Graphs and AlgorIthms Applied to TelecommunicatiOns
GAIATO

Universidade Federal do Ceará – COATI Project, I3S Laboratory (UMR 7271)

Description of the partners

Project COATI, I3S (CNRS & UNS) and INRIA

http://team.inria.fr/coati

COATI (formerly MASCOTTE) is a joint team between INRIA Sophia Antipolis Méditerranée and the laboratory I3S (Informatique Signaux et Systèmes de Sophia Antipolis) which itself belongs to CNRS (Centre National de la Recherche Scientifique) and UNS (University of Nice Sophia Antipolis). Its research fields are Algorithmics, Discrete Mathematics, Combinatorial Optimization and Simulation, with applications to telecommunication networks. The objectives of the COATI project-team are to design networks and communication algorithms. In order to meet these objectives, the team studies various theoretical tools, such as Discrete Mathematics, Graph Theory, or Algorithmics and develops applied techniques and tools, especially for Combinatorial Optimization. In particular COATI used in the last year both these theoretical and applied tools for the design of various networks, such as WDM, wireless (radio), satellite, overlay, and peer-to-peer networks. This research has been done within various industrial and international collaborations.

ParGO and Combinatorics group, Federal University of Ceará

http://www.lia.ufc.br/∼pargo/
http://www.mat.ufc.br/portal2/en/research/research-areas/15-fixed-articles/38-combina

The Federal University of Ceará is the most important and well-known university in Ceará. Two well-connected research groups of this university will be involved in this programme.

The team ParGO (Parallelism, Graphs and Optimisation) of the Artificial Intelligence Laboratory at the Federal University of Ceará has been created a dozen years ago. Its activity are related to solving problems in Discrete Mathematics with techniques from combinatorial algorithmics and integer linear programming. It has an important expertise in sequential, parallel and distributed algorithms, as well as in graph theory and combinatorial optimisation. The probems studied by ParGO have applications to wireless networks and to parallelizing multicore shared memory architectures. The researchers of team ParGO are strongly involved in the master and doctoral programme.

Recently, (in 2011), a research group in combinatorics has been created at the Department of Mathematics of the Federal University of Ceará. (http://www.mat.ufc.br/) Two assistant professors were hired, as well as PhD students and postdoctoral fellows, and the group is meant to grow is the near future. This group is mainly interested in graph theoretical problems. It collaborates deeply with the team Pargo.

History of the collaboration between the teams

The collaboration between the two teams originated in the nineties when Ricardo Correa accomplished his PhD under the supervision of Afonso Ferreira, a former member of Mascotte, and Claudia Linhares-Sales was in the same laboratory. Since then, contact was maintained. For a decade, the collaboration has been stepping up. From 2002 to 2008, members of COATI regularly visited the Federal University of Ceará (in average three two-weeks visits per year) while the visits of the brasilians researchers have been less regular but of longer term. Ricardo Correa spent in Sophia-Antipolis 2 months in 2003 and 3 months in 2006, Claudia Linhares-Sales the whole academic year 2006/2007. Since 2007, we decided adjoin to this visits student exchange between the two partners. Long term visits of french PhD students have been made in Fortaleza and more importantly, many brasilian students went to Mascotte. First Napoleão Nepomuceno did his PhD thesis (defend in December 2010) in cosupervision. Several students (L. Sampaio, J. Araujo, R. Soares, A.-K. Maia made a long term visit (between two and four months) in Mascotte while preparing their master's thesis. They all pursued in attending PhD in Mascotte: two of them are co-supervised and the two others are funded by a Capes grant. This student exchange is still very active: two master students, namely C. Carvallho and R. Dantas, will make a three month internship in Mascotte from mid-november 2013 to mid-february 2014.

The members of the two teams are now used to work with each other and share a common interest for the same kind of research problems.

Our collaboration has been partly funded by an Equipe Associée INRIA (EWIN 2009-2011) and an INRIA-FUNCAP programme (ALERTE 2011-2013). More importantly, this collaboration is primordial for the two teams because they are in opposite situations with regards to hiring students. On the one hand, the Master's programme of the Department of Computer Science in Fortaleza trains many very good students, but has very few PhD grants to offer, because their PhD programme just started. On the other hand, COATI project has difficulties to find good students. Therefore, our collaboration, and in particular students exchange and cosupervision, helps ParGO to build its PhD programme and COATI to face the (hopefully temporary) lack of good french PhD candidates in theoretical computer science.

