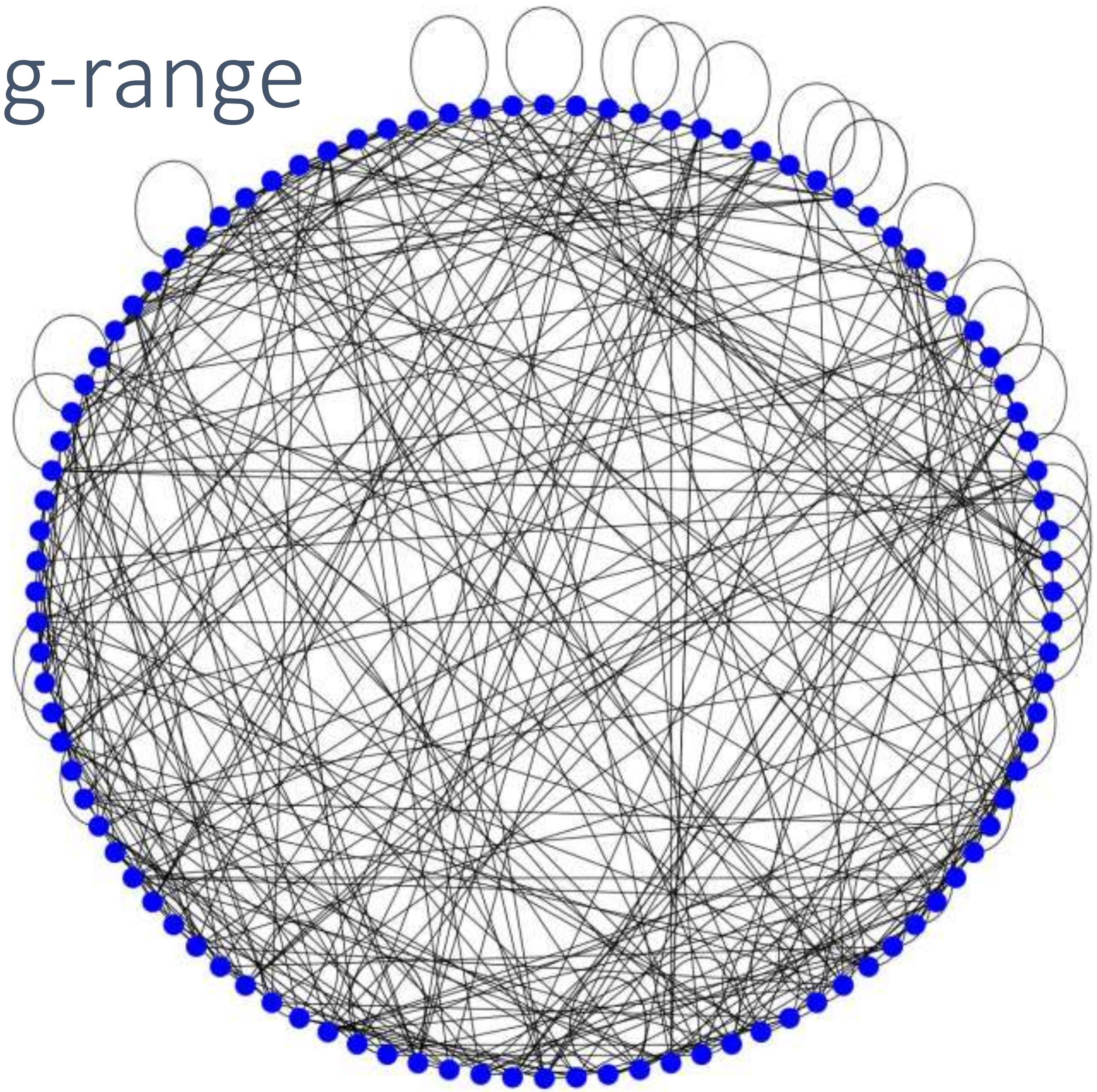
Circular layout for QLDPC codes:

- Place the qubits on a circle
- Implement the color-based circuit using long-range gates.

Issue: Crossing gates may induce correlated errors and degrade the performance.

