

# Curvilinear Structure Modeling and Its Applications in Computer Vision

**Seong-Gyun JEONG**

seong-gyun.jeong@inria.fr

In collaboration with Dr. Josiane ZERUBIA and Dr. Yuliya TARABALKA

AYIN, INRIA Sophia Antipolis Méditerranée, France

Publications available on <https://team.inria.fr/ayin/publications-hal/>



# Outline

## 1. Introduction

- Motivations
- Previous work
- Overview

## 2. Stochastic model

- Curvilinear features
- Stochastic model
- Reversible jump Markov chain Monte Carlo
- Integration of line hypotheses

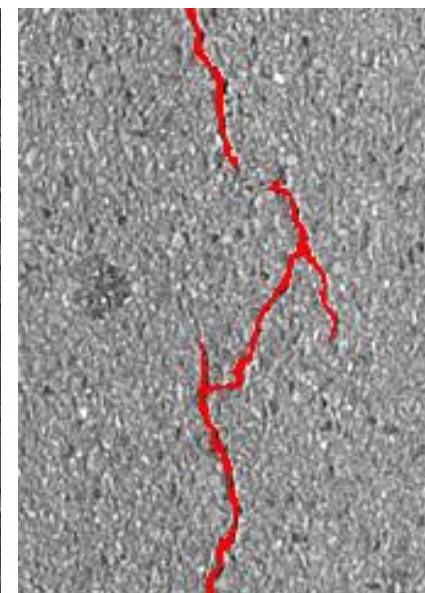
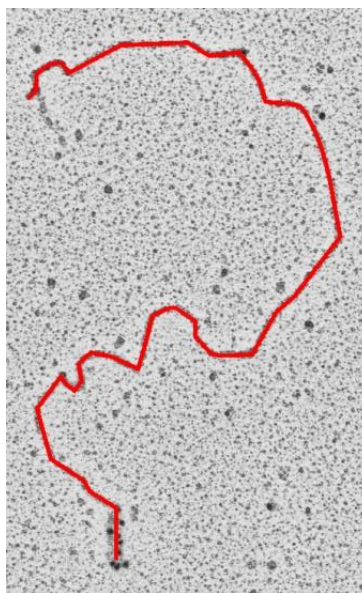
## 3. Structured ranking learning

- Orientation-aware curvilinear features
- Structured ranking learning for curvilinear structure reconstruction
- Progressive curvilinear path reconstruction

# Introduction

# Motivations

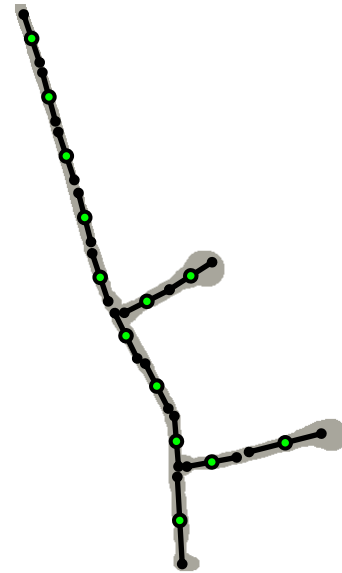
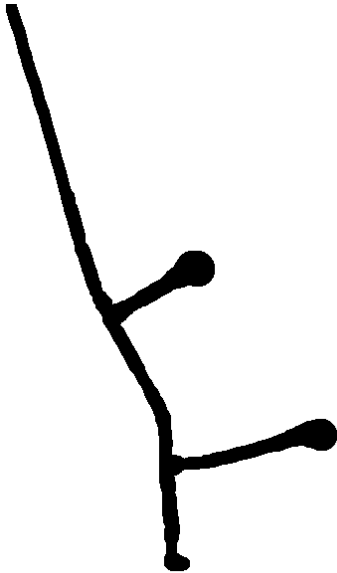
- Appears in various types of natural images  
*e.g., retina, DNA, road network, cracks, facial wrinkles, ...*
- Shows complex geometry
- Low contrast, surrounded by the similar background textures



# Previous work

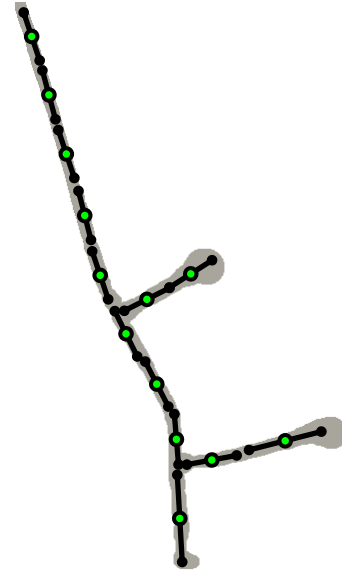
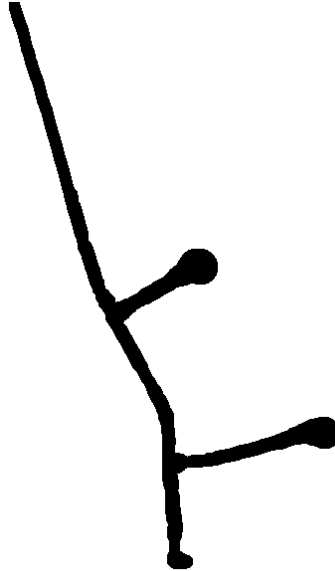
- Blood vessel segmentation [Frangi 98, Staal 04, Law 08, ...]
  - To enhance visibility and aid diagnoses of vascular diseases
- Bioimage analysis [Zhao 11, Wang 11, Peng 11, Turetken 13, ...]
  - To reconstruct physical structure of neural network
- Facial wrinkle detection [Batool 12, Jeong 14, ...]
  - To evaluate skin condition for beauty and dermatology
- Road network extraction [Lacoste 05, Hu 07, Valero 10, ...]
  - To extract geographical information from satellite images
- Defects in the asphalts [Iyer 05, Chambon 10, ...]
  - To analyze large surfaces safely
- **Issues:** Automatization + Generalization
  - Reduce user-defined parameters
  - Employ machine learning systems

# Overview



- **Goal:** **Unified framework** for curvilinear structure reconstruction
- **Assumption:** can be decomposed into multiple line segments
- Find an **optimal set of line segments** for curvilinear structure reconstruction

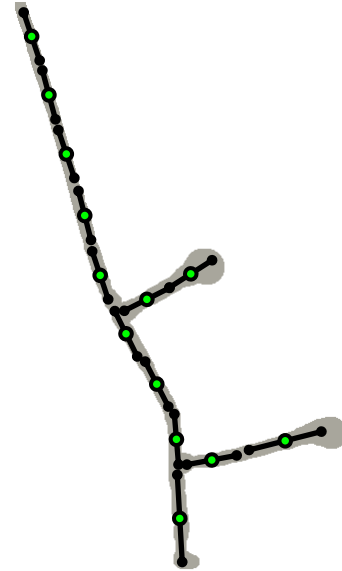
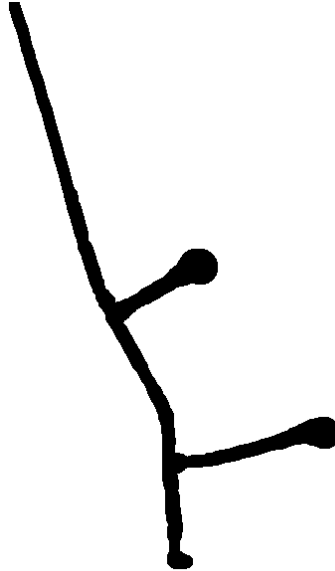
# Overview



- **Stochastic model**

- Maximize **a posterior probability** of line segments for given image
- Data likelihood: *local curvilinear features*
- Prior energy: *constrains local geometry of line segments*

# Overview

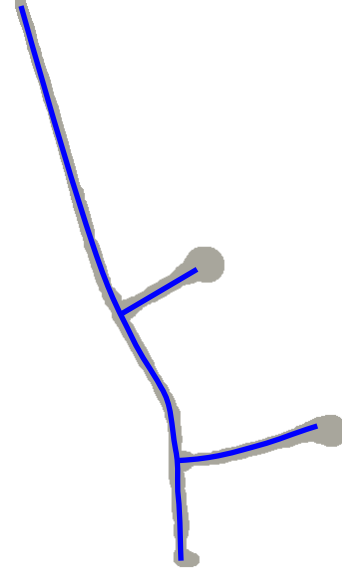
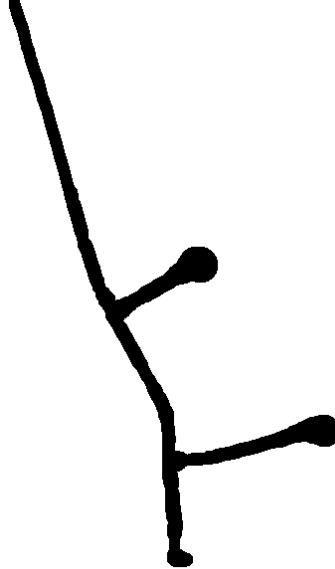


- **Structured ranking learning**

- Learn a function to evaluate correspondence between line segment and the underlying curvilinear structure
- Orientation-aware curvilinear feature descriptor



# Overview



- **Progressive curvilinear path reconstruction**
  - Provide topological features and simplified curvilinear structure
  - Find the longest geodesic paths in the graph

Stochastic model

# Curvilinear features

- Thin / Elongated / Symmetric / Locally oriented
- Show different intensity values compared to its surroundings

## Image gradient

- Can measure local intensity variation
- 2nd derivative of Gaussian kernels



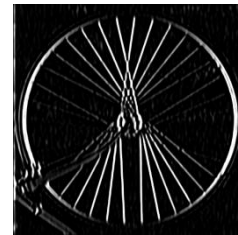
Input



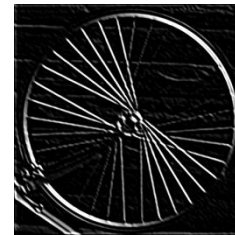
0°



45°



90°



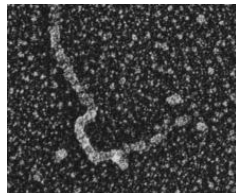
135°

## Morphological filtering [Talbot 07]

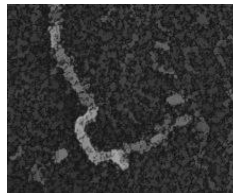
- Highlight structural similarity



Input



Invert



L=10



L=50

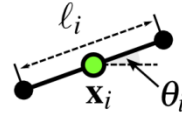


L=100

# Stochastic model

- Line segment as tuple of *pixel*, *length*, and *orientation*

$$s_i = (\mathbf{x}_i, \ell_i, \theta_i) \in \mathbb{R}^2 \times |L| \times |\Theta|$$