Note that we consider our team as a part of the french-brasilian collaboration in combinatorics. Therefore, we organized the First Brasilian-French Workshop on Graphs, Combinatorial Optimization and Applications in Redonda, Ceará, Brasil, in May 2012. We are planning to organize a second one in 2015.

1 Scientific description of the collaboration

Our collaboration aims at designing algorithms to solve efficiently some networks problems. Lots of these problems can be naturally modelled as graph-theoretical or combinatorial problems. Because realistic problems are very complex, we first study simplified models. These models have then to be more and more refined so as to be closer and closer to reality. Our work can be decomposed into three tasks.

Theoretical results. The goal of this task is to obtain algorithmically meaningful theoretical results. They could be of several kinds.

- *Structural results*, such as decomposition theorems or intrinsic properties of graphs in a given class are often precisous guides for algorithms design.
- *Combinatorial bounds* allow evaluating the quality of partial solutions. In particular, they enable to cut branches in branching algorithms and to establish approximation ratio for

approximate algorithms.

- *Complexity results* steer algorithm design. Knowing if the problem is NP-complete, approximable or not determine the type of algorithms we will design.

Algorithms design and analysis. This task is the core of the project. However it relies on the above task and benefits from the experimentation feedback. Several techniques are envisaged, but they must be thought as a way of identifying differents aspects of a same problem in order to obtain an efficient mixed algorithm.

Implementation and experimentation. This task aims at testing the practical efficiency of the algorithms we design in order to compare them with existing algorithms. We are paying a particular attention to the reproductibility and our experimentations, in particular by providing free access to our algorithms and data sets. PARGO and COATI have experienced developing common tools for implementation and experimentation. In particular, we developed a common interface to the two libraries Mascopt (http://www-sop.inria.fr/mascotte/mascopt) and Parego. Mascopt is a JAVA library developped by Mascotte (now COATI) and distributed under LGPL license. It is dedicated to optimisation in graphs and networks. Parego is a graph library developped by PARGO. It is dedicated to representation of set relations. Nowadays, COATI is developing a new library of graph algorithms (http://grph.inria.fr/).

In the last five years, we mainly focused on optimisation and dynamic routing in wireless networks and on various channel assignment problems related to graph colouring. For the next few years, we plan to continue to work on some these problems, but also would like to work on some others. Forthwidth we detail the topics on which we will focus.

1.1 Graph colouring and allocation problems

We study various problems of channel assignment that may be modelled as graph colouring problems. For each problem, the final aim is to design the most efficient algorithm possible. Most of these problems are NP-hard. So we may not expect to have polynomial-time algorithms to solve them. Hence we will try several approaches which may be seen as different compromises between the quality of the solution and the (theoretical) running time of the solutions: approximation algorithms, exact exponential-time algorithms, fixed parameterized algorithms, heuristics, ...

In the last few years, we worked on a problem, posed by Alcatel Space Technologies, about sending information from a satellite to receivers on earth [37, 47, 29, 20].

Another problem models channel assignment in radio networks. A usual constraint is a minimum separation between frequencies which depends on the distance between the transmitters (vertices) to which the are assigned. A first model use the concept of $L(p_1, \dots, p_k)$-labelling, which is positive integer-valued function f such that $|f(x) - f(y)| \geq p_i$ if the vertices x and y are at distance at most i in the graph. This model is the particular case (for powers of graphs) of a more general one in which the edge set of a graph is partitionned into k sets $E_1, \dots, E_k$. The aim is then to find a $(p_1, \dots, p_k)$-backbone colouring that is a function f such that $|f(x) - f(y)| \geq p_i$ if the edge xy is in E_i. We will give bounds and design algorithm for finding $L(p_1, \dots, p_k)$-labelling or $(p_1, \dots, p_k)$-backbone colouring of graphs. We are particularly interested in classes of graphs that arise in real-life, like planar graphs, disk graphs or line graphs. We gave a series of NP-hardness results as well as upper bounds for (p_1, p_2)-backbone colouring for planar graphs, when the set E_2 induces various types of forests (matching, galaxy, spanning tree, ...) [13, 51, 52]. There are still many open problems and conjectures in the area.

