The Barát-Thomassen Conjecture

Julien Bensmail, Ararat Harutyunyan, Tien-Nam Le, Martin Merker, Stéphan Thomassé

Université Nice-Sophia-Antipolis, France

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Introduction
Decomposing graphs

$G$: (undirected simple) graph.

$H$: (undirected simple) graph with $|E(H)|$ dividing $|E(G)|$ (implicit).

Definition: $H$-decomposition

An $H$-decomposition of $G$ is a partition $E_1, \ldots, E_k$ of $E(G)$ such that each $G[E_i]$ is isomorphic to $H$. 

$S_4$-decomposition

$P_3$-decomposition

When does $G$ admit $H$-decompositions?
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![S₄-decomposition](image)

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![P₃-decomposition](image)

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When does $G$ admit $H$-decompositions?
What for $H$ being a tree?
Tree decompositions

What for $H$ being a tree?

**Theorem [Wilson, 1976]**

For every tree $T$ and large enough $n$, graph $K_n$ admits $T$-decompositions.

$\Rightarrow$ Intuitively, need large degree + some edge-connectivity (2nd $\Rightarrow$ 1st).
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$\Rightarrow$ Intuitively, need large degree + some edge-connectivity (2nd $\Rightarrow$ 1st).

For instance, no $P_3$-decomposition of:
The Barát-Thomassen Conjecture

**Conjecture [Barát, Thomassen, 2006]**

For every tree $T$, there exists $k_T$ such that every $k_T$-edge-connected graph admits $T$-decompositions.

**General remark:**

Large edge-co. $\not\implies H$-decompositions (e.g. $H = C_4$: need close cut edges)
Progress towards the conjecture

Was verified for $T$ being:

- a star [Thomassen, 2012],
- the tree with degree sequence $(1, 1, 1, 2, 3)$ [Barát, Gerbner, 2014],
- a bistar of the form $S_{k,k+1}$ [Thomassen, 2014],
- of diameter at most 4 [Merker, 2017],
- among some family of trees with diameter 5 [Merker, 2017],

and...

- the path of length 3 [Thomassen, 2008],
- the path of length 4 [Thomassen, 2008],
- a path of length $2^k$ [Thomassen, 2014],
- any path [Botler, Mota, Oshiro, Wakabayashi, 2017].
Main result

Theorem [B., Harutyunyan, Le, Merker, Thomassé, 2017]

The Barát-Thomassen Conjecture is true.

Please: Do not ask me about $k_T$. 😊
Proof
Say hello

Our toy $T$ for today:
Going bipartite

First tool:

**Theorem [Thomassen, 2013]**

It is sufficient to prove the conjecture for $G$ bipartite.

**Idea:** Take a max cut and “clean”.

$\text{edge-co: } \sim k_T/2$
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edge-co: $\sim k_T/2 \rightarrow$

\[ |E(T)| \text{ fixed} \Rightarrow \text{constant amount of consumed edge-connectivity.} \]
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![Diagram of bipartite graph elements](image)
Going bipartite

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It is sufficient to prove the conjecture for $G$ bipartite.

**Idea:** Take a max cut and “clean”.

$|E(T)|$ fixed $\Rightarrow$ constant amount of consumed edge-connectivity.

$\Rightarrow$ Use $R +$ cut-edges to make further copies of $T$. 

$$\text{edge-co: } \sim k_T/2 \rightarrow$$
Going bipartite (cont’d)

Theorem [Thomassen, 2013]

It is sufficient to prove the conjecture for $G = (A, B)$ bipartite, with the further assumption that all degrees in $A$ are divisible by $|E(T)|$.

**Idea.** Decompose $G$ into $G_1, G_2$ with large edge-connectivity, where the desired property in $G_1$ (resp. $G_2$) is fulfilled in $A$ (resp. $B$).
Going bipartite (cont’d)

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2. $\Rightarrow$ Decompose $G$ into $G_1, G_2, G_3$ with large edge-connectivity.
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Want: $A$-degrees in $G_1$ divisible by $|E(T)|$, $B$-degrees in $G_2$ divisible by $|E(T)|$. 
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Want: A-degrees in $G_1$ divisible by $|E(T)|$, B-degrees in $G_2$ divisible by $|E(T)|$.