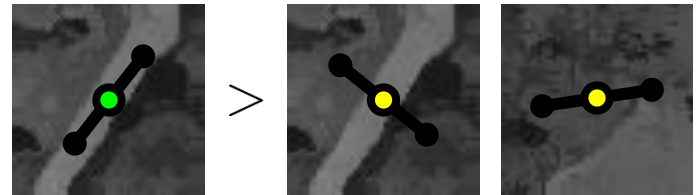


- Find a set of line segments that maximize a posterior probability

$$\hat{\mathbf{s}} = \underset{\mathbf{s} \in \Psi}{\operatorname{argmax}} p(\mathbf{s}|I) = \underset{\mathbf{s} \in \Psi}{\operatorname{argmax}} p(I|\mathbf{s})p(\mathbf{s})$$

## Data likelihood

$$p(I|s_i) = \frac{1}{Z} \exp \left( - \sum_{\mathbf{x}_j \in s_i} E_{\text{data}}(\phi(\mathbf{x}_j)) \right)$$



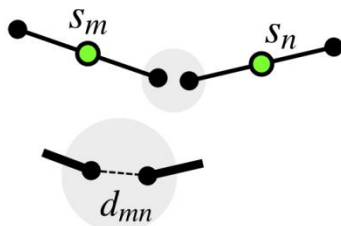
- To localize a line segment for a given image
- Curvilinear feature vector  $\phi(\mathbf{x})$  evaluates a pixel whether it is on the curvilinear structure according to the image gradient and morphological filtering

# Stochastic model

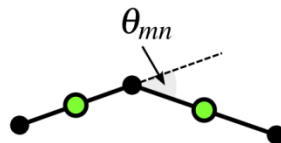
## Prior energy

$$p(\mathbf{s}) = p(s_1, s_2, \dots, s_N) = \prod_{s_m \sim s_n} p(s_m | s_n) = \sum_{s_m \sim s_n} E_{\text{prior}}(s_m, s_n)$$

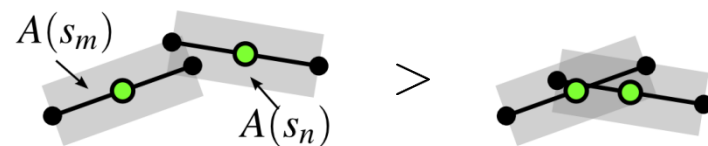
- Define **local geometry** of line segments (*line segments interact if they are close enough*)
- Smoothly connected line segments are desired
  - Connectivity: *end-to-end distance of line segments*
  - Curvature: *angle difference between adjacent line segments*
- Reject congestion of lines within local configuration
  - Measured by the proportion of pixels falling in the same areas



Connectivity



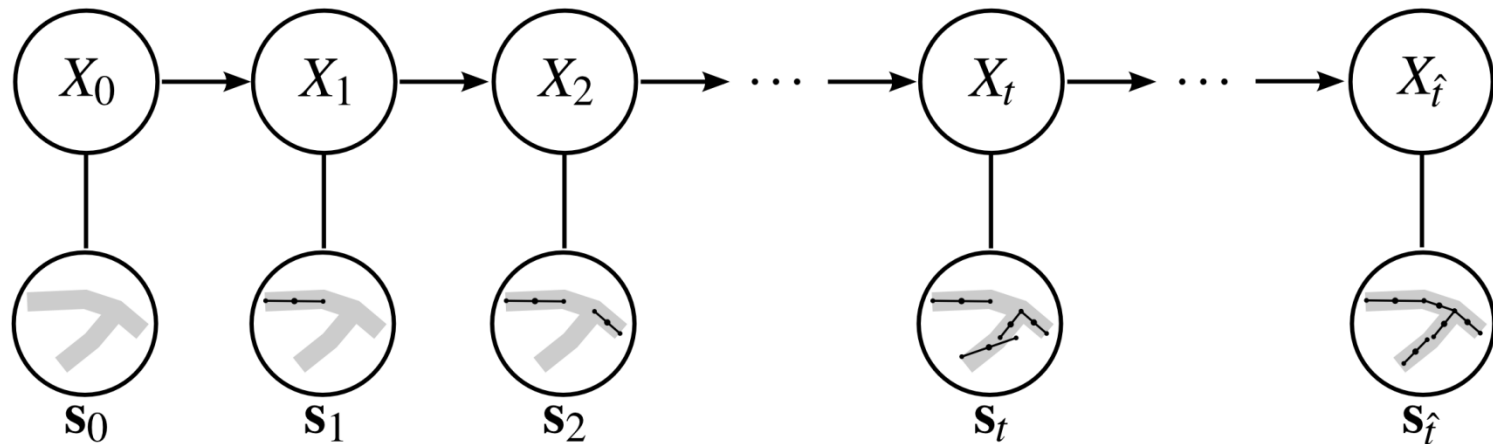
Curvature



Congestion

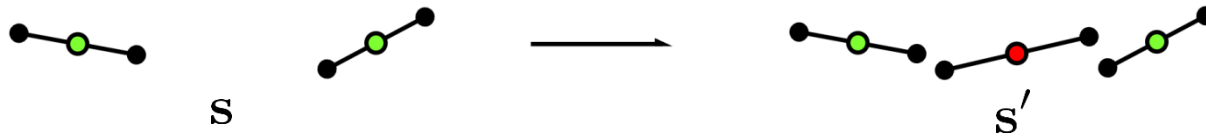
# RJCMCMC [Green 95]

- Reversible jump Markov chain Monte Carlo
- Each state of a discrete Markov chain  $(X_t)_{t \in \mathbb{N}}$  corresponds to a random configuration on  $\Psi$
- Markov chain eventually reaches an equilibrium state which maximizes the proposed density function

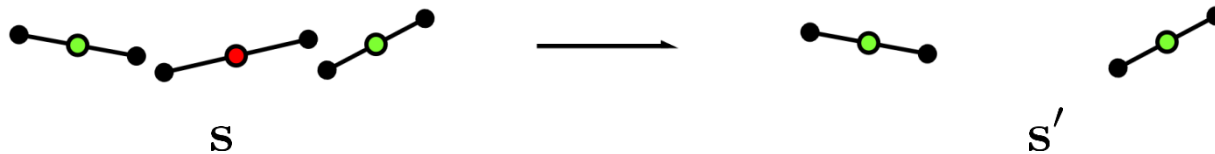


# RJMCMC

- Simulate a discrete Markov chain over the configuration space via **sub-transition kernels** *i.e.*, *Birth, Death, Affine transform*
- **Birth kernel** *proposes a new segment*
  - $s_i = (\mathbf{x}_i, \ell_i, \theta_i) \in \mathbb{R}^2 \times |L| \times |\Theta|$
  - Randomly select location, length, and orientation from sample space

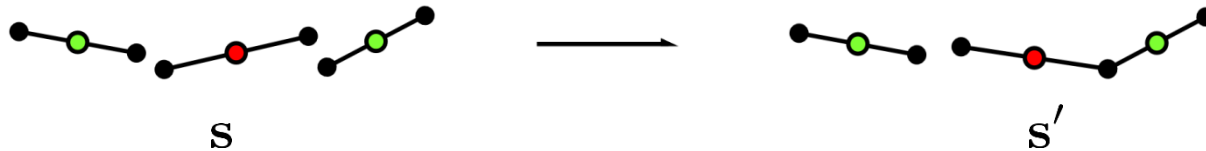


- **Death kernel** *removes a segment*
  - Randomly select an existing line segment from the current configuration

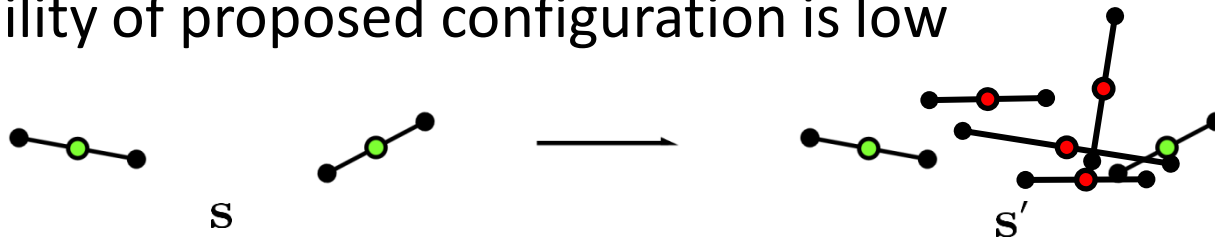


# RJMCMC

- Simulate a discrete Markov chain over the configuration space via **sub-transition kernels** *i.e.*, *Birth, Death, Affine transform*
- ***Affine transform*** updates intrinsic variables of the segment
  - Select a line segment and update its location, length, and orientation randomly
  - $s_i = (\mathbf{x}_i, \ell_i, \theta_i) \rightarrow s'_i = (\mathbf{x}_i \pm \Delta \mathbf{x}, \ell_i \pm \Delta \ell, \theta_i \pm \Delta \theta)$



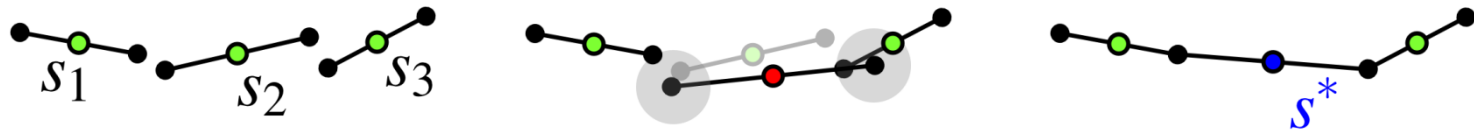
- Markov chain will ***remain at the current configuration***, if the probability of proposed configuration is low





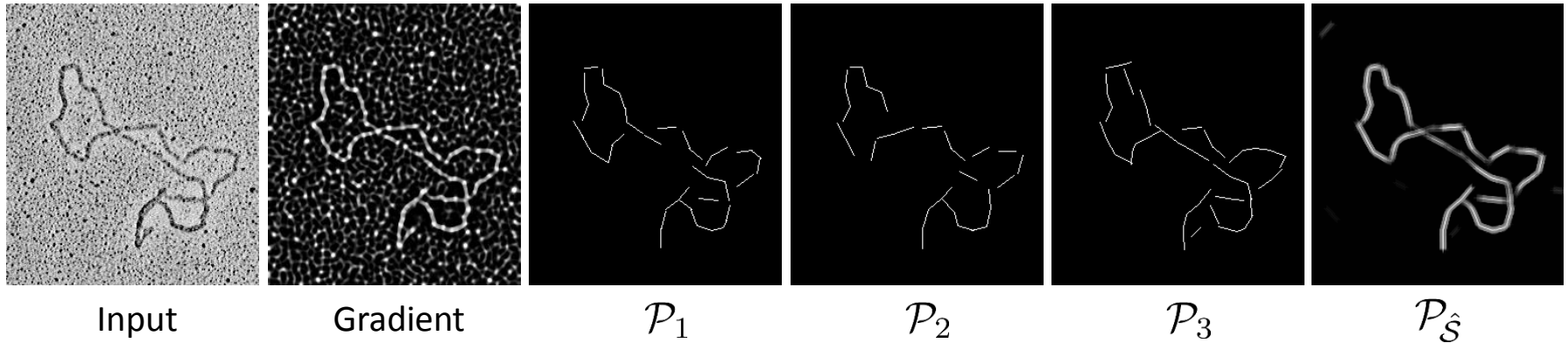
# Delayed Rejection [Green 01]