Another problem related to shared resources allocation is the Weighted Colouring Problem [64]. A *weighted graph* (G, w) consists of a graph $G = (V, E)$ and a weight function $w : V \to \mathbb{R}_+$ over the vertices of G. The weight of a colour is the maximum weight of a node with this colour, and the weight of a colouring is the sum of the weights of its colours. The *weighted chromatic number* of (G, w) is the minimum weight of a proper colouring of (G, w). For instance, this problem has been defined to study practical applications such as the Dynamic Storage Allocation Problem and the Distributed Dual Bus Network Media Access Control Protocol, which is a standard IEEE802.6 for metropolitan networks [64]. In an on-going work, we study the long-standing open problem of the complexity of this Weighted Colouring in trees [1]. We propose a new reduction technique to prove that no polynomial-time algorithm can solve this problem unless the Exponential Time Hypothesis fails. We aim at applying our technique for similar problems. In particular, to the problem of minimizing the maximum sum of contiguous subsequence that has been studied by some members of ParGO group for its applications to buffer minimisation in radio networks [50]. There are also many open problems and conjectures related to these problems.

In addition, time is unidirectional, past events cannot be reversed and the future is uncertain. So on-line algorithms must be designed. In addition, such algorithms form an elegant framework for analyzing algorithm with incomplete information or incomplete access to the input. Hence we will study on-line algorithms. We obtained many results [35, 43, 28, 18, 19, 3, 42] for classical proper colouring. We will now focus on colourings arising from the above-mentionned applications.

1.2 Dectecting faults in multiprocessor networks and identifying codes in graphs

Identifying problems by tests are very common in various domains such as pattern recognition, group test in biology, failure detection, localization, or key searching in databases [62, 65, 68, 71] We are more specifically interested in a problem that models fault diagnosis in multiprocessor systems. In these systems, it may happen that some of the processors become faulty, in a way that depends on the purpose of the system. We wish to detect and replace such processors, so that the system can work properly. We assume that our hardware is of such a quality that, at any time, at most k of the processors of the system are faulty, where k is a fixed constant. Let us assume that each processor p of the system is able to run a procedure `test(p)`, which checks its own state as well as the state of its neighbouring processors $N(p)$. This procedure returns only binary information; e.g. 0 if p or a processor of its neighborhood $N(p)$ is faulty, and 1 otherwise. This information is returned to a central controller, which is not considered to be part of the system. Note that the procedure does not reveal the identity of the faulty processor: If `test`(p) outputs 0, then all we can say is that p and/or some of its surrounding processors in $N(p)$ is faulty. We wish to devise a subset of processors C such that:

(1) If all the processors of C return 1, then none of the processors of the network is faulty.

(2) If at least one, but at most k of the processors are malfunctioning, then the central controller is able to locate them using C.

Formally, we model the multiprocessor system by a graph $G = (V, E)$, whose vertices are processors and whose edges are links between these processors. For a vertex $v \in V$, the *closed neighbourhood*, denoted $N[v]$, is $N(v) \cup \{v\}$. Let $C \subseteq V$ be a subset of vertices of G, and for all subsets of at most k vertices $X \subseteq V$, let us denote $I(X, C) = \bigcup_{x \in X} N[x] \cap C$. If all the $I(X, C)$'s are distinct, then we say that C *separates* the sets of at most k vertices of G, and that C is a *k-identifying code*.

Not every graph has a k-identifying code. In fact, a graph admits a k-identifying code if and only if for every pair of subsets $X \neq Y$, $|X|, |Y| \leq k$, we have $N[X] \neq N[Y]$, where $N[S]$ denotes $\bigcup_{s \in S} N[s]$. In the case, where G admits a k-identifying code, then $C = V$ is alaways a k-identifying code. We are usually interested in finding a k-identifying code of minimum cardinality, or when the graph is infinite, of minimum density.