3. Orient $G_3$ so that the convenient degrees modulo $|E(T)|$ are attained (i.e. $|E(T)| - d_{G_1}(v)$ for $v \in G_1$, and $|E(T)| - d_{G_2}(v)$ otherwise).
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4. Add all arcs from $A$ to $B$ to $G_1$, to $G_2$ otherwise.
Decomposition strategy

$G, T$ bipartite $\Rightarrow$ Make the bipartitions coincide:

1. Edge-colour $G$ with $\{\ldots\}$;
2. Repeatedly combine $\ldots$ to form a copy of $T$.

Problems:
1. # of $\ldots$ should locally be the same.
2. We do not necessarily get a copy isomorphic to $T$. 

12 / 26
Decomposition strategy

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Strategy:

1. Edge-colour \( G \) with \{ / , / , / , / , / , / \};
2. Repeatedly combine a / , a / , a / , a / and a / to form a copy of \( T \).
Decomposition strategy

$G, T$ bipartite $\Rightarrow$ Make the bipartitions coincide:

![Graph 1](image1)
![Graph 2](image2)

**Strategy:**
1. Edge-colour $G$ with \{/, /, /, /, /, /, /\};
2. Repeatedly combine a /, a /, a /, a / and a / to form a copy of $T$.

**Problems 😞:**
1. # of /'s, /'s, /'s, /'s and /'s should locally be the same.
2. We do not necessarily get a copy **isomorphic** to $T$:

![Graph 3](image3)
Dealing with Issue 1

\[ \nu \in V(G) \text{ and } t \in V(T) \text{ compatible} = \text{Same side of the bipartitions.} \]
Dealing with Issue 1

\( v \in V(G) \) and \( t \in V(T) \) **compatible** = Same side of the bipartitions.

To deal with Issue 1:

**Definition: \( T \)-equitability**

An edge-colouring \( E(G) \rightarrow E(T) \) is **\( T \)-equitable** if, for every compatible vertices \( v \in G \) and \( t \in T \), we have \( d_i(v) = d_j(v) \) for any two edges \( i, j \) of \( T \) incident to \( t \).
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What will save us:

**Theorem [Merker, 2017]**

If \( G = (A, B) \) is a bipartite graph with
- sufficiently large edge-connectivity, and
- all degrees in \( A \) are divisible by \( |E(T)| \),

\( \Rightarrow \) \( T \)-equitable edge-colouring where all coloured degrees are “huge”.

\( \Rightarrow \) May assume \( G \) is edge-coloured in a \( T \)-equitable way.
Locally, “palettes” of colours are good, now 😊.

Construct copies of $T$:

1. For each $v \in G$ that can play the role of $t \in T$:
   - choose one edge of each colour;
   - create a star centred at $v$.

2. Identify stars to create copies.
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Dealing with Issue 2

Remember: \( + + + + + + \) may not give a “real” copy of \( T \):

![Diagram](image-url)
Remember: \( + / + / + / + / + / \) may not give a “real” copy of \( T \):

\[ \text{Collection } \mathcal{H} := \mathcal{G} \cup \mathcal{B}, \text{ where } \mathcal{G} \text{ (resp. } \mathcal{B}) \text{ contains “real” (resp. “bad”) copies.} \]

\( \mathcal{G} \) will be used to “repair” \( \mathcal{B} \).
Let $B \in \mathcal{B}$, and label vertices following a BFS.

In $B$, vertices $v_1, \ldots, v_5$ are good. Edge $v_4v_6$ is problematic.
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**Repairing process:**

1. Pick $R \in \mathcal{G}$ s.t. $B$ and $R$ intersect only intersect in $v_4$; and
2. “Switch” the subgraph “rooted” at the edge $v_4v_6$. 
Illustration
On the repairing operation

Schematized:

\[ B \]

\[ v_4 \rightarrow v_5 \rightarrow v_6 \]

\[ R \]

\[ v_4' \]

Remarks:

B and R might be bad (because of later vertices) / ... but their first six vertices are good.

Whole repairing strategy:

1. Repair all bad copies where the edge simulating \( t_1 \) is problematic;
2. Then, those where the edge simulating \( t_1 \) is problematic;
3. etc.

\(|G| \gg |B|\) (+ intersection property) \( \Rightarrow \) Repair everything.
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Schematized:

Remarks:

- \( B \) and \( R \) might be bad (because of later vertices) ☹ ...
- ... but their first six vertices are good ☺ .

