High order boundary conforming adaptive meshing, and boundary conditions (PhD Thesis – AIRBUS/INRIA)

Context

Safer and cheaper aircrafts require new concepts, and increasingly complex geometrical, physical, and numerical modeling. These new models must integrate multi-physical interactions between aerodynamics, propulsion, structures and materials. The systematic verification and validation of these models allows to integrate CFD in the certification process, reducing our reliance on costly experimental testing.

It is with this goal in mind that AIRBUS, DLR, and ONERA have launched a new common CFD software project, called CODA. This new CFD platform, designed for efficient parallel and heterogeneous architectures, permits the integration of advanced interoperable CFD components, including in particular Finite Volume (FV) as well as Finite Element (FE) methods. DG schemes are novel finite element techniques providing a natural framework for high-orders of accuracy associated with good numerical properties (low dissipation and dispersion) on general meshes, enabling control over the computational errors, which is an essential ingredient of the digital certification process. For both FV and DG approaches, CODA provides an extension towards high-orders of accuracy in the form of the consideration of approximating polynomial degrees greater than 1 for DG and high-order k-exact formulations for FV. Both classes of schemes have demonstrated a strong potential for aerodynamic problems of applied interest [BAS16, ANT17, PON17].

Contrary to low order FV schemes commonly used in the industry, DG and k-exact FV schemes provide high-order of accuracy on unstructured meshes, furthermore, high-order approximations offer a flexible framework in which the spatial resolution can be conveniently adapted, not only by local mesh modification (h-adaptive), but also by locally varying the degree p of the polynomial reconstruction. Local p-adaptation can reduce dissipation and dispersion errors in regions where the solution is smooth thus allowing for the accurate resolution of flow phenomena with a lower number of degrees of freedom (DOFs) as compared to standard FV methods. Regarding DG methods, thanks to their compactness, they can also easily handle complex geometries and irregular meshes with hanging nodes, which simplifies the implementation of h-adaptive techniques [HAR02, KUR16].

An important issue in aircraft design concerns the accuracy and efficiency of predictive numerical tools for cruise conditions. The potential of adaptive high order methods in providing error reduction, as well as automated error control and related CPU time savings in flow simulations is nowadays well established [WAN13]. There is however an essential element which must absolutely be taken into account to tap into this potential : the proper treatment of boundary conditions. This involves several independent aspects. The first is the availability of an appropriate high order geometrical representation of the boundaries. Although several questions are open in this respect, in this project this is somehow expected to be available from the design process under the form of appropriate CAD descriptions, with underlying spline approximation (cf. below). A second necessary aspect is the ability to produce a discrete approximation of such geometry with accuracy compatible with the high order approximation used to discretize the flow equations. The last one is a numerical approximation of the boundary condition itself. This approximation must be equally compatible with the error levels of the discretization scheme.

The availability of a high quality curved mesh is a necessity to obtain the desired accuracy, and is still one of the bottlenecks to allow the adoption of higher order techniques as operational tools in an industrial environment [WAN13]. For realistic applications involving complex curved 3D geometries (e.g. ONERA M6 wing, of full wing-bod models as the CRM or XRF1), especially in the transonic regime, the impact the geometrical accuracy on the correct prediction of shock structures and boundary layer separation -may be enormous. A lot of progress has been made in recent years on different techniques allowing to obtain with an acceptable level of automation good quality curved meshes. These techniques involve either curving straight faced meshes [DEY01, LUO04, SAH10,ABG14,FOR16, MOX16], or the use of some optimization or variational approach [GAR15,TOU13,TUR16]. An important aspect is the ability of combining the above techniques with metric based mesh adaptation techniques built from error indicators extracted directly from the flow solver. These techniques have been verified for and applied to wide families of problems [GAL20,ALA21], and seem particularly fit for complex geometries and industrial applications.

Concerning the approximation of the boundary condition itself, when dealing with finite elements, the most classical approach is to work with an iso-parametric approximation in which the geometry as well as the flow solution are approximated by some high order polynomial [ZIE71]. Standard approaches range from the use of various maps based on some local interpolated or modal polynomial approximation of the curved geometry, to the more recent use of rational B-spline or NURBS approximations used in the so called iso-geometric analysis (IGA) in which the polynomial spaces describing the geometry are also used to expand the flow variables [HUG05]. These polynomials can be integrated by resorting to the appropriate transformation to a reference integration domain. Other ideas have been proposed to account for the curvature of the geometry, based on appropriately designed corrections of the boundary fluxes allowing to compensate for possible geometrical errors even on straight faced meshes [KRI06,CIA23]. Hybrid approaches can be imagined, allowing to relax the constraints on the accuracy of the curved mesh, with corrections for fine scale errors.