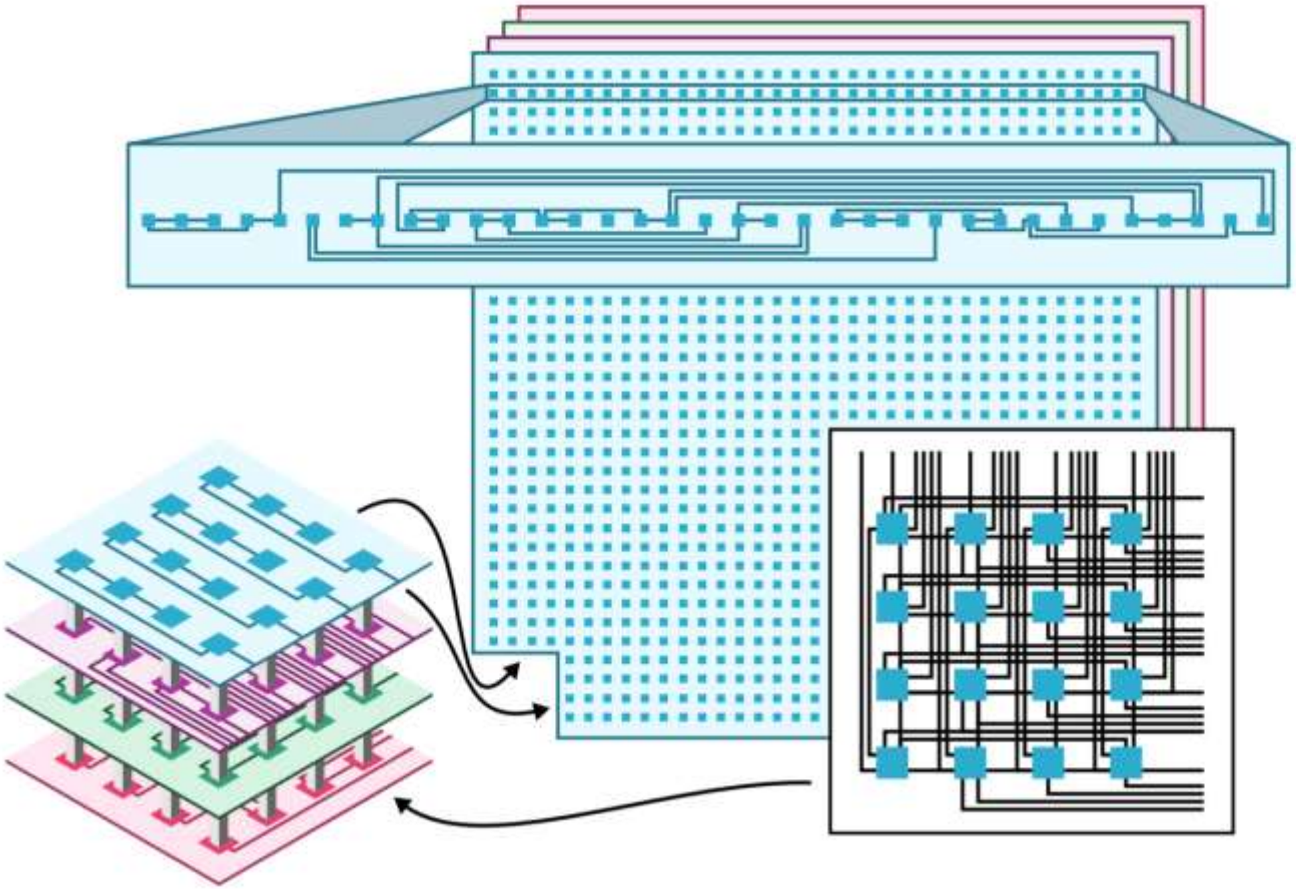
Goal: Design a syndrome extraction circuit with

- Short depth.
- A small number of crossing gates.



