Scheduling with Component Health Index: Complexity Study

E. Foussard    M.-L. Espinouse    G. Mounié    M. Nattaf

Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, LIG, 38000 Grenoble, France

April 2022
Who am I?

2nd year PhD student
- G-SCOP ROSP with M.-L. Espinouse and M. Nattaf
- LIG Datamove with G. Mounié: one day per week, usually Tuesday (office 428)

Research interests:
- Planning and Scheduling problems
- MILP, Constraint Programming, Heuristics and Approximations, Complexity theory...
- Application to industrial maintenance planning
General problem

Machine with $G$ components
→ Health index $H_g \in [0, 100]$
per component

Set of jobs $N$
→ Duration $p_i$
→ Wear $w_{ig}$
→ Health thresholds $s_{ig}$

Maintenance Operations $M$
→ Duration $p_m$
→ Regeneration $reg_{gm}$ on component $g$

Objectives: $C_{max}, \sum C_j...$

Figure: Example for one component without thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Hypotheses: 1 component, perfect maintenance, no thresholds
Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Greedy example

Hypotheses: 1 component, perfect maintenance, no thresholds
Scheduling with unavailability

Literature reviews: Kaabi and Harath (2014), Ma et al. (2010)

Non-resumable case:\(^1\):
- Fixed unavailabilities: Lee (1996), Sadfi et al. (2005)
- Periodic unavailabilities: Qi et al. (1997)
⇒ Temporal approach of maintenance scheduling

Main originalities
- Multi-component problem
- Time/wear decoupling ⇒ Predictive approach

\(^1\)tasks may not be interrupted by maintenance
Degradation models

Classical approaches:
- Virtual Age-based stochastic approaches: de Jonge and Scarf (2020), Moghaddam and Usher (2011)
- Rate-modifying activities: Strusevich and Rustogi (2017)

Equipment Health Index:
- Kao et al. (2018)
- Master thesis of L. Penz (Prix Master ROADEF 2021)
Reductions

Source: Pinedo (2012)
Machine with \( G \) 1 composant
\[ \rightarrow \text{Health index } H \in [0, 100] \text{ per component} \]

Set of jobs \( \mathcal{N} \)
\[ \rightarrow \text{Duration } p_i \]
\[ \rightarrow \text{Wear } w_i \]
\[ \rightarrow \text{Health thresholds } s_{ig} \]

Maintenance operations \( \mathcal{M} \)
\[ \rightarrow \text{Duration } p_m \]
\[ \rightarrow \text{Regeneration } reg_g \text{ on component } g. \text{ Perfect maintenance on the unique composant} \]
Existence of a feasible solution

No limits on the number of maintenance operations ⇒ Greedy algorithm

Bounded amount of maintenance operations ⇒ Strong sense NP-complete
3-PARTITION (Garey & Johnson 1990)
- Set $A$, $|A| = 3m$
- Size $s(a) \in \mathbb{Z}$ per object
- Bound $B$, such that $B/4 < s(a) < B/2$ and $\sum_{a \in A} s(a) = mB$.

Is there a partition $A_1 \ldots A_m$ such that for all $1 < i < m$, $\sum_{a \in A_i} s(a) = B$?

Reduction:
1 component, perfect maintenance of duration $p_m = 1$

3m tasks:
- duration $p_a = 1$
- wear $w_a = \frac{100 \cdot s(a)}{B}$

Is there a feasible schedule with at most $m - 1$ maintenance operations?
**Example**

**Instance of 3-PARTITION:**

<table>
<thead>
<tr>
<th>a</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>s(a)</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

avec $B = 50$

**Corresponding instance:**

<table>
<thead>
<tr>
<th>a</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_a$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$w_a$</td>
<td>26</td>
<td>28</td>
<td>30</td>
<td>32</td>
<td>40</td>
<td>44</td>
</tr>
</tbody>
</table>

with a single maintenance operation of duration $p_m = 1$

$A_1 = \{2, 4, 5\}$ and $A_2 = \{1, 3, 6\}$ is a valid partition.
⇒ If $A_1 \ldots A_m$ valid partition, reproduce the partition by affecting the tasks to the $m$ corresponding batches ($m - 1$ maintenance operations)

$$\forall i \in \{1 \ldots m\} \quad \sum_{a \in A_i} w_a = \frac{100}{B} \cdot \sum_{a \in A_i} s(a) = 100$$

⇐ If there exists a feasible schedule with $m - 1$ maintenance operations (thus $m$ batches).

$$\left\{ \begin{array}{l}
B/4 < s(a) < B/2 \\
\sum_{a \in A} s(a) = mB
\end{array} \right. \quad \Rightarrow \text{Batches of 3 jobs st } \sum_{a} w_a = 100$$

A valid partition can be extracted from the batches.
$C_{\text{max}}$ (#maint. unbounded)

Corresponding decision problem equivalent to the existence of feasible schedule with limited maintenance $\Rightarrow$ Strong sense NP-hard even with 1 component.

Relations with Bin-Packing $\Rightarrow$ Bin-Packing approximation algorithms have a performance ratio $1 + \rho_{BP}$ if $\sum p_i \geq p_m$

Maintenance operations minimization $\Rightarrow (2 - \epsilon)$-inapproximable
Reductions

\[ \sum_{j} C_{j} \]
\[ C_{\text{max}} \]
\[ L_{\text{max}} \]
\[ \sum_{j} w_{j} C_{j} \]
\[ \sum_{j} T_{j} \]
\[ \sum_{j} U_{j} \]

Source: Pinedo (2012)
Strong sense NP-hard even with 1 component (reduction from Qi et al. 1997).

One known polynomial sub case: constant wear $\rightarrow$ SPT optimal

Known properties on the shape of the solutions:
- SPT order within each batch
- Batches ordered by non-decreasing $\frac{p_{m}+p(K_i)}{|K_i|}$

The "hard part" is defining the batches
\[ \sum C_j \text{ (#maint. unbounded)} \]

- Number of scheduled maintenance operations not necessarily minimal \( \Rightarrow \) not the same as \( C_{\text{max}} \)

\[ \sum C_j = 13 \quad \sum C_j = 12 \]

- Exchanging two tasks may break the schedule
Reductions

Source: Pinedo (2012)
Perspectives and ongoing projects

- Branch&Bound with column generation for the $C_{max}$ problem with thresholds, study of classical BP heuristics (Gérémi Bridonneau’s internship)

- Exact methods: study of MILP formulations and valid inequalities, Constraint Programming models...

- More results on approximation algorithms and performance of online algorithms
Thanks you for your attention!
Any questions?