Initial work on metric based hp adaptation techniques within CODA for DG discretizations on unstructured meshes has been successfully applied to a range of flow configurations and models, including 2D and 3D detached turbulent and laminar flows using Navier-Stokes, RANS and ZDES model equations [BAS21, BAS22a, BAS22b, BAL21].

Objectives

The objective of this PhD is to develop adaptive techniques compatible with high order boundary approximations. The aim is to be able to efficiently reduce the error associated to the approximation of the boundary conditions, via some reliable technique to obtain curvilinear meshes, and with some efficient numerical integration of the boundary conditions, while still being able to exploit metric based h-adaptive methods. The primary objective is the improvement of simulations of flows around curved shapes, allowing to obtain optimal accuracy, while minimizing the associated computational cost. To do this, this project will pursue the following scientific objectives :

- Develop a robust method to generate high order (at least quadratic) surface meshes as well as high quality volume meshes conformal with the curved surface mesh

- Propose some metric based h-adaptation method compatible with the above curved mesh generation processes

- Assess and extend existing boundary integration methodologies in CODA. We will evaluate hybrid approaches in which correction terms to a Pk solution are included to account for geometrical errors on a Pm mesh. The objective is to seek the combination of degrees k and m providing the lowest error at a given cost

- Verification and validation of the above techniques on state of the art benchmarks, as e.g. those proposed within the International Workshops on High Order CFD Methods (HiOCFD), and on aircraft configurations with particular emphasis on the prediction of

    - supercritical wings with strong trailing edge aerodynamic loads, as those used in the PhD by G. Sporschill [SPO21], plus some confidential Airbus configurations;

    - high-lift configurations both clean, and including complex ice accretion shapes.

The curved mesh technological bricks will be developed within the Flowsimulator environment of Airbus in order to be used in the h-adaptive process initiated during the PhD of F.Basile.

Organisation des travaux de recherche

[M1-M6] CODA software development project immersion at AIRBUS. Learn and train on CODA software and its environment, namely development environment (IDE, debugger), pre/post-processing tools, mesh generation, AIRBUS HPC environment. Set-up and simulate simple to industrial relevant test cases. Become autonomous with regards to the CODA software development process by developing and integrating a RANS turbulent wall law.

[M1-M6] Thorough literature review of PDE based and optimization based mesh curving technologies, prospection on their implementation in– Flowsimulator, and selection of target methods to implement.

[M1-M12] Evaluate existing boundary integration strategies for high-order DG in CODA on HiOCFD benchmarks, on various transonic configurations, and setup of targeted final aircraft configurations ;

[M6-M18] Implementation and comparison of selected mesh curving strategies in Flowsimulator. Verification on appropriately chosen benchmarks from HiOCFD, and validation on selected aircraft configurations ;

[M12-M24] Implementation of boundary integral corrections to account for mesh unresolved geometric features in CODA. Evaluation of the best compromise in terms of relative accuracy of mesh and –boundary integration strategy ;

[M18-M27] Coupling of mesh curving method with metric based h-adaptation in Flowsimulator. Evaluation on appropriately chosen benchmarks from HiOCFD, and proof of concept on appropriately chosen benchmarks from HiOCFD, and validation on selected aircraft configurations ;

[M18-M27] The student will publish at least one A ranked scientific journal paper and present his/her work to at least one international scientific conference. The student will assemble and discuss all his/her research activities relative to mesh curving and boundary integral correction in his/her PhD manuscript.

[M28-M36] The student will reintegrate the AIRBUS team to carry out the technological transfer of his/her thesis outputs and take benefit of AIRBUS expertise for the application of the developed high fidelity near curved wall flow description capabilities on industrial configurations.

The time of the student will be split between Airbus and INRIA as follow (TBD) :

★ [M1-M6] at Airbus in Toulouse (6 months).

★ [M7-M24] at INRIA (18 months).

★ [M25-M36] at Airbus in Toulouse (12 months).

Contacts

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