- Gives a *second chance* to a rejected configuration *by enforcing the connectivity*



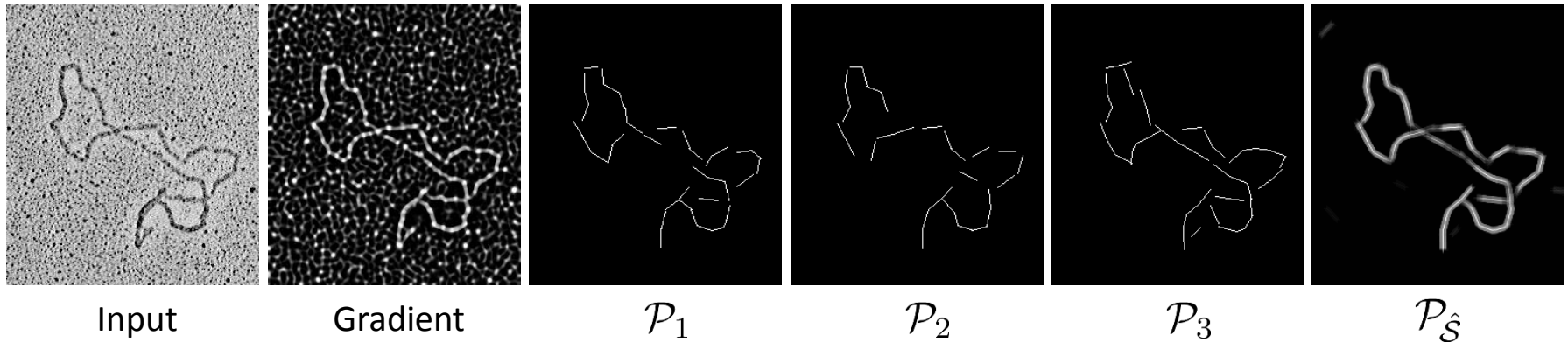
1. Let  $\mathbf{s}=\{s_1, s_2, s_3\}$  be the current configuration
2. Propose a new configuration via affine transform kernel
3. If  $\mathbf{s}'$  is rejected, DR kernel searches for the nearest end points in the rest of the line segments
4. An alternative line segment  $\mathbf{s}^*$  will enforce the connectivity

# Create line hypotheses



- Stochastic model is **sensitive** to the selection of hyperparameter
- Learning is NOT feasible
  - Ground truth is given as a **binary segmentation map**
- To avoid estimating hyperparameter,
  1. We build line hypotheses with respect to  **$K$  different hyperparameter vectors**
  2. Integrate line hypotheses to reduce sample space
  3. Find the most promising line hypothesis and use its hyperparameter vector

# Integrate line hypotheses

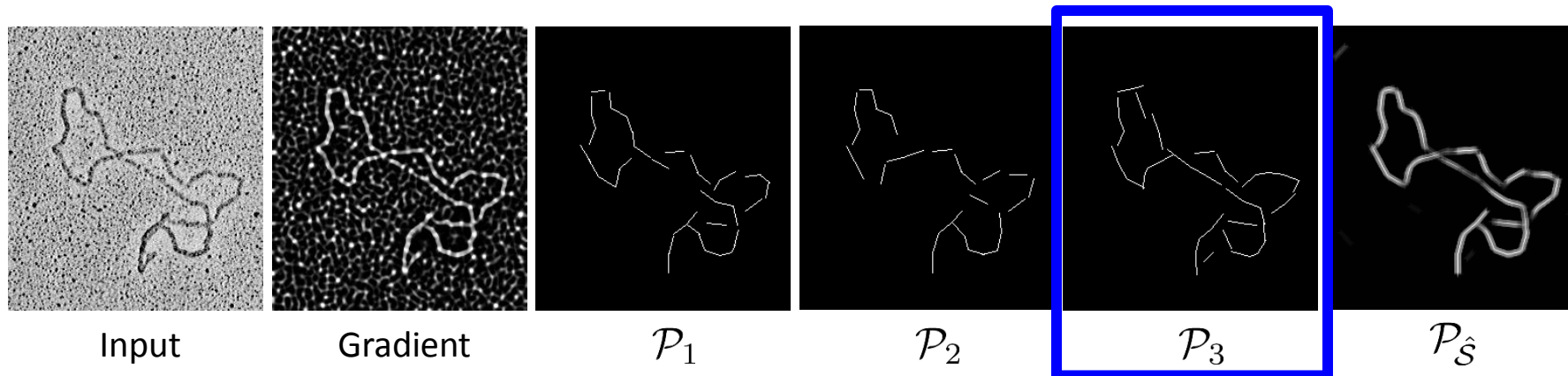


- Assumption
  - Prominent line segment will be observed more frequently
- Mixture density

$$\mathcal{P}_{\hat{S}} = \frac{1}{K} \sum_{k=1}^K \mathcal{P}_k$$

- Shows consensus between line hypotheses
- Reduce sampling space
- Criterion for hyperparameter vector selection

# Integrate line hypotheses



- Update data likelihood

$$E'_{\text{data}}(s_i) = E_{\text{data}}(s_i) - \log \mathcal{P}_{\hat{S}}(s_i)$$

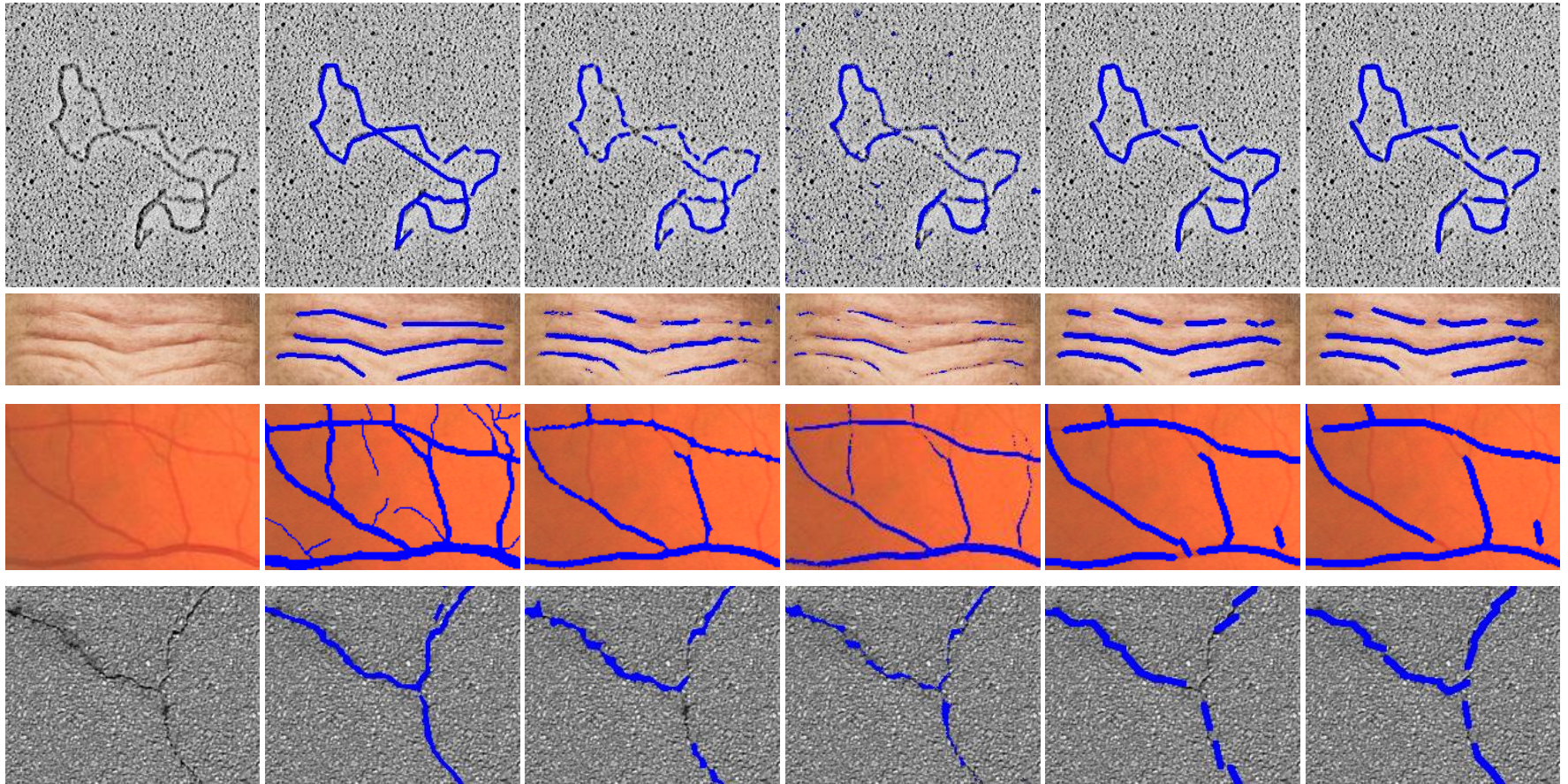
- Induce reduction of sampling space
- Find the most promising hyperparameter vector

$$\hat{k} = \underset{k=\{1,\dots,K\}}{\operatorname{argmax}} \operatorname{CC}(\mathcal{P}_{\hat{S}}, \mathcal{P}_k)$$

- Re-Simulate Markov chain

$$\mathbf{s}^* = \underset{\mathbf{s} \in \mathbb{S}}{\operatorname{argmin}} \sum_{i=1}^{\#(\mathbf{s})} E'_{\text{data}}(s_i) + \sum_{s_m \sim s_n} E_{\text{prior}}(s_m, s_n; \boldsymbol{\omega}_{\hat{k}}).$$

