Mesh adaptation for embedded boundary meshes and Generation and visualization of high-order meshes

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GAMMA3 and POEMS

19 March 2018
2nd year Ph.D. student seminar
GAMMA3 - MΞDISIM - POEMS

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LABEX Mathématique Hadamard.

Outline of the presentation

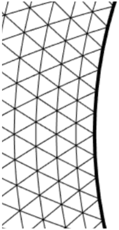
I will present my work as Ph.D. student in the GAMMA3 Project.

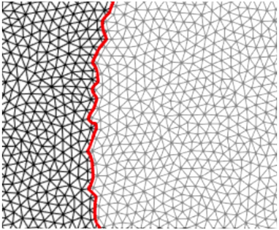
Outline

1. Mesh adaptation for embedded boundary meshes
2. High-order mesh generation
3. High-order meshes and solutions visualization
4. Perspectives
5. Proceedings

Embedded boundary meshes

- Issues with the explicit representation of an object inside a mesh.
- Embedding a geometry may be a solution.
- Application of the method in CFD for the Euler equations.
- Example of a body-fitted and embedded boundaries.

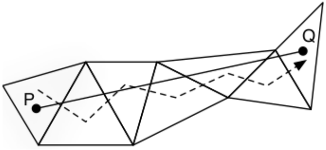




[1] Fidkowski et al. *Triangular cut-cell adaptative method for Navier-Stokes equations*, 2007
[2] Fahrat et al. *Algorithms for interface treatment and load computation in embedded boundary methods*, 2011
[3] Löhner et al. *Adaptative embedded unstructured grid methods*, 2004

Embedded boundary meshes

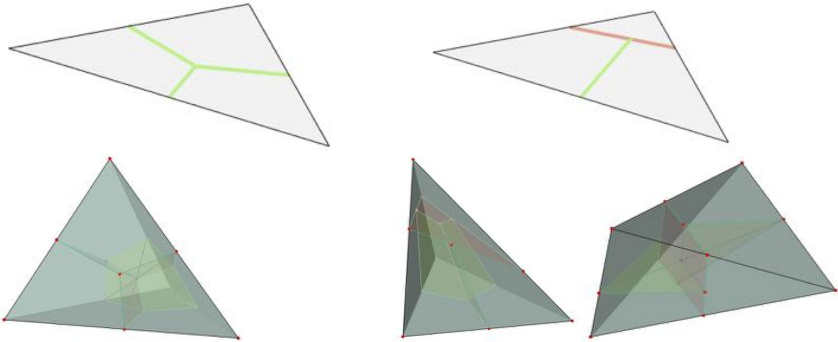
- First step: find out where the geometry intersects the mesh. Construction via path along the elements. The first considered element is one containing a vertex of the edge/triangle of the embedded geometry.



- Second step: Determine if a vertex is *covered* or not by the geometry. Choice of a first vertex known as a non covered by the geometry and then tag all vertices linked by a non intersected edge to a known non-covered vertex.

Embedded boundary meshes

Local modifications of Finite Volume median cells: Cut-cell method.