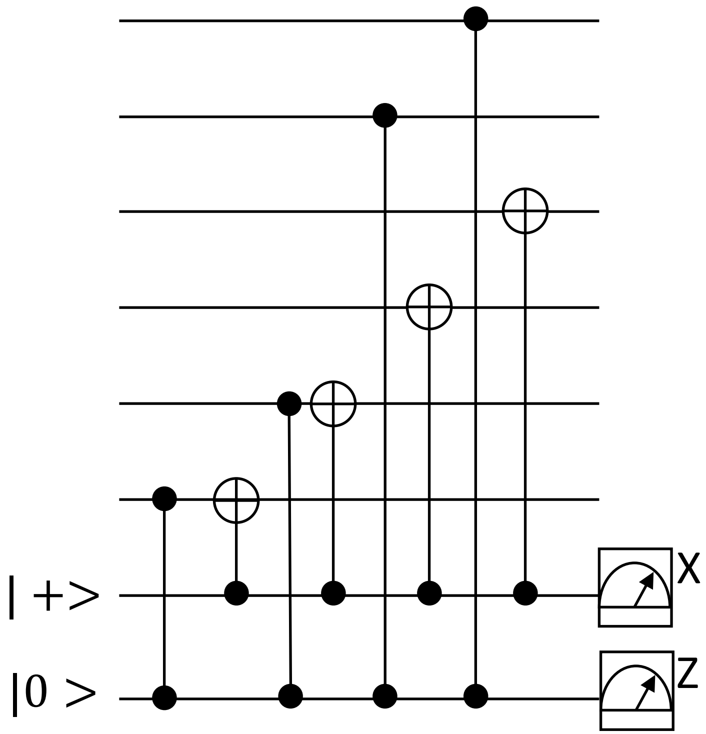
Circular layout for a graph with 100 vertices with degree 8

ℓ-planar layout

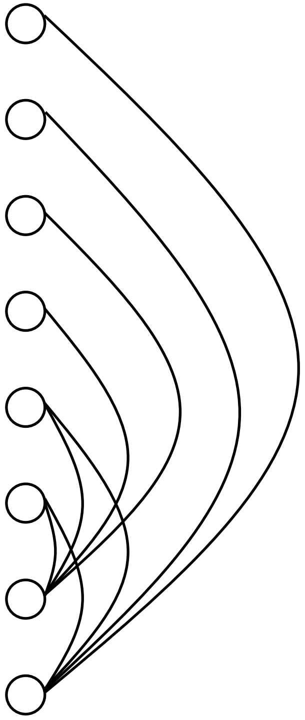


Theorem 1. Let Q be a CSS code such that each stabilizer generator has weight at most δ and each qubit is involved in at most δ stabilizer generators. Then, one can implement the measurement of all the stabilizer generators of Q with a circuit with depth $2\delta + 2$ using a $\lceil \delta/2 \rceil$-planar layout.