# Experimental Results



Input

Ground truth

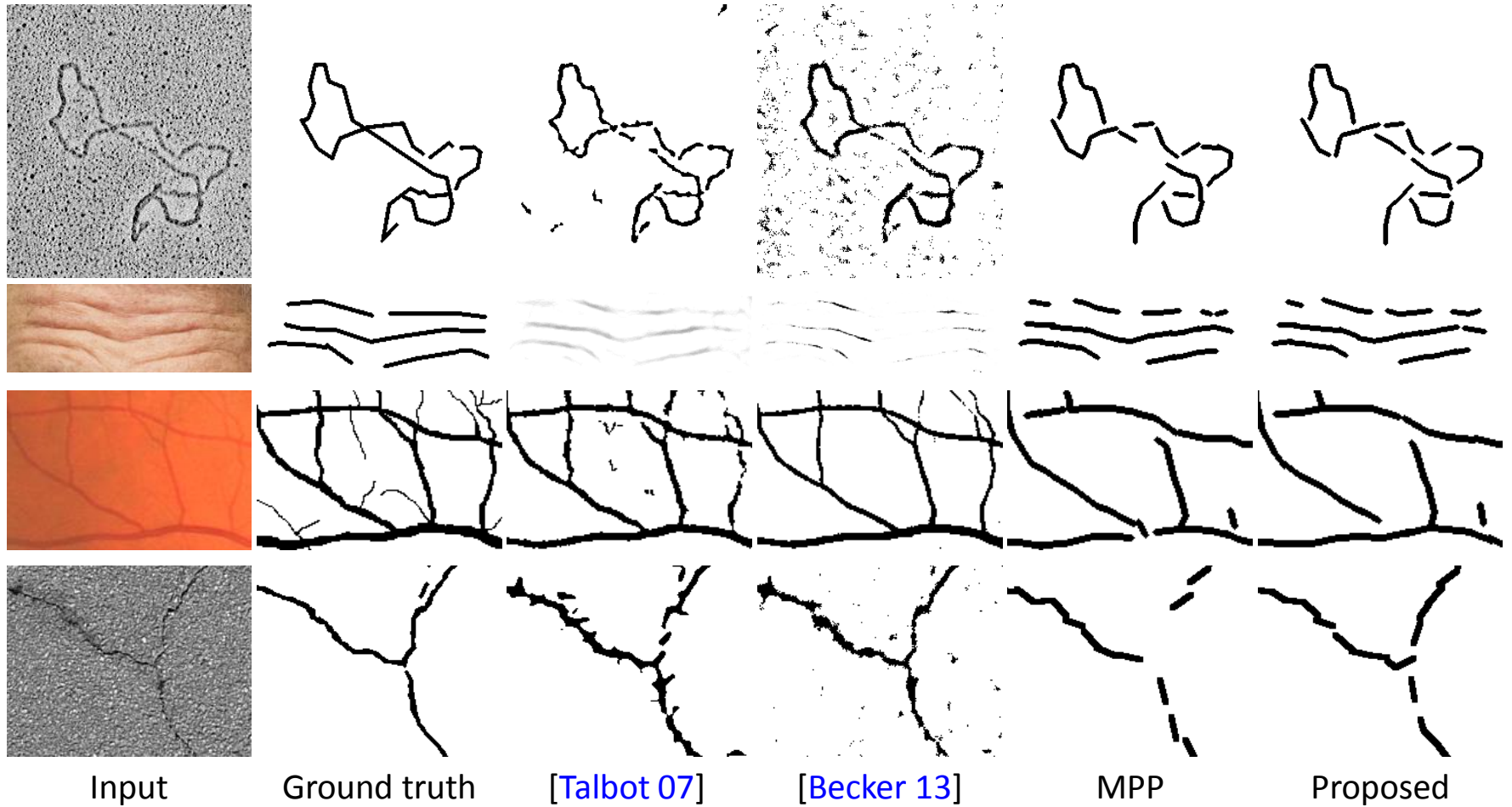
[Talbot 07]

[Becker 13]

MPP

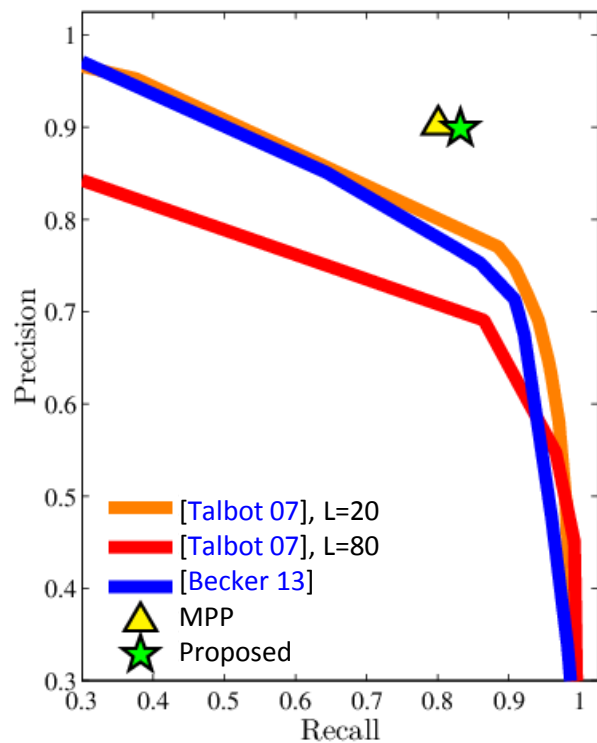
Proposed

# Experimental Results

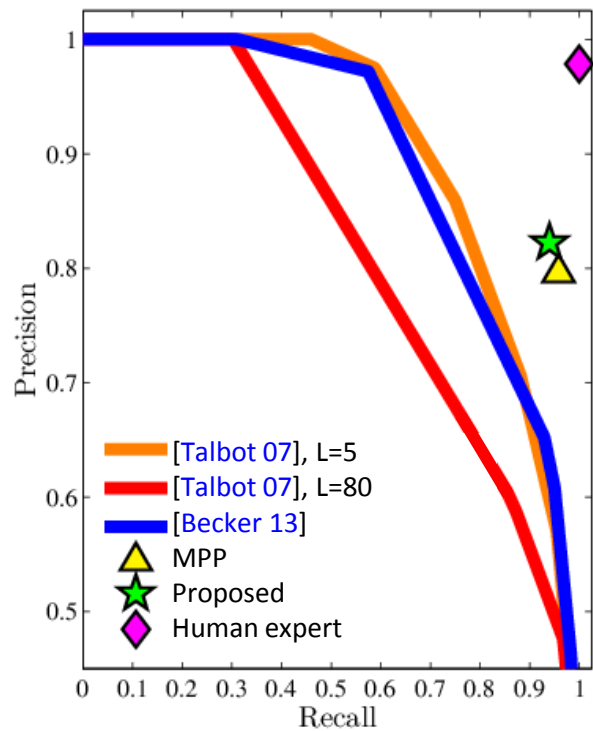




# Experimental Results



DNA



Wrinkles

# Summary

- **Stochastic model** for curvilinear structures
  - *e.g., Wrinkles, DNA filaments, road cracks, blood vessels, ...*
  - **Data term:** Image gradient & morphological filtering responses
  - **Prior term:** To provide smoothly connected lines
  - **Simulation:** RJMCMC with delayed rejection
- **Reduce parameter dependencies** of the stochastic modeling with hypotheses integration

- **Limitation**

Heuristically designed prior energy

Fails to find varying thicknesses

Heavy computation





# Inference of Curvilinear Structure

# Supervised machine learning

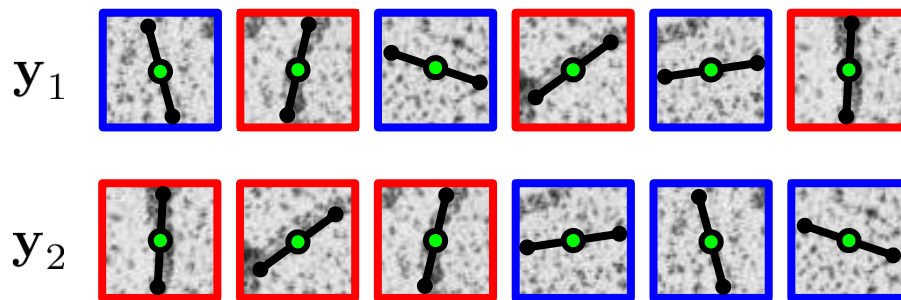
- **Goal:** obtain a function  $h : \mathcal{Z} \mapsto \mathcal{Y}$  which maps an input space  $\mathbf{z} \in \mathcal{Z}$  to an output space  $y \in \mathcal{Y}$
- *Supervised machine learning algorithm* evaluates the quality of hypothesis  $\hat{y} = h(\mathbf{z}; \mathbf{w})$  with labeled examples

## Machine learning vs. Structured learning

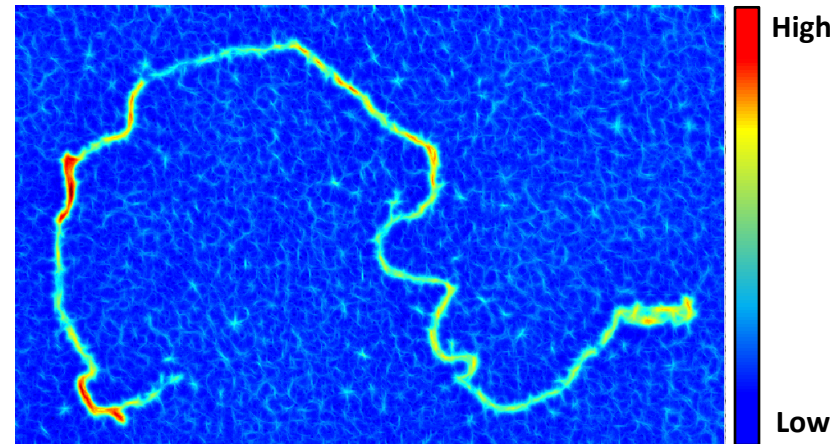
- Inputs can be any kind of objects (**Both**)
- Output is a real number (*Machine learning*)  
*e.g., Classification, regression, ...*
- Outputs are complex / structured objects (*Structured learning*)  
*e.g., Segmentation, protein sequence, NLP, ranking, ...*