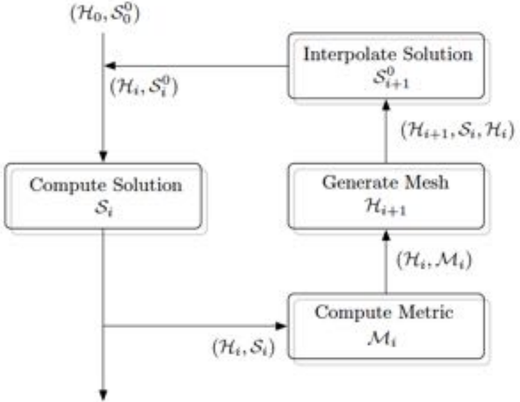


Embedded boundary meshes

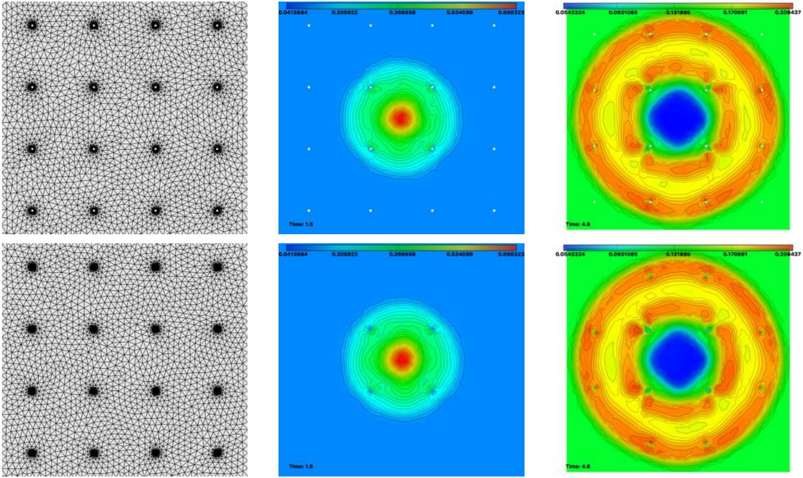
- Modifications of the flow solver.
- A slipping boundary condition is induced on intersected edges.
- For visualization and adaptation purposes, a constant value is imposed for covered vertices and computations are cancelled for these points. (No penalization)
- Local modifications for gradient computation.
- Works for implicit and explicit time integration and also for order 1 and 2 in space.
- Works as well for steady and unsteady simulations.

Embedded boundary meshes

- Coupling with mesh adaptation

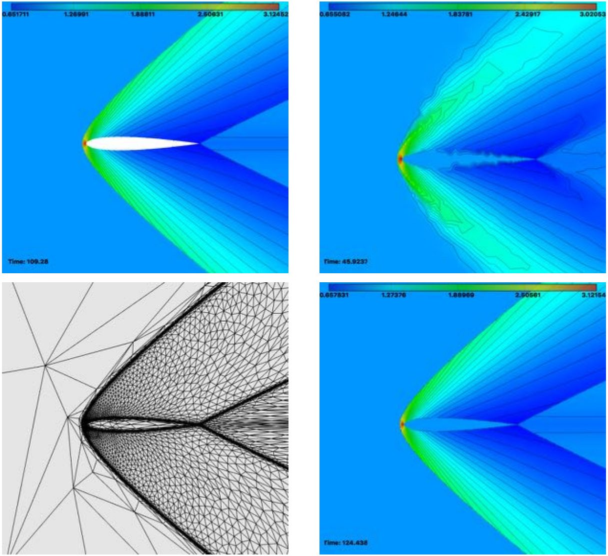


Case of an unsteady simulation.



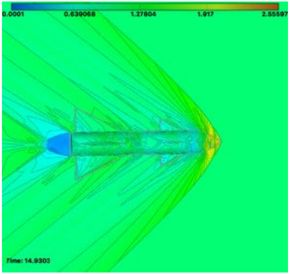
Examples

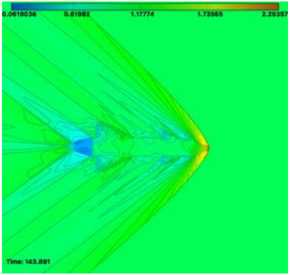
Case of an embedded supersonic NACA0012 airfoil.

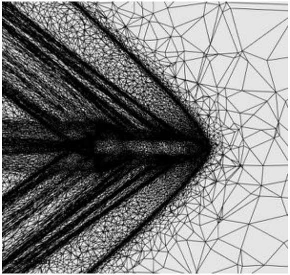


Examples

Case of an embedded supersonic missile.







High-order mesh generation

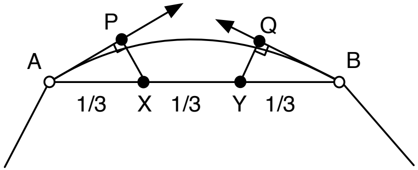
High-order mesh generation

- High-order meshes are more and more used.
- Generation of a curved mesh from an initial $\mathbb{P}_1$ straight mesh.
- Based on a high-order (of the order of the wanted curved mesh) finite element resolution of the linear elasticity equation.
- The input is the pre-curved boundary mesh, interpreted as a Dirichlet boundary condition.
- The output is a valid curved high-order mesh.

