A 2 Horizontal Dimension numerical scheme for the new generalized BBM-BBM equations on unstructured meshes.

Description of the internship:

Boussinesq-type models are asymptotic depth averaged models which lead to numerical models that have practical use, especially in the coastal community, since they are computationally less expensive than those produced by more complex mathematical models. Recently in [1], a generalized version of *abcd*-Boussinesq type (BT) models for variable mild bottom topography were developed and benchmarked in 1 Horizontal Dimension (HD). The advantage of these models is that they have better properties on the boundaries than the classical ones [4,6] and that suitable energy functionals are conserved in some cases. The numerical results, obtained using classical benchamrk tests, showed that they are not affected by the assumptions on the topography. We would like to confirm that the assumptions made for the topography are not overly restrictive and that applications of the new systems , in more than one dimension, can include a wide range of scenarios, including the study of tsunamis, tidal waves and waves in harbors and lakes.

The objective of this internship is firstly to implement the new BBM-BBM system [1] in 2 HD and secondly to compare its numerical results with the existing ones of the new Nwogu system [5] and the classical Nwogu's [2, 4], on standard academic benchmark test cases. The idea is to re-write the mathematical model in a way were a hybrid strategy can be applied. More precisely, following the literature, we propose a two step solution. In the first step the non-hydrostatic terms are recovered by inverting an elliptic operator. In the second step, the hyperbolic shallow water equations with the source term computed in the previous step, are solved. The above strategy will be implemented in a Finite Volume framework on unstructured meshes. More precisely the student will have to extend the code of an existing implementation of a model close to *abcd*- BT models. Finally the student will test the resulting code using standard benchmark cases [2] .

Skills

- Programming experience.
- A Course on numerical analysis and/or hyperbolic PDEs.
- Interest on applied mathematics and scientific computing.
- Experience on Finite Volume schemes is valued but not required.

References:

1. Israwi, Samer, Khalifeh, Yousseff and Mitsotakis, Dimitrios, Equations for small amplitude shallow water waves over small bathymetric variations, 2023, https://arxiv.org/abs/2303.11556
2. M. Kazolea and A.I. Delis, I.K. Nikolos and C.E. Synolakis, "An unstructured finite volume numerical scheme for extended Boussinesq-type equations", *Coastal Engineering*, 69,42-66, 2012
4. O. Nwogu, An alternative form of the Boussinesq equations for nearshore wave propagation, *J. Waterw. Port Coast. Ocean Eng*. 119 (1994) 618–638
5. M. Kazolea and D. Mitsotakis," A new Nwogu system with moving topography in 2 horizontal dimensions", In preparation.
6. M. Kazolea, & M. Ricchiuto, (2023). Full nonlinearity in weakly dispersive Boussinesq models: luxury or necessity?, *Journal of Hydraulic Engineering,* DOI: 10.1061/JHEND8/HYENG-13718

Contact/supervisor: Maria Kazolea maria.kazolea@inria.fr , https://mskazolea.wixsite.com/personal