Ordonnancement deux-agents avec augmentation de ressources

Le Dual fitting pour un algorithme d'approximation

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Problème

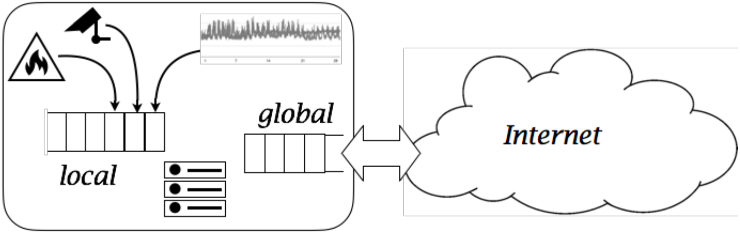


Figure: Schematic view of a smart building with multiple sensors (fire detector, motion sensor, video surveillance, sound analysis, etc.) and a link to the Internet (Cloud services, weather forecast data, information collected around in the smart city, etc.). The tasks associated to the sensors are collected in a local queue and the external tasks are put in a global queue. Both are managed by the computing units of the smart building.

Modélisation

Deux types de tâches:

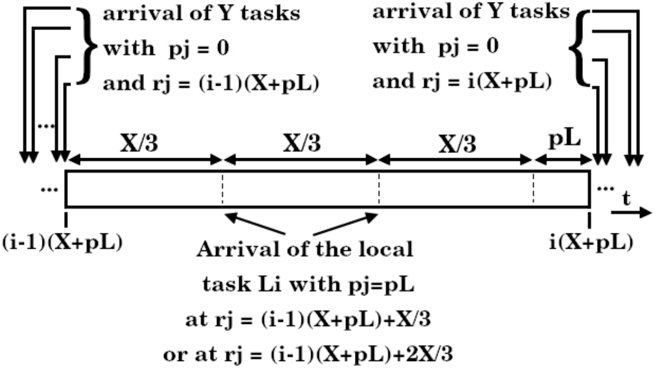
- L'agent "Local" dont les tâches sont générées par les capteurs et doivent être traitées le plus rapidement possible. (p_j connu at r_j, Online, ppmt, min$\sum$Fj)
- L'agent "Global" composé de tâches externes soumises par batch. (Offline, pas de preemption, rejection autorisée batch ulterieur, deadline commune)

ϵ_s

trivial séparément

Borne d'Inapproximation

- une tâche globale de durée G/3
- une tâche globale de durée 2G/3
- une tâche locale de durée L
- une deadline des globales = G+L
- une speed augmentation ϵ_s
- le droit de ne pas executer la tâche G/3



$O\left((1 - \frac{2k}{n^{\mathcal{G}}})\mathcal{W}\right)$

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Algorithme $\mathcal{A}$

- Si plus de $\frac{1}{\epsilon_r}$ tâches locales sont en train d'attendre qu'une tâche globale k termine son execution, on considère que le Flowtime est trop dégradé et on décide de rejeter la tâche globale k.
- SRPT pour les locales
- LIST pour les globales (garantie du respect de la deadline d^G)

Dégradations $Delta_j$

If k($\in \mathcal{G}$) is rejected:

$$\Delta_j = -\frac{p_k^{rem}(r_j)}{1+\epsilon_s} \cdot |\mathcal{Q}_i^{\mathcal{L}}(r_j) - 1| + \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} \frac{p_l^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} \frac{p_j}{1+\epsilon_s}$$

If k($\in \mathcal{G}$) is not rejected:

$$\Delta_j = \frac{p_k^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} \frac{p_l^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} \frac{p_j}{1+\epsilon_s}$$

If k($\in \mathcal{L}$) is preempted:

$$\Delta_j = \frac{p_j}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} \frac{p_j}{1+\epsilon_s}$$

If k($\in \mathcal{L}$) is not preempted:

$$\Delta_j = \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} \frac{p_l^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} \frac{p_j}{1+\epsilon_s}$$