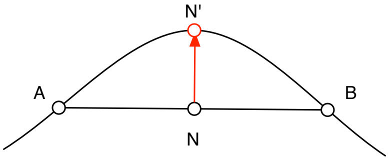
[1] Toulorge et al. *Robust untangling of curvilinear meshes*, 2016
[2] Karman et al. *High-Order Mesh Curving Using WCN Mesh Optimization*, 2016
[3] Turner et al. *A Variational Framework for High-order Mesh Generation*, 2017

High-order mesh generation

- The curvature of the boundary is either created from the $\mathbb{P}_1$ mesh or deduced from a CAD model.
- A cubic reconstruction can be deduced from a $\mathbb{P}_1$ mesh.
  The two constructed control points define a $\mathbb{P}_3$ curve.



- The distance to the curved boundary is a Dirichlet boundary condition for the linear elasticity problem.



High order FE resolution of the linear elasticity equation

- A $\mathbb{P}_k$-Lagrange shape function on a simplex and its gradient are a polynomial combination of the barycentric coordinates of this simplex.
- Exact formula for shape function and shape function gradient products can be analytically found at any order on a straight element.

$$\int_{K_d} \prod_{i=1}^{d+1} \lambda_i^{\beta_i} \mathrm{d}\Omega = \frac{\prod_{i=1}^{d+1} \beta_i!}{\left(d + \sum_{i=1}^{d+1} \beta_i\right)!} d!|K_d|, \quad \forall (\beta_i) \in \mathbb{N} \quad i \in [\![1, .., d+1]\!]$$

$$\nabla_{\mathbf{x}} \lambda_i = \frac{1}{d!|K_d|} \mathbf{n}_i$$

- Quadrature formulas are only used on non-straight elements.
- Provides a significant speed-up in the matrix assembling.

Validity and quality criteria for the HO mesh

- An element is valid if and only if its jacobian is strictly positive evreywhere inside it.
- For an element of degree d, the jacobian is of degree p ($p \geq d$). It can be written it into a Bézier form:

$$J(u, v, w) = \sum_{i=1}^{p} N_i B_i^p(u, v, w)$$

- If $J \leq 0$ for a given (u, v, w) then $min_i N_i \leq 0$. But the opposite is not necessarily true.
- A possible quality function:

$$Q = \alpha \frac{h S_k max(V_1, V_k) N_{max}^{1/n}}{V_k min(V_1, V_k) N_{min}^{1/n}}$$

- $Q = 1$ when the element is regular and is the standard quality function when the element is straight.

Validity of the HO element

- The validity of an high-order element is not that intuitive !
  The following element is valid even if the control polygon overlaps itself. (Control points are the coefficients of the mapping in the Bernstein basis.)

High-order mesh untangling

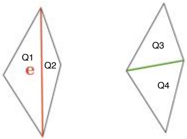
- Sometimes, the generated mesh can be invalid: we need to untangle it where needed.
- Toulorge et al. use a *log-barrier* to untangle blobs of an high-order mesh:

$$min_x F(x,\epsilon) \text{ where } F(x,\epsilon) = \sum_{e=1}^{Nbe_b} \sum_{i=1}^{p} \big[\log(\frac{N_i(x) - \epsilon J_0^e}{J_0^e(1-\epsilon)})\big]^2 + \Big(\frac{N_i(x)}{J_0^e} - 1\Big)^2$$

- J_0^e is the $\mathbb{P}_1$ jacobian of e, ϵ is chosen as $\min_{i,e} \frac{N_i}{J_0^e} - 0.01|\min_{i,e} \frac{N_i}{J_0^e}|$.
- Each term of the sum is a convex function, infinite when $N_i = \epsilon J_0^e$ and minimal when $N_i = J_0^e$.