In a recent work [49], we used the discharging method (which we intensively used and develop in our work on graph colouring) to obtain optimal 1-identifying code in the square grid of height 3. The use of the discharging method for finding and proving (near) optimality of identifying codes seems very promising. In a first stage, we will focus try use it for k-identifying codes in infinite square grids with bounded height. It is in particular the subject of R. Dantas' internship (Mid november 2013- Mid February 2014). In a second stage, we plan to study many other families of graphs. Hopefully, we would be able to find a general method, possibly compter-assisted, to find optimal identifying codes, for large classes of graphs.

1.3 Capacity planning of reliable fixed broadband wireless networks

Fixed point-to-point wireless communications is a particular sector of the communication industry that holds great promise for delivering private high-speed data connections by means of microwave radio transmission [56, 69]. Microwave, in the context of this research, refers to terrestrial fixed point-to-point digital radio communications, usually employing highly directional antennas in clear line-of-sight and operating in licensed frequency bands from 6 GHz to 38 GHz. Historically, microwave was mainly used by incumbent network operators to carry trunk telephony traffic, and by broadcasters to link remote broadcast transmitters to studios. Today, demand is driven by the infrastructure requirements of mobile networks, where microwave is used to provide interconnectivity between base stations, controllers, and switches [60].

Thanks to the ability for microwave links to be rapidly and cost-effectively deployed, fixed point-to-point wireless networks have become a common alternative to provide broadband communications. Despite recent advances in fixed point-to-point wireless communications, a variety of questions remain unaddressed in this area. Particularly, capacity planning in wireless networks is quite different from wired network planning. In fact, the radio frequency spectrum is a limited natural resource which has been regulated worldwide to promote its efficient use. Moreover, environment conditions, such as weather, play an important role in wireless communications since they can introduce instantaneous variations into the communication channel, likely leading to outage events.

Our research team has consistently investigated on network optimization problems related to the design and configuration of wireless microwave backhaul [32] and, particularly, on the problem of conceiving reliable fixed point-to-point wireless networks under outage probability constraints [22, 23, 7, 8]. In [22], we introduce a joint model of data routing and bandwidth assignment that minimizes the total renewal fees of licenses. We present chance-constrained mathematical formulations and their ILP counterparts, and we study the price of reliability. In [23], we extend our earlier study of planning fixed broadband wireless networks under unreliable channel conditions and generalize two classes of cutset-based valid inequalities. In [7], we present a budget constrained formulation to the problem. To improve the solving performance, we propose new valid inequalities and a primal heuristic. The outperformance of the novel model compared to more traditional approaches is documented. Our computational results evaluate the performance of the valid inequalities as well as the primal heuristic. Finally, we present a reliability analysis of fixed point-to-point wireless networks based on different budgets. In [8], to reduce the need for overprovisioning bandwidth during network planning, we propose a model for which routing decisions are made according to channel conditions. Besides, we propose a column generation approach to handle larger problems.

Much of this work has been done in collaboration with the SME 3Roam [54], University of Nice-Sophia Antipolis and Federal University of Ceará (Currently one of the researchers is working at University of Fortaleza.). There is still much room for research in this emerging area. As future work, we intend to model this problem as a tri-level two-player game, where the network operator decides the bandwidth assignment, then external random factors cause the deterioration of the performance of some links, and finally, the network operator attempts to find a feasible flow over the residual capacity of the network. In fact, bandwidth assignment and network flow decisions take place in different time and, therefore, we can hopefully save bandwidth utilization allowing dynamic routing. In addition, to improve the reliability of the network, we envisage a study on the impact of traffic fluctuations.

1.4 Graph searching

Pursuit-evasion games involve a team of mobile agents that aims at capturing a set of escaping agents that hide in a network modelled by a graph. These are natural games arising from a wide range of applications, from the practical ones such as search and rescue (e.g., rescuing a speleologist in a cave, [58, 59], surveillance, monitoring, military strategy to trajectory tracking, etc. to the handling of abstract mathematical and theoretical computer science concepts (e.g., Graph Minor Theory by Robertson and Seymour [73]). Hence, Pursuit-evasion games have been studied by various divergent disciplines, e.g., graph theory, differential games, robotics, control theory, geometric algorithms, etc.