$\sum \Delta_j$

$$\sum_{j \in \mathcal{L}} F_j^A = \sum_{j \in \mathcal{L}} \Delta_j$$

$$\leq \sum_{j \in \mathcal{L}} \left(\sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} \frac{p_l^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l \in \mathcal{Q}_i^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} \frac{p_j}{1+\epsilon_s} \right) + \sum_{j \in \mathcal{G}} \frac{p_j}{\epsilon_r(1+\epsilon_s)}$$

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Ratio competitif

Ratio d'approximation : $max_{\forall I \in Instance} \left\{ \frac{\text{Algorithm's Solution}}{\text{Optimal solution}} \right\}$

Ratio competitif : $max_{\forall I \in Instance} \left\{ \frac{\text{Online Algorithm's Solution}}{\text{Offline Optimal solution}} \right\}$

On s'interesse à : $max_{\forall I \in Instance} \left\{ \frac{\text{Online Augmented Algorithm's Solution}}{\text{Offline Optimal solution}} \right\}$

- 11 adultes et 5 enfants à la mer
- Un bateau "x" coûte 10€ et peut sauver 3 adultes et 1 enfant
- Un bateau "y" coûte 5€ peut sauver 1 adulte et 2 enfants
- On ne dispose que de 4 bateaux "x"
- Quels bateaux envoyer pour sauver tout le monde en payant le moins cher ?

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- On ne dispose que de 4 bateaux "x"
- Quels bateaux envoyer pour sauver tout le monde en payant le moins cher ?

$$
\begin{array}{rr}
 & min\ 10x + 5y \\
(adultes) & 3x + y \geq 11 \\
(enfants) & x + 2y \geq 5 \\
(nbrmax) & x \leq 4 \\
 & x, y \geq 0
\end{array}
$$

$$
\begin{array}{lr}
 & min\ 10x + 5y \\
(adultes) & 3x + y \geq 11 \\
(enfants) & x + 2y \geq 5 \\
(nbrmax) & x \leq 4 \\
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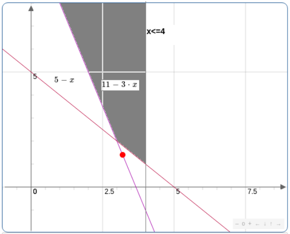
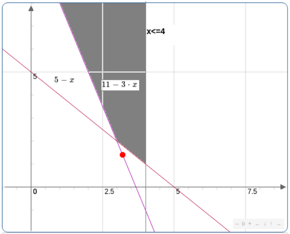
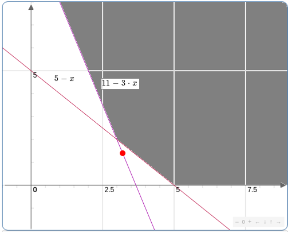
\end{array}
$$

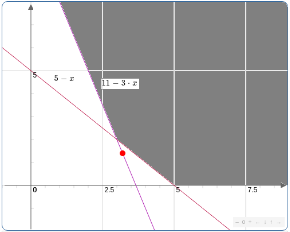


$$\begin{array}{rr} & min\ 10x + 5y \\ (adultes) & 3x + y \geq 11 \\ (enfants) & x + 2y \geq 5 \\ (nbrmax) & x \leq 4 \\ & x, y \geq 0 \end{array} \qquad \begin{array}{rr} & min\ 10x + 5y \\ (adultes) & 3x + y \geq 11 \\ (enfants) & x + 2y \geq 5 \\ \cancel{(nbrmax)} & \cancel{x \leq 4} \\ & x, y \geq 0 \end{array}$$



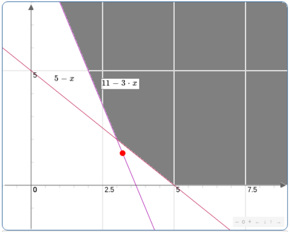


$$\begin{aligned} & \quad min\ 10x + 5y \\ (adultes) & \quad 3x + y \geq 11 \\ (enfants) & \quad x + 2y \geq 5 \\ & \quad x, y \geq 0 \end{aligned}$$