Towards a generalization of the edge swapping (in 2D)

- $\mathbb{P}_1$ case : a swap occurs if $max(Q1, Q2) \geq max(Q3, Q4)$



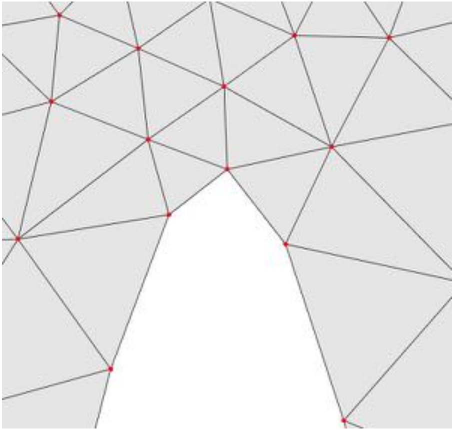
- $\mathbb{P}_2$ case : a swap occurs if $max(Q1, Q2) \geq min_x max(Q3(x), Q4(x))$.
- x is the coordinates of the node of the swapped edge that can be optimized if the cavity is curved.
- Quality function and maximum function are not smooth enough for differentiable optimization.
- Smooth maximum function:

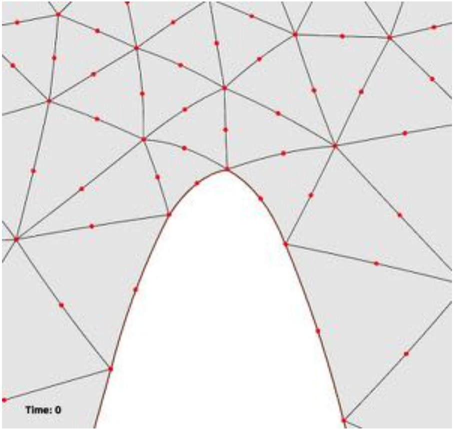
$$S_\alpha(x_1, .., x_n) = \frac{\sum_{i=1}^{n} x_i e^{\alpha x_i}}{\sum_{i=1}^{n} e^{\alpha x_i}} \text{ with } \alpha \gg 1$$

- Deduction of a smooth quality function $\tilde{Q}$.
- A swap occurs if $max(\tilde{Q}1, \tilde{Q}2) \geq min_x S_\alpha(\tilde{Q}3(x), \tilde{Q}4(x))$?

Examples

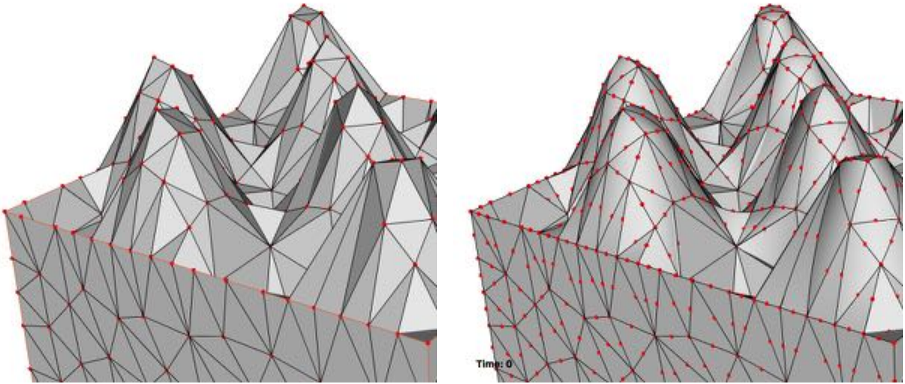
Illustration of a $\mathbb{P}_2$ curved mesh with the initial mesh in 2D.





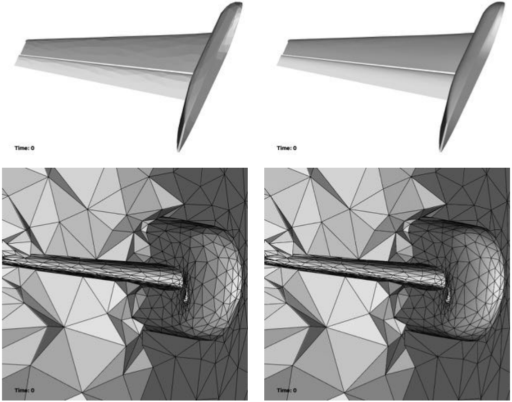
Examples

Illustration of a $\mathbb{P}_2$ curved mesh with the initial mesh in 3D.



Examples

Illustration of a $\mathbb{P}_2$ curved mesh with the initial mesh in a more complex case of a high-lift 3D. Surface and volume mesh.