# Structured ranking learning

- Learn a ranking function to evaluate correspondence between line segments and feature maps
- Curvilinear structure will be reconstructed by the high ranked line segments
- Score function  $H(s, y; w)$



$$H(s, y_1; w) < H(s, y_2; w)$$

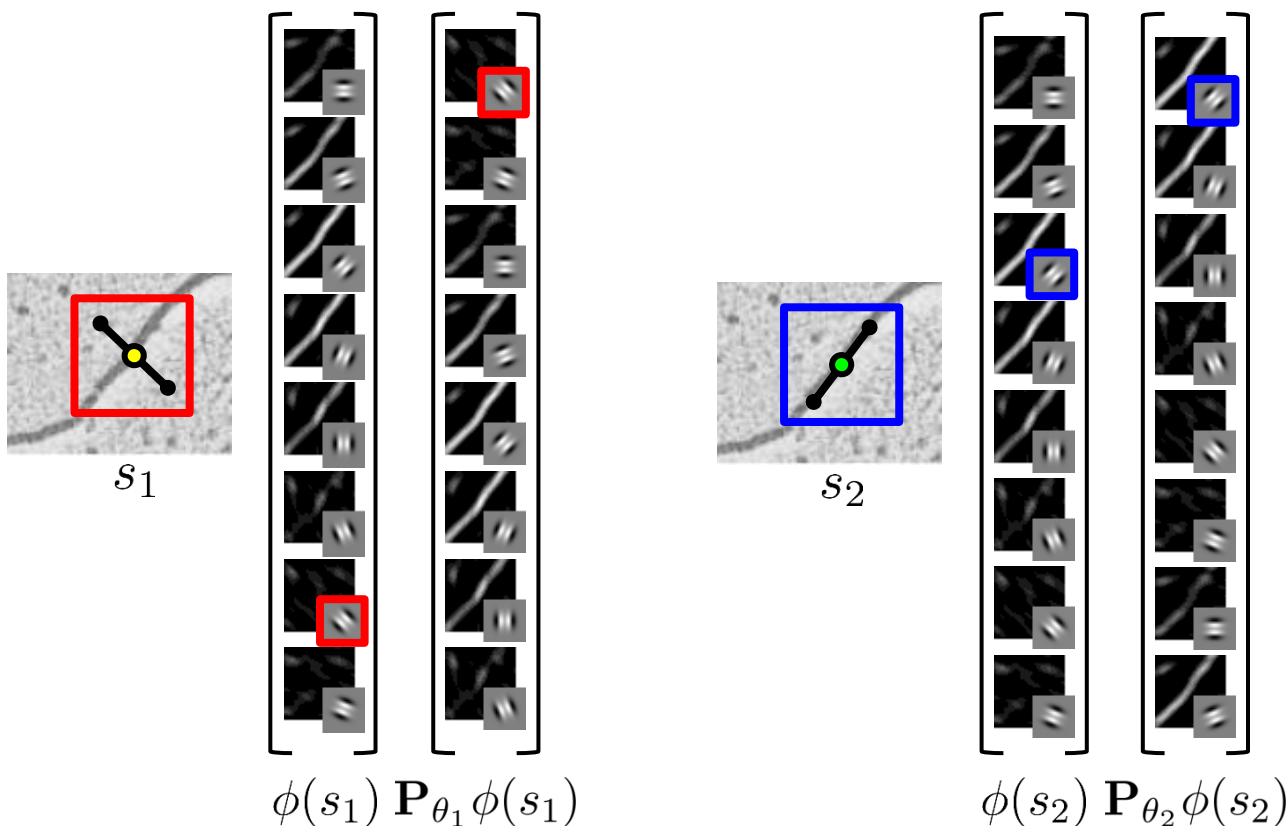


Output ranking scores

- Orientation of line segments?

# Orientation-aware curvilinear feature

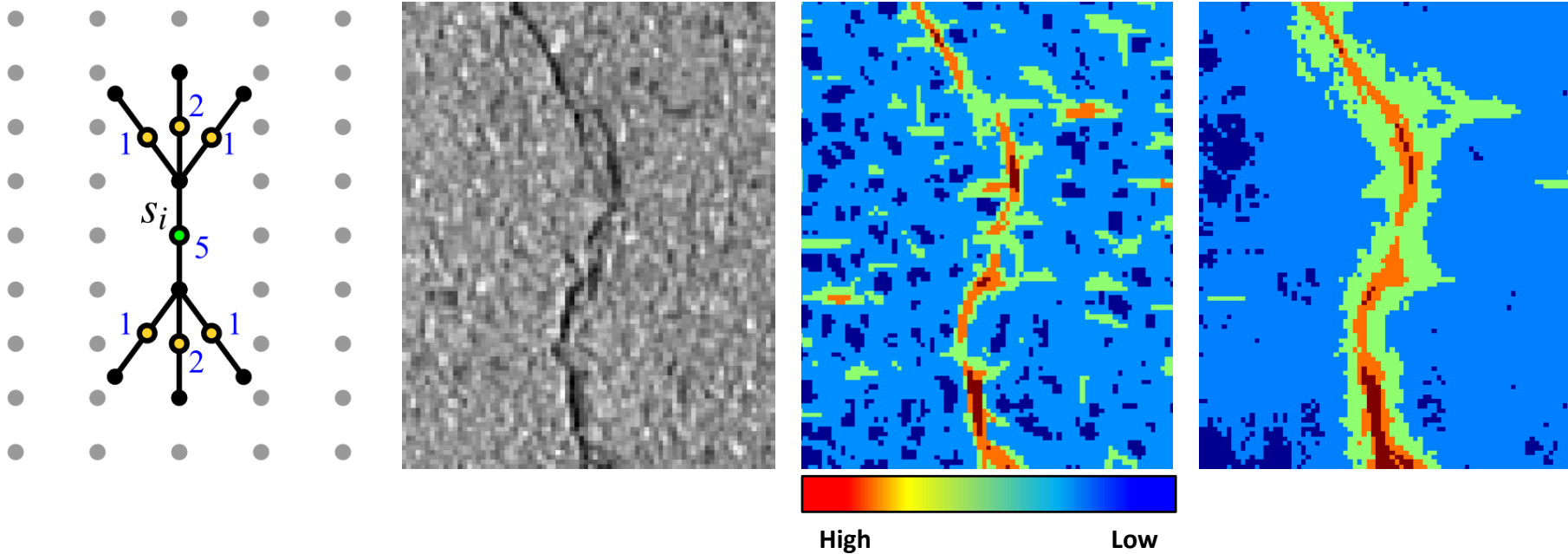
- Model parameter  $w$  determines the relative importance of feature maps
- Permute the elements in the feature vector according to the given orientation



# Spatial grouping of the features

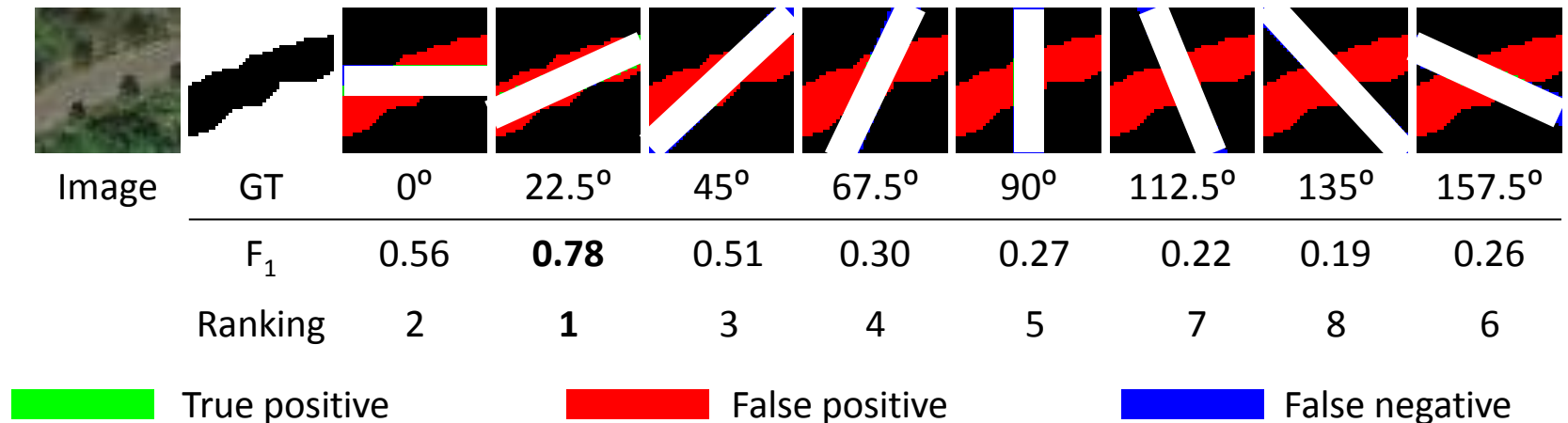
- Enhance spatial coherence of the output ranking scores
- Weighted sum with neighboring set of line segments

$$\bar{\phi}(s_i) = \frac{\sum_{j \in \mathcal{N}_i} \omega_j \phi'(s_j)}{\sum_{j \in \mathcal{N}_i} \omega_j}$$



# Learning

- We need a training dataset  $\mathcal{D} = \{(s_i, y_i)\}_{i=1}^K$ 
  - A list of line segments (**Easy**)
  - The relevant raking values (?)
- Ground truth (GT) is given as a binary segmentation map
  - No shape information w.r.t. line segments, *i.e.*, *length*, *orientation*, *thickness*
- Evaluate the shape dissimilarity ( $F_1$ ) between the line segment and the corresponding image patch from ground truth



# Learning

- Prediction is performed by finding rankings that maximize the score function

$$\hat{\mathbf{y}} = \operatorname{argmax}_{\mathbf{y} \in \mathcal{Y}} H(\mathbf{s}, \mathbf{y}; \mathbf{w}) = \operatorname{argmax}_{\mathbf{y} \in \mathcal{Y}} \mathbf{w}^\top \Psi(\mathbf{s}, \mathbf{y})$$

- Joint feature map

$$\Psi(\mathbf{s}, \mathbf{y}) = \sum_{ij} y_{ij} (\bar{\phi}(s_i) - \bar{\phi}(s_j)) ,$$