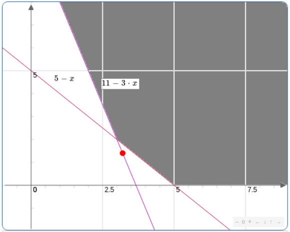
$$\begin{aligned} 10x + 5y \geq 3x + 1y \geq 11 & \quad (1a) \\ 10x + 5y \geq 6x + 2y \geq 22 & \quad (2a) \\ 10x + 5y \geq 1x + 2y \geq 05 & \quad (1e) \\ 10x + 5y \geq 2x + 4y \geq 10 & \quad (2e) \\ 10x + 5y \geq 5x + 5y \geq 21 & \quad (2e + 1a) \end{aligned}$$

$$\begin{array}{lr} & \min\ 10x + 5y \\ (adultes) & 3x + y \geq 11\ (x\ \alpha) \\ (enfants) & x + 2y \geq 5\ (x\ \beta) \\ & x, y \geq 0 \end{array}$$



$$\begin{array}{ll} & \min\ 10x + 5y \\ (adultes) & 3x + y \geq 11\ (x\ \alpha) \\ (enfants) & x + 2y \geq 5\ (x\ \beta) \\ & x, y \geq 0 \end{array}$$

$$\begin{array}{ll} & \max\ 11\alpha + 5\beta \\ (bateau\ x) & 10 \geq 3\alpha + 1\beta \\ (bateau\ y) & 5 \geq 1\alpha + 2\beta \\ & \alpha, \beta \geq 0 \end{array}$$



$$\begin{aligned} 10x + 5y &\geq (3\alpha + 1\beta)x + (\alpha + 2\beta)y \\ &= (3x + y)\alpha + (x + 2y)\beta \\ &\geq 11\alpha + 5\beta \end{aligned}$$

$$\begin{aligned} &\mathit{min}\ 10x+5y\\ &3x+y\geq 11\\ &x+2y\geq 5\\ &x\leq 4\\ &x,y\geq 0 \end{aligned} \qquad \begin{aligned} &\mathit{min}\ 10x+5y\\ &3x+y\geq 11\\ &x+2y\geq 5\\ &x,y\geq 0 \end{aligned} \qquad \begin{aligned} &\mathit{max}\ 11\alpha+5\beta\\ &10\geq 3\alpha+1\beta\\ &5\geq 1\alpha+2\beta\\ &\alpha,\beta\geq 0 \end{aligned} \qquad \begin{aligned} &11\alpha^A+5\beta^A\\ &10\geq 3\alpha^A+1\beta^A\\ &5\geq 1\alpha^A+2\beta^A\\ &\alpha^A,\beta^A\geq 0 \end{aligned}$$

$$\text{OPT(LP)} \geq \text{OPT(LP R)} \geq \text{Opt(Dual(LP R))} \geq \text{Sol(Dual(LP R))}$$

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Modelisation du Problème $\mathcal{P}$

Soit un modèle ($\mathcal{P}$) qui modélise l'optimal (pas le droit d'accélérer, pas le droit de rejeter)
$x_j(t) = 1$ if j is executed at time t, and 0 otherwise.

$$\begin{aligned}
\text{min.} \quad & \sum_{j \in \mathcal{L}} F_j & & \\
\text{s.t.} \quad & \int_{r_j}^{\infty} x_j(t)dt \geq p_j & \forall j \in (\mathcal{L} \cup \mathcal{G}) & \qquad (1) \\
& \sum_{j \in (\mathcal{L} \cup \mathcal{G})} x_j(t) \leq 1 & \forall t \geq 0 & \qquad (2) \\
& C_j \leq d^{\mathcal{G}} & \forall j \in \mathcal{G} & \qquad (3) \\
& x_j(t) \in \{0; 1\} & \forall j \in (\mathcal{L} \cup \mathcal{G}),\ \forall t \geq 0 & \\
x_j \text{ doit être "continu"} & & \forall j \in \mathcal{G} &
\end{aligned}$$