High-order meshes and solutions visualization

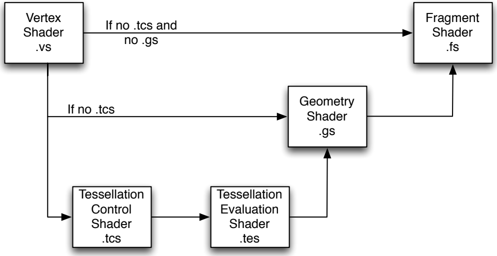
High-order meshes and solutions visualization

- A real need to visualize high-order meshes as their development is ongoing.
- High-order solvers provide high-order solutions that need to be properly displayed.
- Most of the techniques use elements subdivision (even when they are straight on CPU) to render high-order solutions.
- Other techniques use ray-tracing that may be greedy in computations.
- We provide a solution that does not subdivide straight entities at all and subdivides curved entities on the fly on GPU.

[1] Remacle et Al, *Efficient visualization of high-order finite elements*, 2006
[2] Peiro et al., *High-Order Visualization with ElVis*, 2015

Curved surface elements visualization

- The visualization process relies on the use of the OpenGL 4.0 graphic pipeline that can be customized with up to five different shader stages.



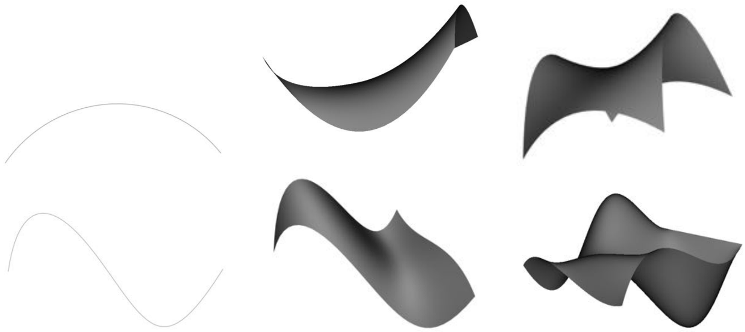
- The accuracy of the geometric approximation can be controlled thanks to a tesselation shaders.
- Among built-in variables, we can pass trough our own variables:

$\Longrightarrow$ for a pixel we then know: (x, y, z), or (u, v), or primitive ids.

- Textures are used to stored raw data (HO-Solutions).

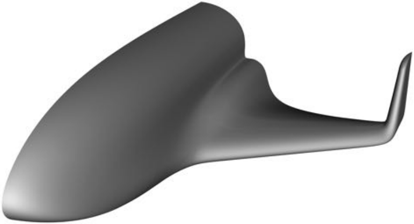
Curved surface elements visualization

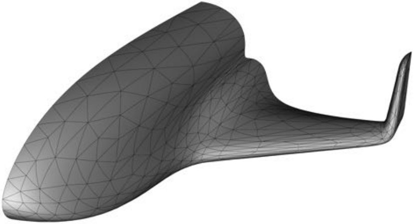
- Elements of degree 2 (up) and degree 3 (bottom) displayed with Vizir.
- From left to right: edge, triangle and quadrilateral.



Curved surface elements visualization

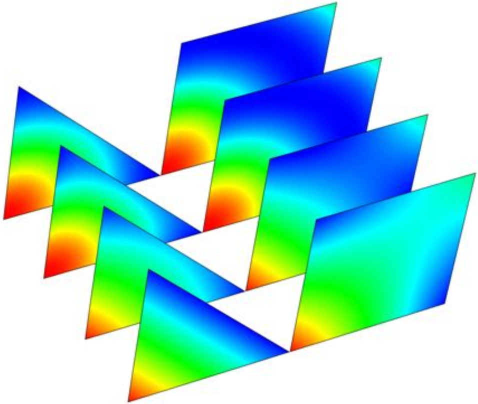
Illustration of an anisotropic $\mathbb{P}_2$ surface mesh approximating a shuttle NURBS with 2nd order elements.





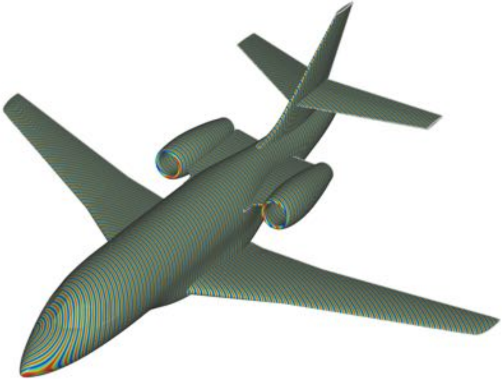
Almost pixel exact solution rendering

- Rendering of high order solutions from degree 0 to degree 4.
- Pixel exact rendering on straight elements.
- Rendering of an interpolated solution $cos(\pi(x^2 + y^2))$ on a triangle and a quadrilateral from degree 1 to degree 4.