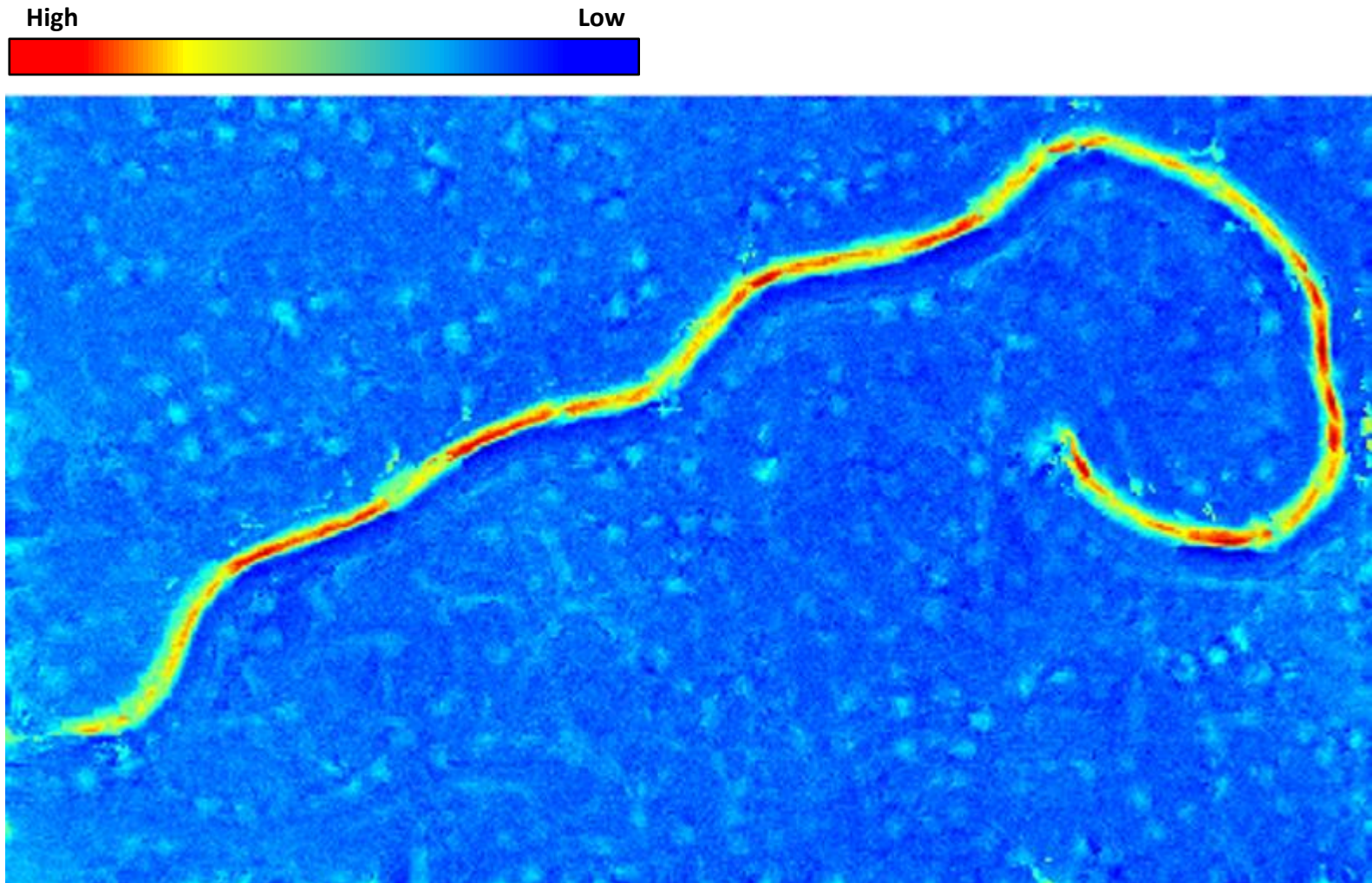
- Ranking matrix

$$y_{ij} = \begin{cases} +1 & \text{if } y_i > y_j, \\ -1 & \text{otherwise.} \end{cases}$$

- Optimize constrained objective function via cutting plane algorithm [[Joachims 09](#)]

# Output ranking score map

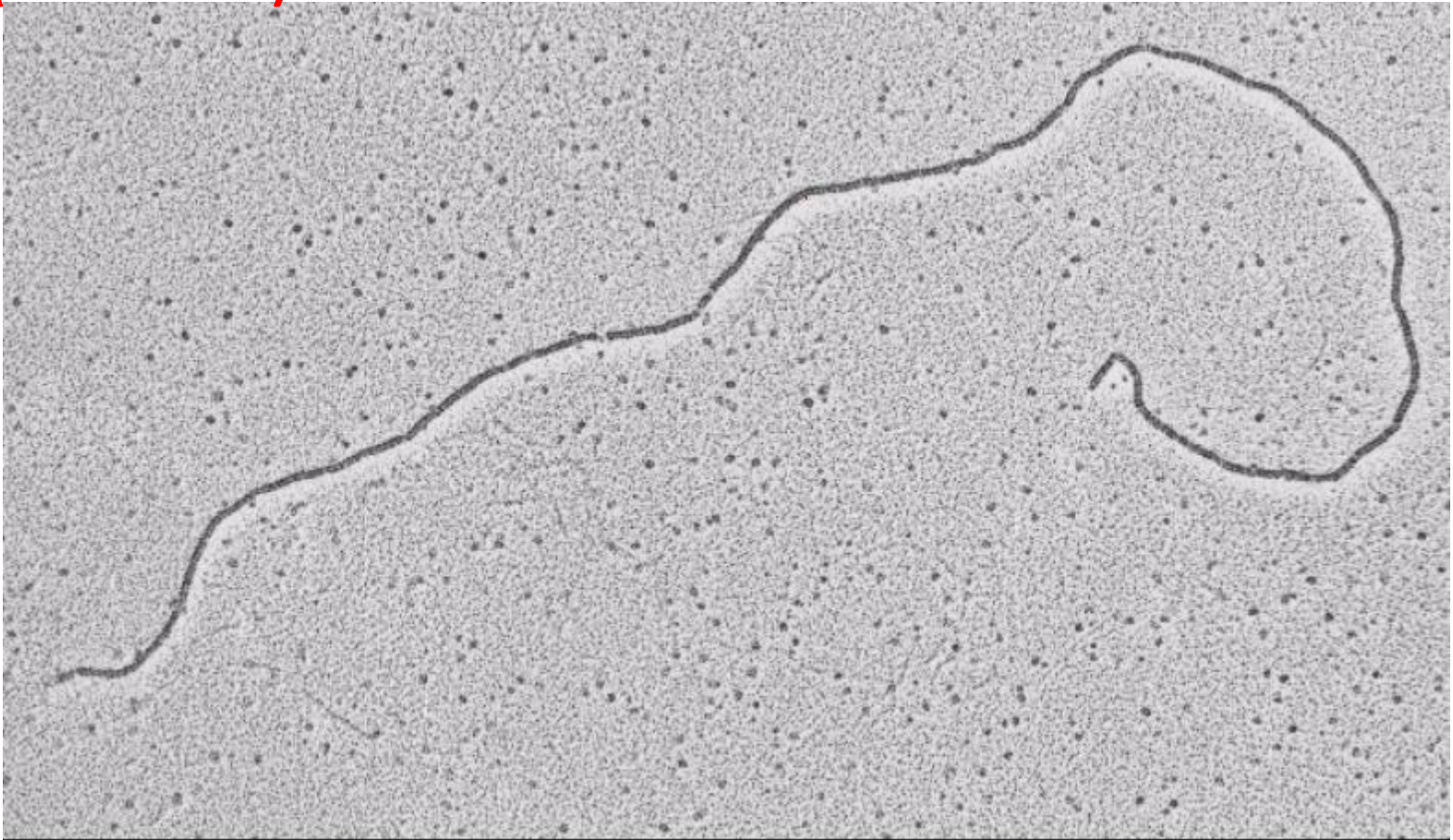
- Output ranking scores highlight the latent curvilinear structure





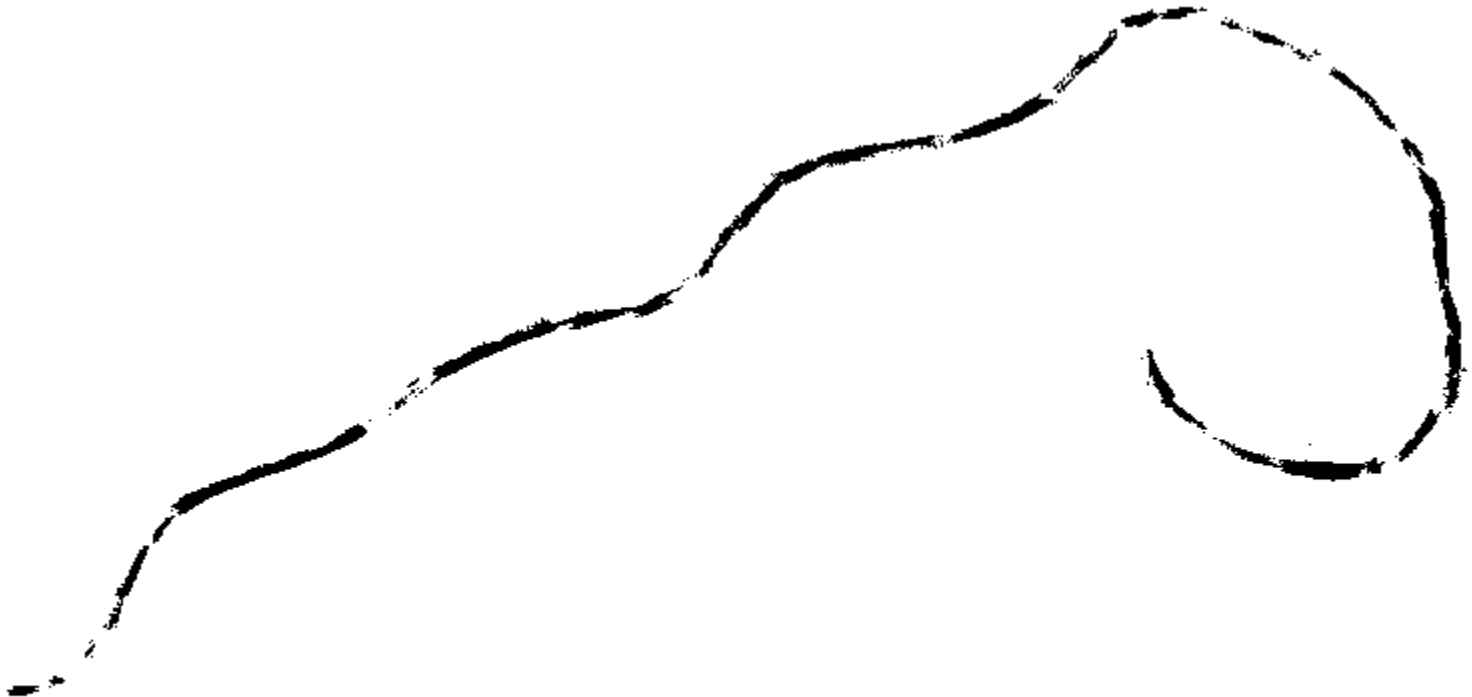
# Binary segmentation map

- Compute the average proportion of pixels being part of the curvilinear structure from training images for **stop criterion** ( $\approx$ threshold)