There are many variants of Pursuit-evasion games studied in the literature. During his Ph.D. thesis, Ronan P. Soares studied the so-called graph searching games for their application to routing in communication networks [14, 12].

We aim at studying the cops and robber games where a set of cops (controlled by one player) tries to capture the robber (controlled by the opposing player) [57]. The cops and the robber are restricted to the vertices of a graph and they move each round to neighbouring vertices. The smallest number of cops needed to capture the robber is the *cop number* $c(G)$ of the graph G. Such a simple-sounding game leads to a complex theory with many results and beautiful conjectures.

An interesting result, due to Aigner and Fromme [55], is the fact that, in every planar graph, the cop number is at most 3. Moreover, Quilliot [72] proved that, in every graph with genus g, the cop number is at most $2g+3$ (the genus of a planar graph is 0) and Shroeder improved this to $3g/2+3$ [75]. The main conjecture in this topic is that the cop number is at most $c(G) \leq g+3$ [75].

Another interesting conjecture, due to Meyniel [63], states that, if G has n vertices, then the cop number is $c(G) = O(\sqrt{n})$. That is, for n sufficiently large, there is a constant $d > 0$ such that $c(G) \leq d\sqrt{n}$. The best upper bound known so far is that $c(G) = O(\frac{n}{2^{\sqrt{\log n}}})$ in any n-node graph G [74]. This illustrates how far we are from proving the Meyniel's conjecture, even the so-called *soft Meyniel's conjecture*, which states that, for a fixed constant $d > 0$, $c(n) = O(n^{1-d})$.

Finally the question of approximating the cop-number of a graph is interesting. It is known that the cop-number problem is NP-hard and that no polynomial-time algorithm with approximation ratio $O(\log n)$ exists unless $P = NP$ [53]. The problem is even known to be PSPACE-hard [70]. However, it is not known whether the cop-number can be approximated up to a ratio $O(n^{1-\varepsilon})$ for some $\varepsilon > 0$.

The PhD student Nicolas Martins, from Universidade Federal do Ceará, is studying these problems. Nicolas Nisse (COATI) is also involved in the study of these games.

1.5 Directed graphs

Significant advances in algorithms on graphs (as in various other combinatorial settings) seem to be closely allied with deep combinatorial structure theory. Hence one of our goal is to obtain new results in graph theory, which will hopefully be of algorithmic use.

Depending on the application, the modelling gives a problem on undirected graphs or directed graphs (digraphs). However, for various reasons, undirected graphs have been studied much more extensively than directed graphs. It seems that the main reason is that undirected graphs form, in a sense, a special class of directed graphs (symmetric digraphs) and hence problems that can be formulated for both directed and undirected graphs are often easier for the latter. While the theory of undirected graphs can be considered now as a developed field of mathematics consisting of several deep areas of its own, the theory of digraphs is still very much an emerging area of research. Our goal is to develop digraph theory and to make progress on some long-standing conjectures and problems.

One of these conjectures is Burr's conjecture which states that every digraph with chromatic number $2k-2$ contains (as a subdigraph) every oriented tree of order k. This conjecture is very interesting for us, because new adavances on this topic could possibly bring more insight on both directed graphs and graph colouring. In [9], we gave some new results towards this conjecture. We are still to pursuing our work on this conjecture. In particular, the forthcoming internship of C. Carvallho (mid november 2013, mid february 2014) will focus on this topic.

Another important problem is the one, called F-Subdivision, of deciding if a digraph D contains a subdivision (not necessarily induced) of a fixed digraph F. Contrary to the undirected case, where the problem is always polynomial-time solvable by Robertson and Seymour minor theory, we found [5] many digraphs F for which the problem is NP-complete. On the other hand, we also give polynomial-time algorithms to solve the problem for many others digraphs. This leads us to conjecturing that there is a sharp dichotomy between NP-complete and polynomial-time solvable instances: according to this conjecture, there are only two kinds of digraphs F: the *hard* ones, for which F-Subdivision is NP-complete, and the *tractable* ones, for which it is polynomial-time solvable. However there is no very clear picture of which graphs should be tractable and which ones should be hard, although some conjectures give some outline. Motivated by directed treewidth and a conjecture of Johnson et al. [66], Seymour posed the conjecture that F-Subdivision is polynomial-time solvable when F is a planar digraph with no big vertices. Bang-Jensen et al. [5] proposed the following sort of counterpart: *F-Subdivision is NP-complete for every non-planar digraph.* We would like to attack the preceding three conjectures and better understand what makes a digraph F tractable or hard. A first step would be to determine the complexity of F-Subdivision for several classes of digraphs. Many graph classes are of interest oriented trees, oriented cycles. To attack the conjecture of Bang-Jensen et al., looking at F-subdivision when F is any orientation of $K_{3,3}$ is a natural question.

