Class on numerical mechanics:

From Lagrangian mechanics to simulation tools for computer graphics

Practicals

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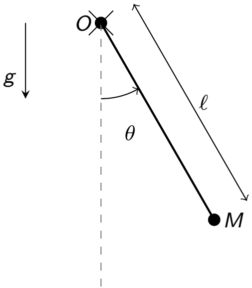
Back to the simple pendulum



Computing the dynamics

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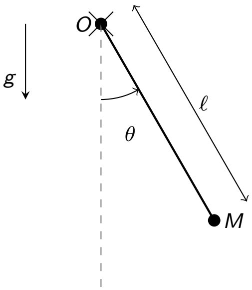

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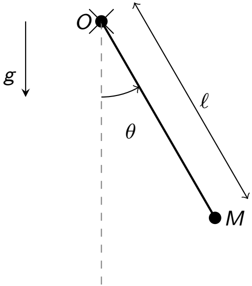

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- $\rightarrow$ Recourse to numerical integration

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- In `Python3` for the prototyping of the numerical methods + `OpenGL` for the visualization
- There is the base code to work on rods and meshes, and to render them, but feel free to modify or to add your own stuff

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The structure for a test case should enable you to easily add your own (see the files `main.py` and `scenes.py`).
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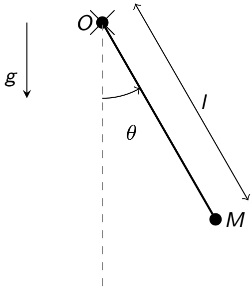
# Add the rendered object
myMeshRenderable = Mesh2DRenderable(myMesh)
viewer.addRenderable(myMeshRenderable)
```

then run `python3 main.py`.

Practical 1 - part 1:
Finite differences (explicit Euler scheme)

Exercise 1: Simple pendulum



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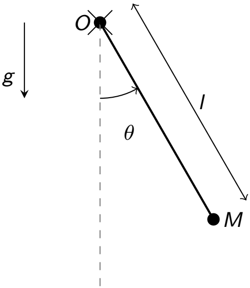

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To simulate the pendulum, you will *discretize in time* the equation of motion using the simple explicit Euler scheme.

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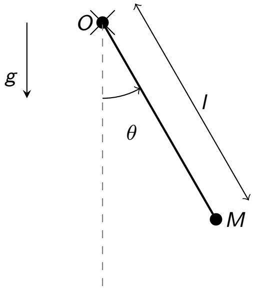

Goals of this practical:

- Get familiar with the code;
- Have a quickly working simulator, to serve as a basis for your future project;
- Study and analyse various integration schemes (if enough time).

More: analysis

More

Explicit vs. Implicit Euler

Exercise 1

Consider the linearized pendulum problem (valid for small angle θ),

$$\ddot{\theta} + \frac{g}{\ell}\theta = 0 \qquad \text{with } \theta(0) = \theta_0 \text{ and } \dot{\theta}(0) = \lambda_0,$$

and express the condition on the time step h for `Explicit Euler` to be stable.

Explicit vs. Implicit Euler

Exercise 2

Same question for `Implicit Euler`.