Chaos, stochasticity and algebraic topology : Novel methods and their applications to climate

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In this talk, the authors present results associated with the convergence between two strains of Henri Poincaré's heritage — dynamical systems theory and algebraic topology — and their joint applications to the climate sciences. Michael Ghil will present the role of the theory of nonautonomous and random dynamical systems (NDSs and RDSs) in a better understanding of the effects of the time-dependent, anthropogenic forcing on the natural, intrinsic climate variability. Bifurcations, tipping points and other surprises can play a major role in these effects (Ghil & Lucarini, 2020).

Denisse Sciamarella will present recent developments in topological data analysis (TDA) that are tailored for the study of dynamical processes. Her Branched Manifold Analysis through Homologies (BraMAH) will be explained, together with the development of the templex, a new mathematical object incorporating directed graphs (Charó et al., 2022a). The associated concepts and tools will be applied to the stochastically perturbed Lorenz convection model (Chekroun et al., 2011), highlighting the effects of noise on the branched manifolds of deterministically chaotic systems (Charó et al., 2022b). Topological tipping points will be defined and their potential role in anthropogenic climate change will be discussed (Ghil & Sciamarella, 2023).

References

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