Objectif F_j

$$\begin{aligned}
&\int_{r_j}^{\infty} \left(\frac{(t - r_j)}{p_j} + \Gamma \right) x_j(t) dt \\
&= 0 + \left[\frac{t^2}{2p_j} \right]_{S_j}^{S_j + p_j} + \left[\frac{\Gamma \cdot p_j - r_j}{p_j} \cdot t \right]_{S_j}^{S_j + p_j} + 0 \\
&= \frac{(S_j + p_j)^2 - S_j^2}{2p_j} + \frac{(\Gamma \cdot p_j - r_j) p_j}{p_j} \\
&= S_j + \frac{p_j}{2} - r_j + \Gamma \cdot p_j \\
&= \text{Flowtime}_j + (\Gamma - \frac{1}{2}) p_j \leq (\Gamma + \frac{1}{2}) \text{Flowtime}_j
\end{aligned}$$

$$\text{Flowtime}_j \geq \frac{\int_{r_j}^{\infty} \left(\frac{(t - r_j)}{p_j} + \Gamma \right) x_j(t) dt}{\Gamma + \frac{1}{2}}$$

Modèle $\Gamma\mathcal{P}$

Soit le modèle $(\Gamma\mathcal{P})$: $(\Gamma + \frac{1}{2})OPT(\mathcal{P}) \geq OPT(\Gamma\mathcal{P})$

$$
\begin{aligned}
\text{min.} \quad & \sum_{j \in \mathcal{L}} \int_{r_j}^{\infty} \left(\frac{(t - r_j)}{p_j} + \Gamma \right) x_j(t) dt & & \\
\text{s.t.} \quad & \int_{r_j}^{\infty} x_j(t) dt \geq p_j & \forall j \in (\mathcal{L} \cup \mathcal{G}) & \quad (4) \\
& \sum_{j \in (\mathcal{L} \cup \mathcal{G})} x_j(t) \leq 1 & \forall t \geq 0 & \quad (5) \\
& C_j \leq d^{\mathcal{G}} & \forall j \in \mathcal{G} & \quad (6) \\
& x_j(t) \in \{0; 1\} & \forall j \in (\mathcal{L} \cup \mathcal{G}), \ \forall t \geq 0 & \\
& x_j \text{ doit être "continu"} & \forall j \in \mathcal{G} &
\end{aligned}
$$

Contrainte C_j

$$
\begin{aligned}
\int_0^\infty \left(\frac{t}{p_j} + \frac{1}{2}\right) x_j(t)dt \\
&= \left[\frac{t^2 + p_j \cdot t}{2p_j}\right]_{S_j}^{S_j+p_j} \\
&= \frac{(S_j + p_j)^2 - S_j^2 + p_j(S_j + p_j - S_j)}{2p_j} \\
&= \frac{(S_j^2 + 2S_j\dot{p}_j + p_j^2) - S_j^2 + p_j^2}{2p_j} \\
&= S_j + p_j = \text{Completion Time}_j
\end{aligned}
$$

Modèle $\Gamma\mathcal{P}$

$$\text{min.} \quad \sum_{j \in \mathcal{L}} \int_{r_j}^{\infty} \left(\frac{(t - r_j)}{p_j} + \Gamma \right) x_j(t)dt$$

$$\text{s.t.} \quad \int_{r_j}^{\infty} x_j(t)dt \geq p_j \qquad \forall j \in (\mathcal{L} \cup \mathcal{G}) \tag{7}$$

$$\sum_{j \in (\mathcal{L} \cup \mathcal{G})} x_j(t) \leq 1 \qquad \forall t \geq 0 \tag{8}$$