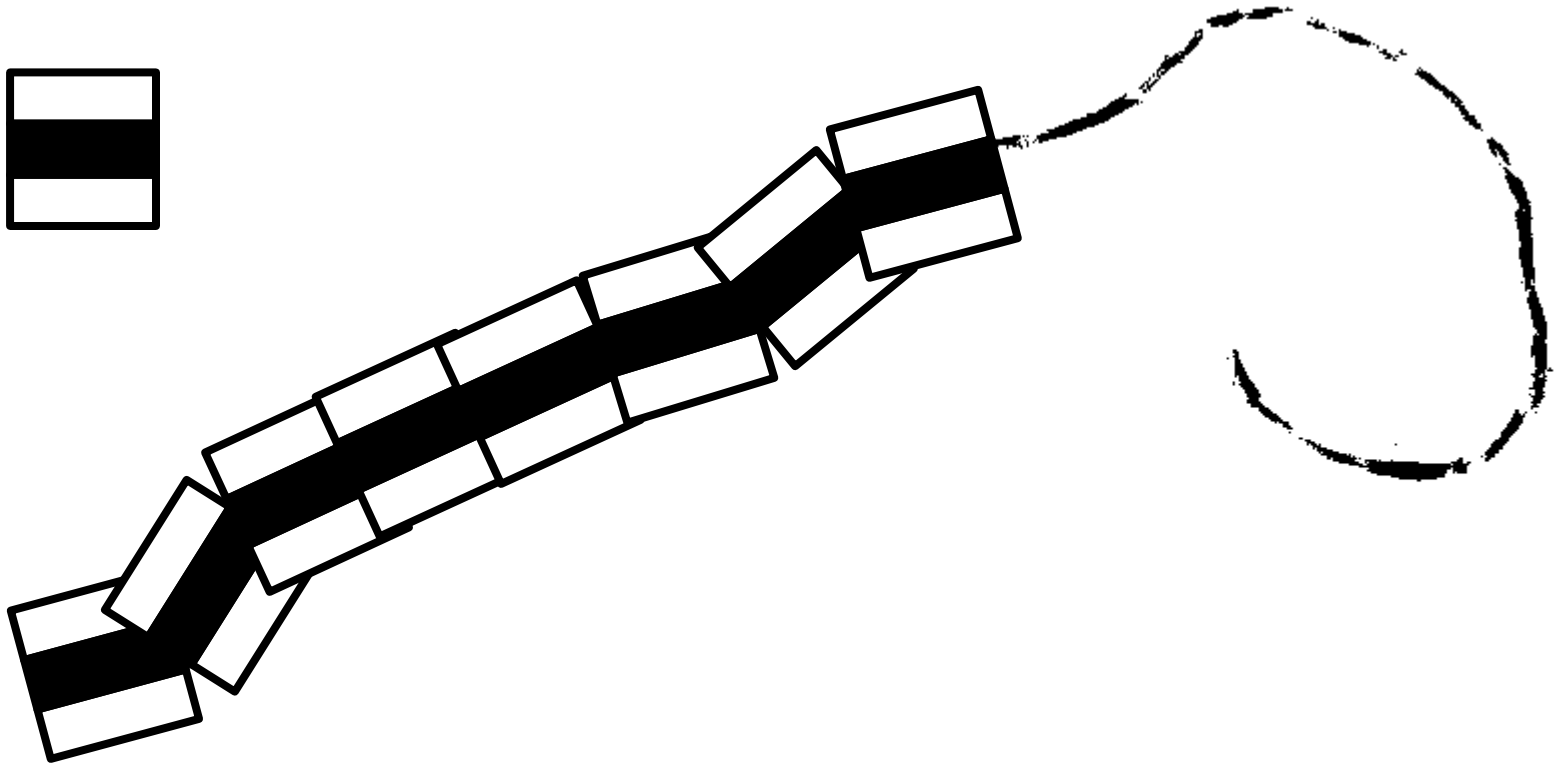
# Binary segmentation map

- Remain pixels according to the output rankings
- Topology can be broken



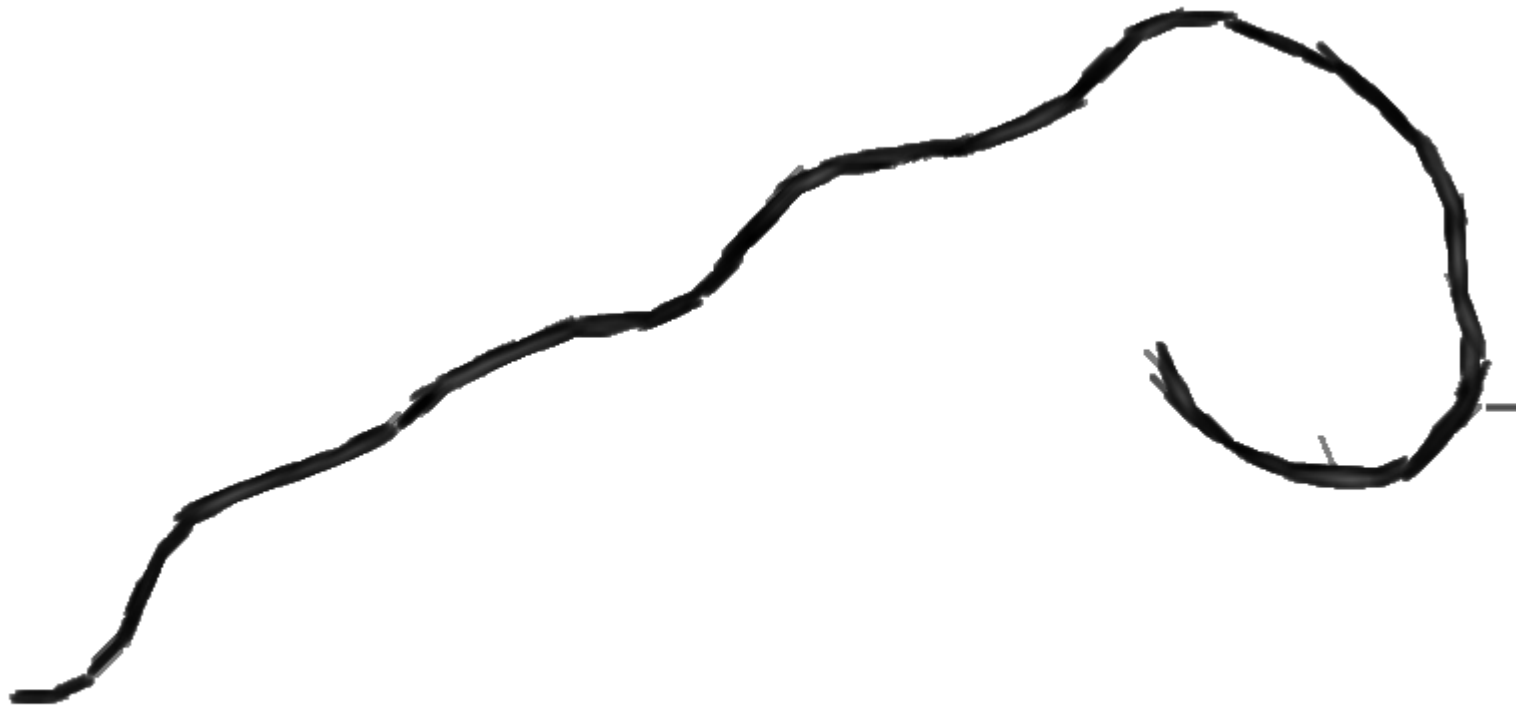
# Dissimilarity score map

- Each pixel encodes shape information (length, orientation, and thickness)



# Dissimilarity score map

- Values are used to generate a graph for curvilinear path reconstruction

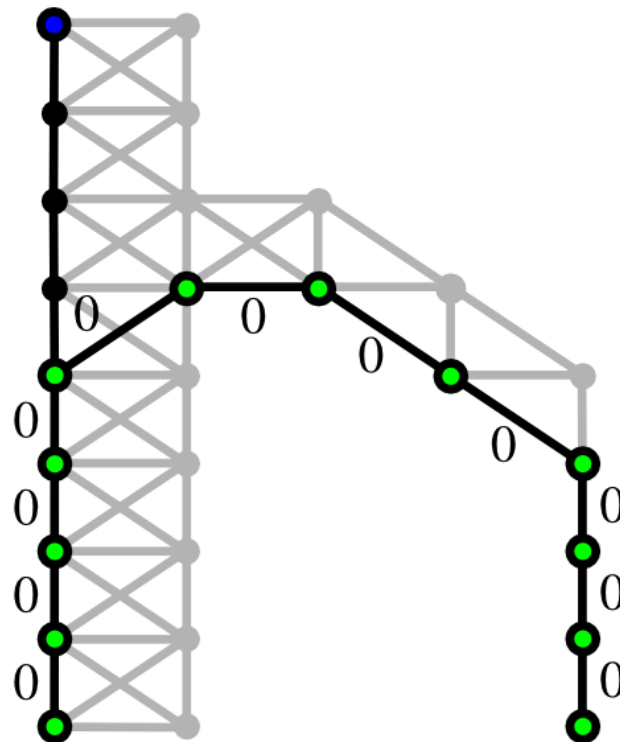


# Progressive curvilinear path reconstruction

1. Induce a subgraph  $\mathbf{G}'$  using structured output ranking scores
2. Randomly select a vertex  $\mathbf{t}$  and find the longest geodesic path ( $\mathbf{t} \rightarrow \mathbf{u}$ )
3. Find the longest geodesic path from  $\mathbf{u}$  to a vertex  $\mathbf{v}$  at the maximum distance from  $\mathbf{u}$

The distance of path  $\mathbf{u} \rightarrow \mathbf{v}$  is a diameter of  $\mathbf{G}'$

4. Assign 0 weight for all edges on this path ( $\mathbf{u} \rightarrow \mathbf{v}$ )
5. Repeat the process step 2 to 4 to add branches which are longer than pre-defined length

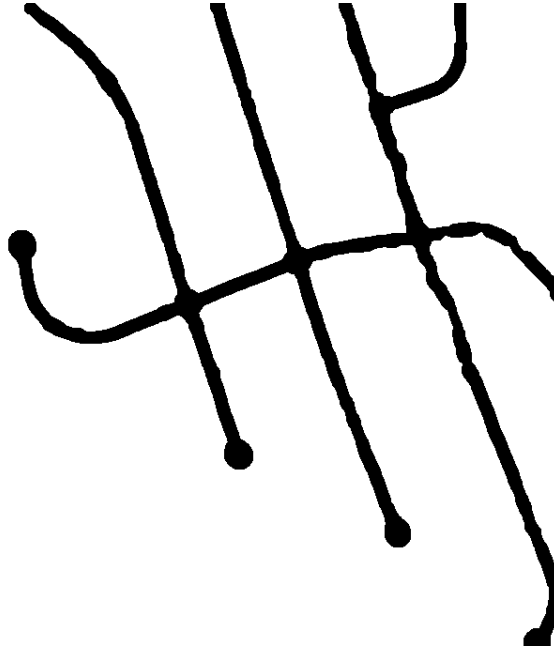


# Progressive curvilinear path reconstruction

- Iteratively find the longest geodesic path in the graph
- Can illustrate topological features in different levels of detail



