Sufficient conditions for a planar digraph to be 2-dicolourable

Internship proposal

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General context : A $k$-colouring of a graph is a partition of its vertex into stable sets (i.e. sets of pairwise non-adjacent vertices). A graph is $k$-colourable if it admits a $k$-colouring, and the chromatic number $\chi(G)$ of a graph $G$ is the minimum $k$ such that $G$ is $k$-colourable. The chromatic number is certainly the most studied parameter in graph theory and many deep results and theories have been developed around this concept. One of the prominent results in this field is the famous Four-Colour Theorem which asserts that every planar graph is 4-colourable. It is NP-complete to decide whether a planar graph is 3-colourable [4], and a vast literature has been devoted to establish sufficient conditions for a planar graph to be 3-colourable For example, Grötzsch’s Theorem states that every planar graphs with no cycle of length 3 is 3-colourable.

For a few years, more and more results are showing that the right concept to generalize chromatic number to directed graphs (a.k.a. digraphs) is the so-called notion of dichromatic number. A $k$-dicolouring of a graph is a partition of its vertices into sets inducing acyclic subdigraphs. A digraph is $k$-dicolourable if it admits a $k$-dicolouring, and the dichromatic number $\vec{\chi}(D)$ of a digraph $D$ is the minimum $k$ such that $D$ is $k$-colourable. Clearly, if $G$ is the underlying graph of $D$ ($uv$ is an edge in $G$ if $(u,v)$ is an arc in $D$ or $(v,u)$ is an arc in $D$) then $\vec{\chi}(D) \leq \chi(G)$, and there is equality if $D$ is a symmetric digraph (if $(u,v)$ is an arc then $(v,u)$ is an arc). In particular, the Four-Colour Theorem implies that every planar digraph is 4-dicolourable. It is NP-complete to decide whether a planar digraph is 2-dicolourable [1]. It is then natural to ask for sufficient conditions for a planar digraph to be 2-dicolourable. In this vein, Neumann-Lara conjectured that every planar digraph with no directed cycle of length 2 is 2-dicolourable. Li and Mohar [2] proved that every planar digraph with no directed cycle of length 3 is 2-dicolourable.

Objectives : The aim of the internship is to find new sufficient conditions for planar digraph to be 2-dicolourable. More precisely, the aim is to identify families of digraphs $\mathcal{F}$ such that every planar digraph with no induced subdigraph in $\mathcal{F}$ is 2-dicolourable. A particular attention will be devoted to families of cycle digraphs which are digraphs that are obtained from an undirected cycle by replacing each edge by either one arc or two opposite arcs.

Références