$$\int_0^{\infty} \left(\frac{t}{p_j} + \frac{1}{2} \right) x_j(t)dt \leq d^{\mathcal{G}} \qquad \forall j \in \mathcal{G} \tag{9}$$

$$x_j(t) \in \{0; 1\} \qquad \forall j \in (\mathcal{L} \cup \mathcal{G}), \ \forall t \geq 0$$

$$x_j \text{ doit être "continu"} \qquad \forall j \in \mathcal{G}$$

Modèle $\Gamma\mathcal{PR}$

$$\text{min.} \quad \sum_{j \in \mathcal{L}} \int_{r_j}^{\infty} \left(\frac{(t - r_j)}{p_j} + \Gamma \right) x_j(t)dt$$

$$\text{s.t.} \quad \int_{r_j}^{\infty} x_j(t)dt \geq p_j \qquad \forall j \in (\mathcal{L} \cup \mathcal{G}) \qquad (10)$$

$$\sum_{j \in (\mathcal{L} \cup \mathcal{G})} x_j(t) \leq 1 \qquad \forall t \geq 0 \qquad (11)$$

$$\int_{0}^{\infty} \left(\frac{t}{p_j} + \frac{1}{2} \right) x_j(t)dt \leq d^{\mathcal{G}} \qquad \forall j \in \mathcal{G} \qquad (12)$$

$$x_j(t) \in \{0; 1\} \qquad \forall j \in (\mathcal{L} \cup \mathcal{G}), \ \forall t \geq 0$$

~~x_j doit être "continu"~~ $\forall j \in \mathcal{G}$

$$(\Gamma + \tfrac{1}{2}) OPT(\mathcal{P}) \geq OPT(\Gamma\mathcal{P}) \geq OPT(\Gamma\mathcal{PR})$$

Split des contraintes

$$\text{s.t.} \quad \int_{r_j}^{\infty} x_j(t)dt \geq \ p_j \qquad \forall j \in (\mathcal{L} \cup \mathcal{G})$$

$$\int_{r_j}^{d^{\mathcal{G}} + \sum_{j \in \mathcal{L}} p_j} x_j(t)dt \geq p_j \forall j \in \mathcal{L} \tag{13a}$$

$$\int_{0}^{d^{\mathcal{G}}} x_j(t)dt \geq p_j \forall j \in \mathcal{G} \tag{13b}$$

Split des contraintes

$$\sum_{j\in(\mathcal{L}\cup\mathcal{G})} x_j(t) \leq 1 \qquad \forall t \geq 0$$

$$\sum_{j\in(\mathcal{G}\cup(\mathcal{L} \text{ with } r_j\leq t))} x_j(t) \leq 1 \qquad 0 \leq t \leq d^{\mathcal{G}} \tag{14a}$$

$$\sum_{j\in\mathcal{L}} x_j(t) \leq 1 \qquad d^{\mathcal{G}} \leq t \leq d^{\mathcal{G}} + \sum_{j\in\mathcal{L}} p_j \tag{14b}$$

Dual

$$
\begin{aligned}
\text{max.} \quad & \sum_{j\in\mathcal{L}\cup\mathcal{G}} \alpha_j p_j - \int_0^{\infty} \beta(t)dt - \sum_{j\in\mathcal{G}} d^G \gamma_j \\
\text{s.t.} \quad & \alpha_j - \beta(t) - \left(\frac{t-r_j}{p_j} + \Gamma\right) \leq 0 && \forall j \in \mathcal{L},\ \forall t \geq r_j && (15)\\
& \alpha_j - \beta(t) - \left(\frac{t}{p_j} + \frac{1}{2}\right)\gamma_j \leq 0 && \forall j \in \mathcal{G},\ \forall t \geq 0 && (16)\\
& \alpha_j \geq 0 && \forall j \in (\mathcal{L}\cup\mathcal{G}) \\
& \beta(t) \geq 0 && \forall t \geq 0 \\
& \gamma_j \geq 0 && \forall j \in \mathcal{G}
\end{aligned}
$$