Whole repairing strategy:

1. Repair all bad copies where the edge simulating \( t_1 t_2 \) is problematic;
2. Then, those where the edge simulating \( t_1 t_3 \) is problematic;
3. etc.
On the repairing operation

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- $B$ and $R$ might be bad (because of later vertices) 😞 ...
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**Assumption:** $G$ is edge-coloured in a $T$-equitable way + Large coloured degrees.
Proof summary

**Assumption:** $G$ is edge-coloured in a $T$-equitable way $+$ Large coloured degrees.

**Main steps:**
1. Combine edges in $G$ to get a decomposition $\mathcal{H} := G \cup B$.
2. Repair bad copies in $B$ step by step, until none remains.
Proof summary

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1. Combine edges in $G$ to get a decomposition $\mathcal{H} := G \cup B$.
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So that Step 2 can be achieved, we need $\mathcal{H}$ to fulfil:
- $|G| \gg |B|$;
- for compatible $v \in V(G)$ and $t \in V(T)$, a wide bunch of copies where $v$ plays the role of $t$, most of which are good, with many different vertices of $G$. 
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⇒ Because

1) $|E(T)|$ is fixed, and
2) the coloured degrees are arbitrarily large,

such an $\mathcal{H}$ exists with non-zero probability.
Probabilistic tools
Building a decomposition

Construct copies of $T$ randomly:

1. For each $v \in G$ that can play the role of $t \in T$:
   - choose one edge of each colour;
   - create a star centred at $v$.

2. Identify stars to create copies.
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McDiarmid’s result

Random variables involved:

\[ X_v(t_i, t_j) := \# \text{ of bad copies with root } v, \text{ and } t_i, t_j \text{ played by a same vertex}. \]

⇒ Expect such \( X_v(t_i, t_j) \)’s to be quite small (due to the degrees):
McDiarmid’s result

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(simplified) McDiarmid’s Inequality

Let \( X \) be a non-negative random variable, determined by \( m \) independent random permutations \( \Pi_1, \ldots, \Pi_m \) satisfying, for some \( d, r > 0 \):

1. interchanging two elements in any \( \Pi_i \) can affect \( X \) by at most \( d \);
2. for any \( s \), if \( X \geq s \) then there is a set of at most \( r s \) choices whose outcomes certify that \( X \geq s \).

Then, for any \( 0 \leq t \leq \mathbb{E}[X] \),

\[ \mathbb{P} \left[ |X - \mathbb{E}[X]| > t + 60d \sqrt{r \mathbb{E}[X]} \right] \leq 4e^{-\frac{t^2}{8d^2 r \mathbb{E}[X]}}. \]
McDiarmid’s result (cont’d)

Our random building is all about permutations:

Building stars at \( v \) (w.r.t. \( t \)) = Permute the /’s, \( /'s \) and \( /'/s \) at \( v \), and combine.
McDiarmid’s result (cont’d)

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Look at McDiarmid’s requirements, for $X_v(t_i, t_j)$:

1. *interchanging two elements in any $\Pi_i$ can affect $X_v(t_i, t_j)$ by at most $d$;*
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McDiarmid’s result (cont’d)

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2. \( X_v(t_i, t_j) \geq s \) \textit{ can be certified by the outcomes of at most } rs \textit{ choices.}
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   ⇒ $v_i = v_j$ can be attested by the outcomes where $v_j$ was chosen. So $r = 1$. 
McDiarmid’s result (cont’d)

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*McDiarmid’s Inequality applies* \[ \Rightarrow \text{There are $\Pi_i$’s for which } |G| \gg |B|. \]
McDiarmid’s result (cont’d)

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```
1 2 3 4 5 6 7
1 2 7 5 3 6 4
5 1 7 3 6 2 4
```

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+ For compatible \( v \in G \) and \( t \in T \), unlikely that two copies where \( v \) plays the role of \( t \) have another common vertex (similar reasoning).
Lovász’s Local Lemma

We have:
- Any $X_v(t_i, t_j)$ is most likely to be quite small;
- Few dependencies between the $X_v(t_i, t_j)$’s.
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$\Rightarrow$ By LLL, non-zero probability that all $X_v(t_i, t_j)$’s are small.

+ Similar arguments for intersections.
Conclusion
Conclusion and perspectives

- Constructive proof?
Conclusion and perspectives

- Constructive proof?
- What is the least $k_T$ guaranteeing $T$-decompositions?
Conclusion and perspectives

- Constructive proof?

- What is the least $k_T$ guaranteeing $T$-decompositions?

- Real importance of huge edge-connectivity over huge degree?

- For $T = P_\ell$, we proved that 24-edge-connectivity and huge degree suffice.

Conjecture [B., Harutyunyan, Le, Thomassé, 2016+]

There is a function $f$ such that, for any fixed tree $T$ with maximum degree $\Delta_T$, every $f(\Delta_T)$-edge-connected graph with sufficiently large minimum degree can be $T$-decomposed.
Conclusion and perspectives

- Constructive proof?
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Thanks!