Almost pixel exact solution rendering

- Example of 2 different solutions rendering on a Dassault Falcon $\mathbb{P}_1$ mesh with $\mathbb{P}_1$ and $\mathbb{P}_3$ solution.



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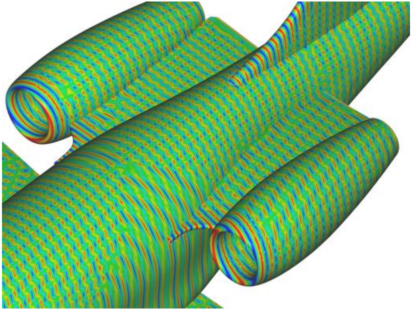

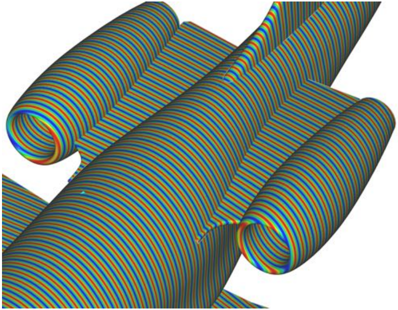

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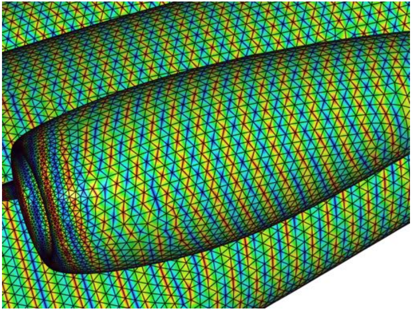

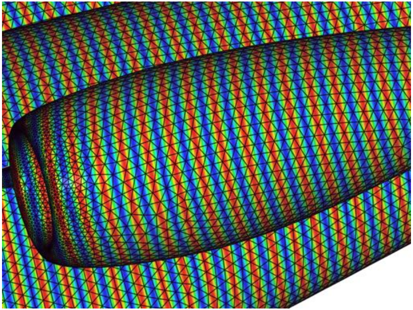

Perspectives

- Mesh adaptation for embedded geometries
  - Finish and validate the embedded mesh adaptation with fixed geometries for the Euler equations.
- High-order mesh generation
  - Reduce zone of elasticity resolution.
  - Once swap is done in 2D, generalize it in 3D and apply all to the high-order moving mesh problems.
  - Curved boundary layer mesh generation.
- High-order mesh visualization
  - Enhancement of the wireframe rendering.
  - Development of high-order cut plane and isolines rendering.

Proceedings

Proceedings

Accepted peer-reviewed proceedings in international conferences

- A. Loseille, **R. Feuillet** Vizir: High-order mesh and solution visualization using OpenGL 4.0 graphic pipeline. *56th AIAA Aerospace Sciences Meeting, AIAA Scitech*. (January 2018)

Submitted peer-reviewed proceedings in international conferences

- AIAA Aviation 2018 (accepted)
  - **R. Feuillet**, A. Loseille, F. Alauzet *High-order moving mesh techniques: Application to curved mesh and high-order curved boundary layers generation*
  - J. Vanharen, **R. Feuillet**, F. Alauzet *Mesh adaptation for fluid-structure interaction problems*

Submitted communications in international conferences

- ECCM - ECFD 2018 (accepted)
  - **R. Feuillet**, A. Loseille, F. Alauzet *Generation and motion of high-order meshes based on a high-order linear elasticity model*
  - J. Vanharen, **R. Feuillet**, F. Alauzet *Mesh adaptation for fluid-structure interaction problems*
- ICOSAHOM 2018
  - **R. Feuillet**, A. Loseille, F. Alauzet *High-order mesh operations based on a linear elasticity model*
- WCCM 2018
  - A. Loseille, **R. Feuillet** *High-order mesh visualization*