Dual Fitting

$$
\begin{aligned}
&(\Gamma + \frac{1}{2}) OPT(\mathcal{P}) && (17)\\
&\geq OPT(\Gamma\mathcal{P}) && (18)\\
&\geq OPT(\Gamma\mathcal{P}\mathcal{R}) && (19)\\
&\geq OPT(Dual(\Gamma\mathcal{P}\mathcal{R})) && (20)\\
&\geq Solution^{H}(Dual(\Gamma\mathcal{P}\mathcal{R})) && (21)
\end{aligned}
$$

Dual Fitting

$$
\begin{aligned}
(\Gamma + \frac{1}{2}) OPT(\mathcal{P}) & \qquad (22)\\
\geq OPT(\Gamma\mathcal{P}) & \qquad (23)\\
\geq OPT(\Gamma\mathcal{P}\mathcal{R}) & \qquad (24)\\
\geq OPT(Dual(\Gamma\mathcal{P}\mathcal{R})) & \qquad (25)\\
\geq Solution^{H}(Dual(\Gamma\mathcal{P}\mathcal{R})) & \qquad (26)\\
\geq \textbf{QuelqueChose} \sum F_j^{\mathcal{A}} & \qquad (27)
\end{aligned}
$$

$$
\frac{(\Gamma + \frac{1}{2})}{\textbf{QuelqueChose}} \geq \frac{\sum F_j^{\mathcal{A}}}{OPT(\mathcal{P})} \qquad (28)
$$

Solution H

$$\alpha_{j\in\mathcal{L}} = \frac{1}{p_j} \cdot \left(\sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} p_l^{rem}(r_j) + \sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} p_j \right) + \Gamma$$

$$\alpha_{j\in\mathcal{G}} = 0$$

$$\gamma_{j\in\mathcal{G}} = 0$$

$$\beta(t) = \|\mathcal{Q}^{\mathcal{L}}(t)\|$$

Contrainte Globale

$$\alpha_{j\in\mathcal{L}} = \frac{1}{p_j}\cdot\left(\sum_{\substack{l\in\mathcal{Q}^{\mathcal{L}}(r_j):\\ p_l^{rem}(r_j)\leq p_j}} p_l^{rem}(r_j) + \sum_{\substack{l\in\mathcal{Q}^{\mathcal{L}}(r_j):\\ p_l^{rem}(r_j)>p_j}} p_j\right) + \Gamma$$

$$\alpha_{j\in\mathcal{G}} = 0 \quad \gamma_{j\in\mathcal{G}} = 0 \quad \beta(t) = \|\mathcal{Q}^{\mathcal{L}}(t)\|$$

Lemma

For every global task j and every time t, $0 \leq t \leq d^{\mathcal{G}}$, the dual constraint (16) is satisfied, that is:

$$\alpha_j - \beta(t) - (\frac{t}{p_j} + \frac{1}{2})\gamma_j \leq 0$$

Contrainte Locale

$$\alpha_{j\in\mathcal{L}} = \frac{1}{p_j} \cdot \left(\sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) \leq p_j}} p_l^{rem}(r_j) + \sum_{\substack{l \in \mathcal{Q}^{\mathcal{L}}(r_j): \\ p_l^{rem}(r_j) > p_j}} p_j \right) + \Gamma$$

$$\alpha_{j\in\mathcal{G}} = 0 \quad \gamma_{j\in\mathcal{G}} = 0 \quad \beta(t) = \|\mathcal{Q}^{\mathcal{L}}(t)\|$$