2 Previsional calendar of the exchange

We plan to have six visits of members of COATI to Fortaleza each year. We anticipate that all these missions will last two weeks. However, the mission of PhD students might last longer (up to three months) in particular if a cosupervised PhD student is hired.

The expected visits for the next three years are the following:

2014 : One visit of F. Giroire, one of F. Havet, one of A. Kodjo, two of A.-K. Maia and one of N. Nisse.

2015 : One visit of J.-C. Bermond, one of D. Coudert, one of F. Havet, one of N. Nisse, and two of PhD students (to be hired).

2014 : One visit of D. Coudert, one of F. Giroire, one of F. Havet, one of N. Nisse, and two of PhD students (to be hired).

The expected visits of brasilian researchers to COATI is the following.

2014 : One two-week visit of M. Campelo and one two-week visit of R. Sampaio.

2015 : One two-week visit of F. Benevides, one two-week visit of N. Nepomuceno, and one three-month intern of a master student.

2014 : One two-week visit of V. Campos, one two-week visit of C. Linhares Sales, one two-week visit of A. S. Silva and one three-month intern of a master student.

In addition to these visits, Pargo intend to organize the second French-Brasilian Workshop on Graphs and Combinatorial Optimisation in 2015.

3 Common PhD students

Student preceded by a $\star$ were co-supervised by a member of Pargo and a member of Coati; students preceded by a + obtained their master at the Federal University of Ceará and were awarded a brasilian PHD grant from CAPES to attend PhD in Coati/Mascotte.

+ A. K. Maia. *Finding a subdivision of a digraph*, since September 2011.

$\star$ R. Soares. *Routing reconfiguration in WDM networks*, since November 2010

+ L. Sampaio. *Algorithmic aspects of graph colouring heuristics.* PhD thesis, University of Nice-Sophia Antipolis, November 2012. Now assistant professor at Universidade Estadual do Ceará.

$\star$ J. Araujo. Graph Coloring and Graph Convexity. PhD thesis, University of Nice-Sophia Antipolis and Federal University of Ceará, September 2012. Now post-doc in COATI project.

$\star$ N. Nepomuceno. *Network optimization for wireless microwave backhaul.* PhD thesis Universit de Nice-Sophia Antipolis, 2010. Now assistant professor at University of Fortaleza.

Common publications (since 2007)

[1] J. Araújo, N. Nisse and S. Pérennes. Weighted Coloring in Trees. *Submitted*, INRIA research report, RR-8249, 2013.

[2] F. Havet and A. K. Maia. On disjoint directed cycles with prescribed minimum lengths. *Submitted*, INRIA research report, RR-8286, April 2013.

[3] F. Havet, A.-K. Maia, and M.-L. Yu. Complexity of greedy edge colouring. *Submitted*, INRIA research report, RR-8171, December 2012.

[4] N. Nisse and R. Soares. On The Monotonicity of Process Number. *Submitted*, INRIA research report RR-7003, October 2012.

[5] J. Bang-Jensen, F. Havet, and A. K. Maia. Finding a subdivision of a digraph. *Submitted*, INRIA research report RR-8024, July 2012.

[6] V. Campos and F. Havet. 5-choosability of graphs with 2 crossings. *submitted*, INRIA research report RR-7618, 2011.

[7] G. Claßen, A. M. C. A. Koster, D. Coudert, and N. Nepomuceno. Chance-constrained optimization of reliable fixed broadband wireless networks. *Submitted*.

[8] B. Jaumard, N. Nepomuceno, D. Coudert, and A. Kodjo. Dimensioning of microwave wireless networks. *In preparation*.

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