Key ingredient: Connectivity graph of a circuit



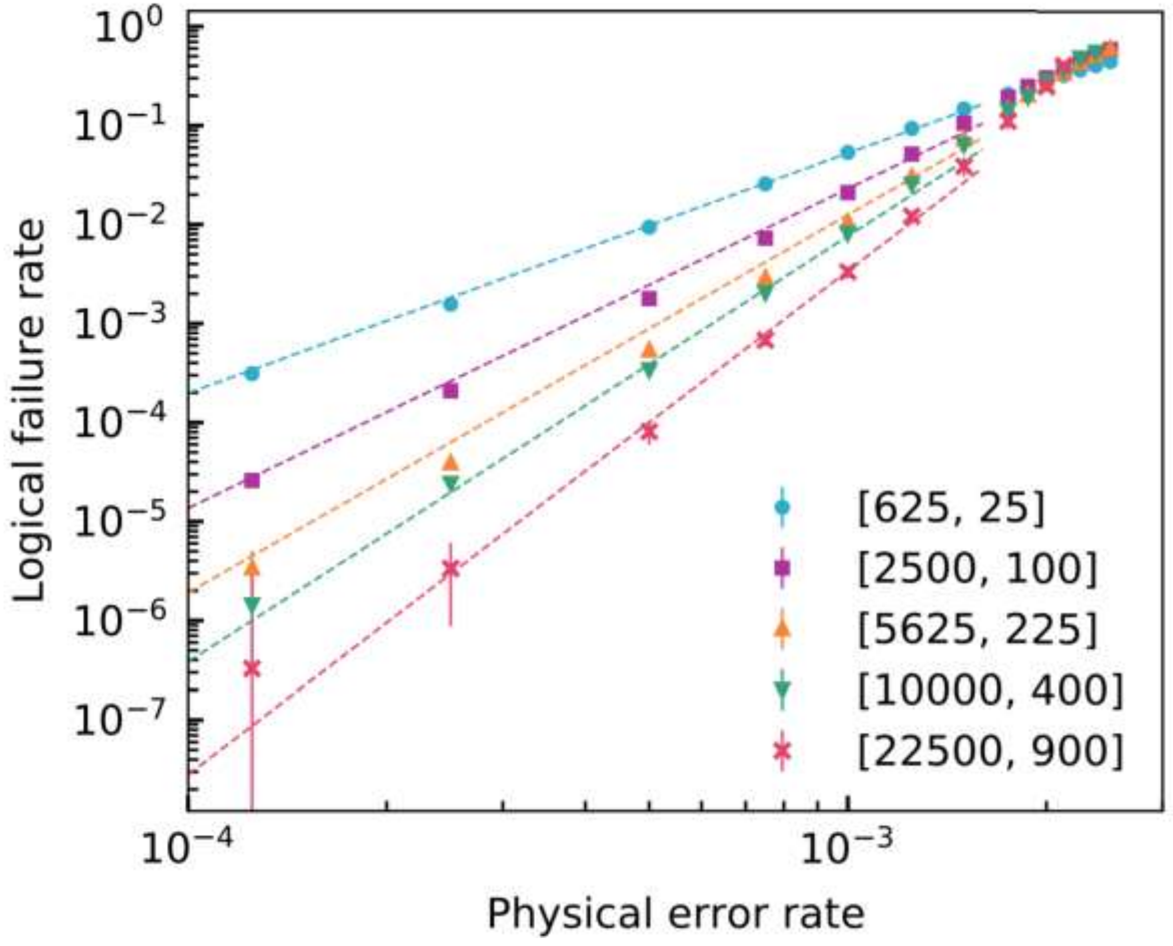
Quantum circuit



Connectivity graph

Proposition 1. Let C be a circuit made with single-qubit and two-qubit operations whose connectivity graph has degree at most δ. Then, C can be implemented with a $\lceil \delta/2 \rceil$-planar layout.