Lemma

For every local task $j \in \mathcal{L}$ and every time t, $r_j \leq t \leq d^{\mathcal{G}} + \sum_{j\in\mathcal{L}} p_j$, the dual constraint (15) is satisfied, that is:

$$\alpha_j - \beta(t) - (\frac{t - r_j}{p_j} + \Gamma) \leq 0$$

Objectif

Lemma

Given our definitions of α_j, $\beta(t)$, γ_j, and Γ the dual objective value verifies:

$$\sum_{j\in\mathcal{L}\cup\mathcal{G}} \alpha_j p_j - \int_0^\infty \beta(t)dt - \sum_{j\in\mathcal{G}} d^{\mathcal{G}}\gamma_j \geq \epsilon_s \sum_{j\in\mathcal{L}} F_j^{\mathcal{A}}$$

$$OBJ = \sum_{j\in\mathcal{L}} \alpha_j p_j - \sum_{\mathcal{L}} F_j^{\mathcal{A}}$$

On rappelle :

$$\sum_{j\in\mathcal{L}} F_j^A \leq \sum_{j\in\mathcal{L}} \left(\sum_{\substack{l\in\mathcal{Q}_i^{\mathcal{L}}(r_j):\\ p_l^{rem}(r_j)\leq p_j}} \frac{p_l^{rem}(r_j)}{1+\epsilon_s} + \sum_{\substack{l\in\mathcal{Q}_i^{\mathcal{L}}(r_j):\\ p_l^{rem}(r_j)> p_j}} \frac{p_j}{1+\epsilon_s} \right) + \sum_{j\in\mathcal{G}} \frac{p_j}{\epsilon_r(1+\epsilon_s)}$$

$$\begin{aligned}
(1+\epsilon_s)\sum_{j\in\mathcal{L}} F_j^A &\leq \sum_{j\in\mathcal{L}} (\ldots) + \sum_{j\in\mathcal{G}} \frac{p_j}{\epsilon_r} \\
(1+\epsilon_s)\sum_{j\in\mathcal{L}} F_j^A - \sum_{j\in\mathcal{G}} \frac{p_j}{\epsilon_r} + \Gamma\sum_{j\in\mathcal{L}} p_j &\leq \sum_{j\in\mathcal{L}} (\ldots) + \Gamma\sum_{j\in\mathcal{L}} p_j \\
&= \sum_{j\in\mathcal{L}} \alpha_j p_j
\end{aligned}$$

$$
\begin{aligned}
OBJ &= \sum_{j\in\mathcal{L}} \alpha_j p_j - \sum_{\mathcal{L}} F_j^{\mathcal{A}} \\
&\geq (1+\epsilon_s) \sum_{j\in\mathcal{L}} F_j^{A} - \sum_{j\in\mathcal{G}} \frac{p_j}{\epsilon_r} + \Gamma \sum_{j\in\mathcal{L}} p_j - \sum_{\mathcal{L}} F_j^{\mathcal{A}} \\
&= \epsilon_s \sum_{j\in\mathcal{L}} F_j^{A} - \sum_{j\in\mathcal{G}} \frac{p_j}{\epsilon_r} + \Gamma \sum_{j\in\mathcal{L}} p_j \\
&= \epsilon_s \sum_{\mathcal{L}} F_j^{\mathcal{A}} \text{ si on choisit } \Gamma = \frac{W}{\epsilon_r}
\end{aligned}
$$

Competitive Ratio

$$
\begin{aligned}
&(\Gamma + \frac{1}{2}) OPT(\mathcal{P}) && (29)\\
&\geq OPT(\Gamma\mathcal{P}) && (30)\\
&\geq OPT(\Gamma\mathcal{P}\mathcal{R}) && (31)\\
&\geq OPT(Dual(\Gamma\mathcal{P}\mathcal{R})) && (32)\\
&\geq Solution^{H}(Dual(\Gamma\mathcal{P}\mathcal{R})) && (33)\\
&\geq \epsilon_s \sum F_j^{\mathcal{A}} && (34)
\end{aligned}
$$

$$
\left(\frac{W}{\epsilon_s \cdot \epsilon_r} + \frac{1}{2\epsilon_s}\right) \geq \frac{\sum F_j^{\mathcal{A}}}{OPT(\mathcal{P})} \quad (35)
$$