Numerical results

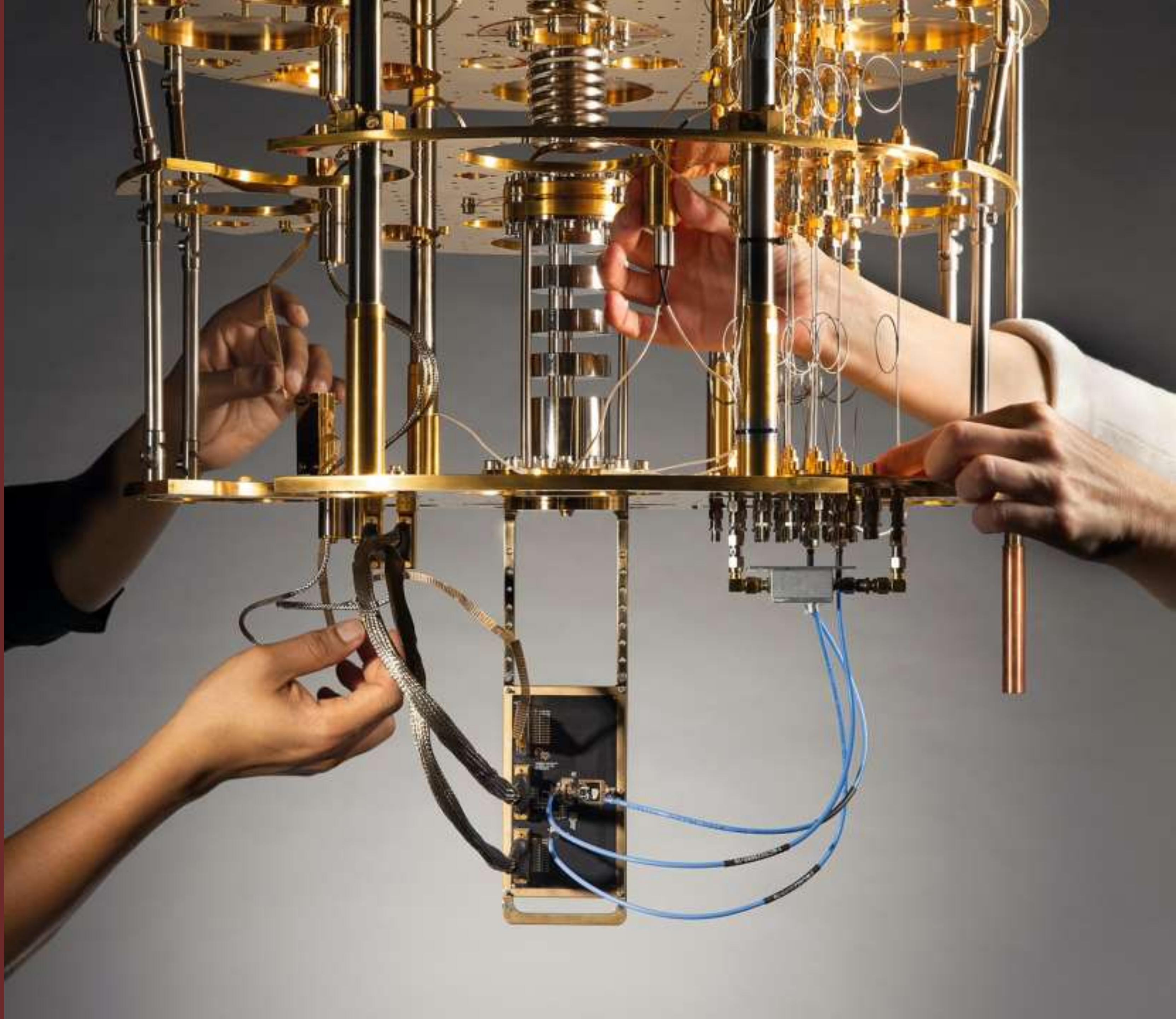


Noise threshold: 0.28%
(instead of 0.7% for the surface codes)

physical qubits per logical qubit: 49
(instead of thousands for surface codes)

Logical failure rate	10^{-9}	10^{-12}	10^{-15}
Logical qubits	1600	6400	18 496
Surface code physical qubits	387 200	2 880 000	13 354 112
HGP code physical qubits	78 400	313 600	906 304
Improvement using HGP codes	4.94×	9.18×	14.73×

Conclusion



Conclusion

Main results:

- LDPC codes beat the surface code with realistic noise (circuit noise).
- LDPC codes implemented on 2D local quantum hardware are not competitive.
- A layout for LDPC codes with a few planar layers on long-range connection in 2D.

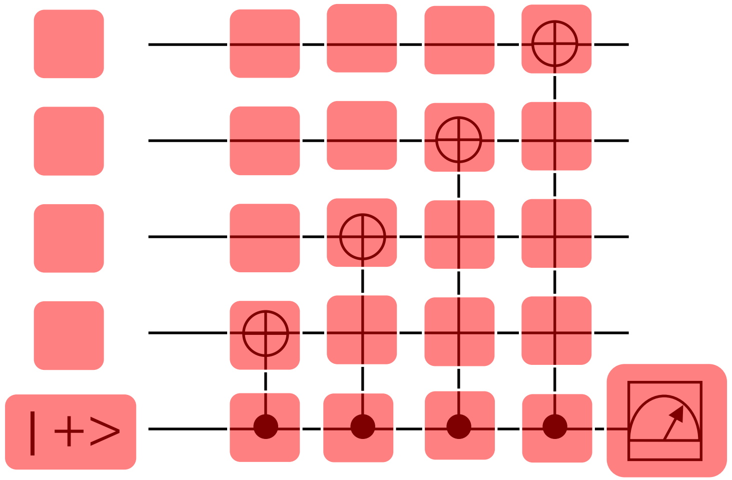
Future work:

- Improve further the performance of QLDPC codes: with better codes, better decoders, better measurement circuits.
- Design reliable long-range connections whose noise rate is independent of the distance between the qubits.
- Design insulated layers of long-range connection with little crosstalk.

Microsoft

Thank you.

Standard Pauli noise models



X plaquette circuit

Perfect measurement model:
- Noise on data qubits

Phenomenological model:
- Noise on data qubits
- Noise on measurements

Circuit noise:
- Noise on data qubits
- Noise on measurements
- Noise on ancilla qubits
- Noise on gates
- Noise on waiting qubits

Local expander graphs

$$\mathrm{h}_\varepsilon(G) = \min_{\substack{L \subseteq V \\ |L| \leq \varepsilon |V|/2}} \frac{|\partial L|}{|L|},$$

A *family of α-expander graphs* is a family of graphs $(G_i)_{i\in\mathbb{N}}$ such that $h(G_i) \geq \alpha$ for all $i \in \mathbb{N}$. We consider a generalization of this notion by considering expansion over small subsets of vertices. A *family of (α, ε)-expander graphs* is a family of graphs $(G_i)_{i\in\mathbb{N}}$ such that $h_\varepsilon(G_i) \geq \alpha$ for all $i \in \mathbb{N}$.

Hypergraph Product Codes