Input image



Ground truth



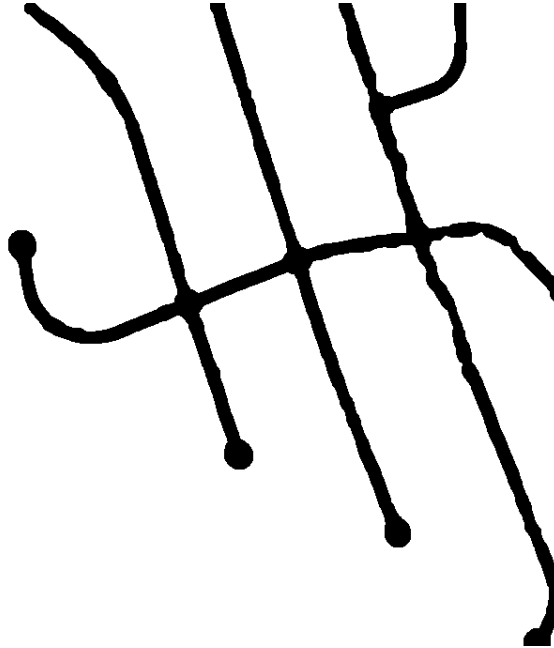
Proposed

# Progressive curvilinear path reconstruction

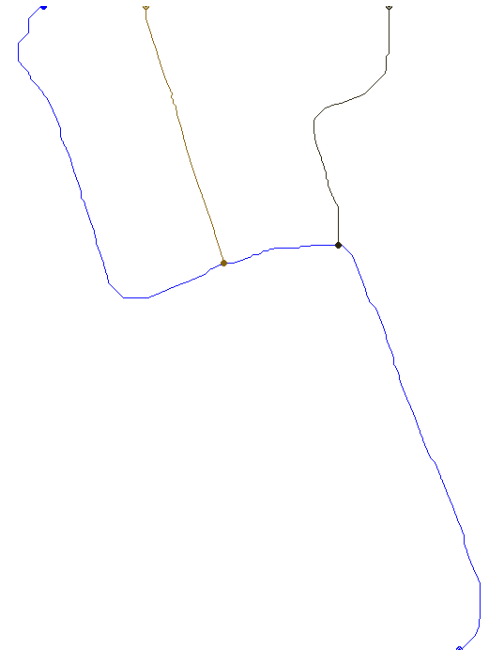
- Iteratively find the longest geodesic path in the graph
- Can illustrate topological features in different levels of detail



Input image



Ground truth



Proposed

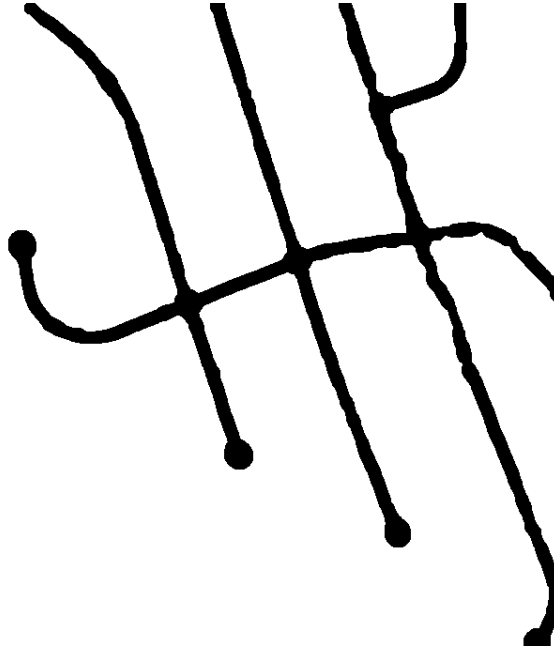


# Progressive curvilinear path reconstruction

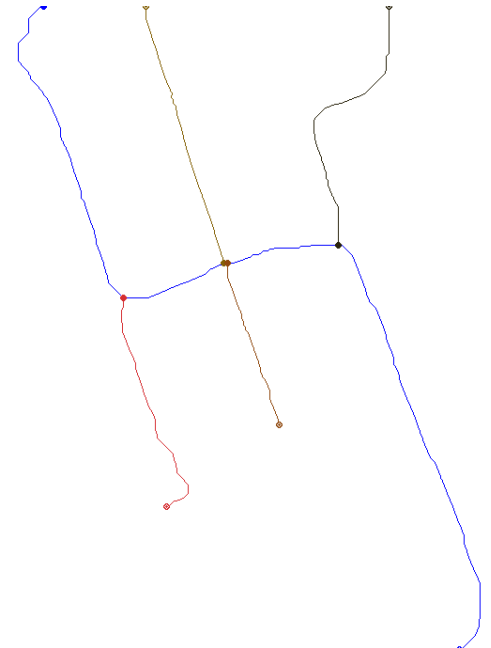
- Iteratively find the longest geodesic path in the graph
- Can illustrate topological features in different levels of detail



Input image



Ground truth



Proposed

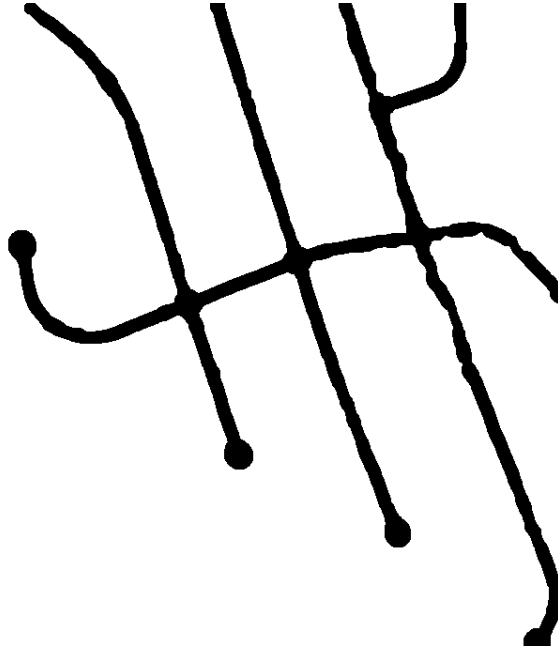


# Progressive curvilinear path reconstruction

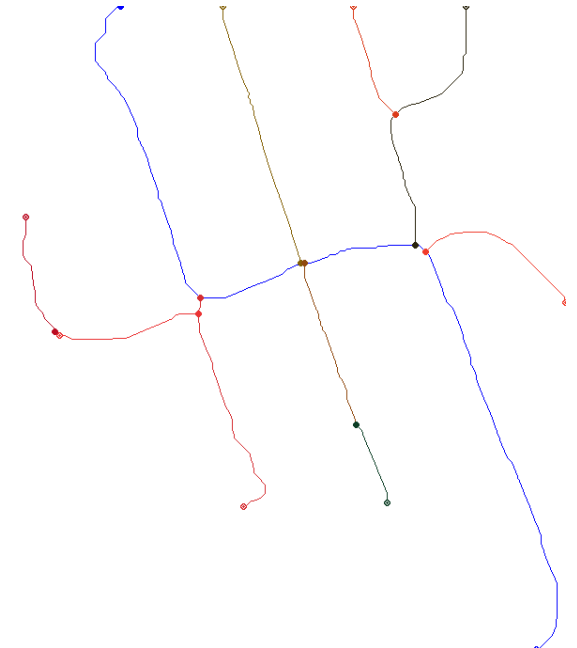
- Iteratively find the longest geodesic path in the graph
- Can illustrate topological features in different levels of detail



Input image



Ground truth

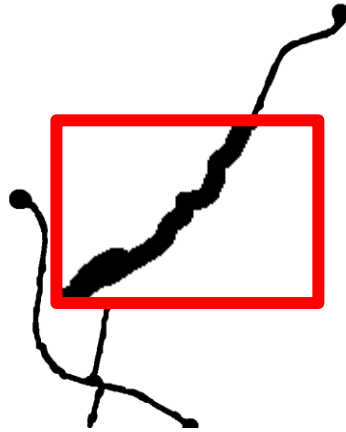


Proposed

# Experimental results: Aerial



Input



Ground truth



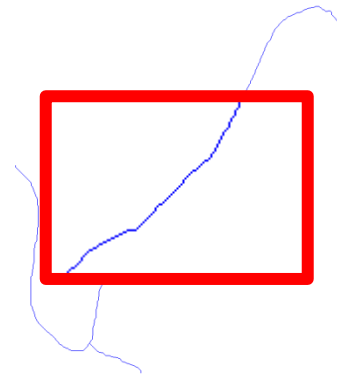
[Law 08]



[Becker 13]

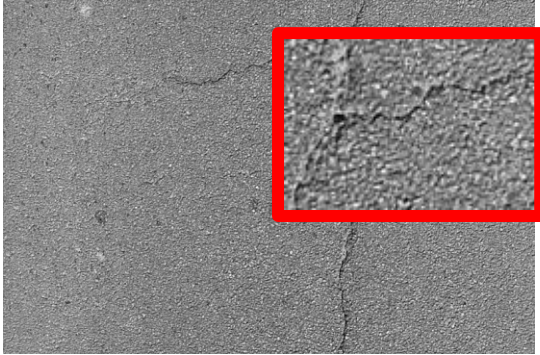


[Sironi 14]

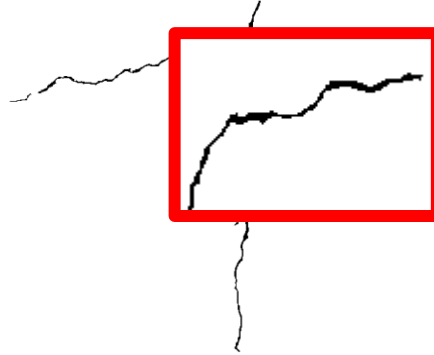


Proposed

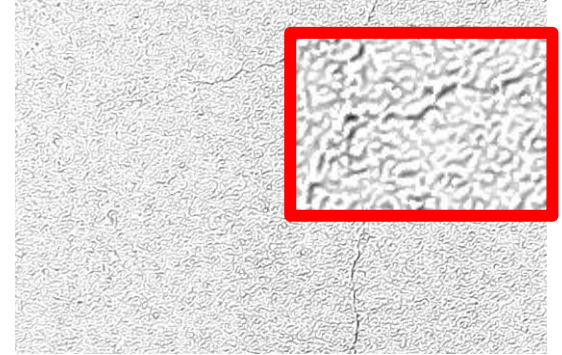
# Experimental results: Cracks



Input



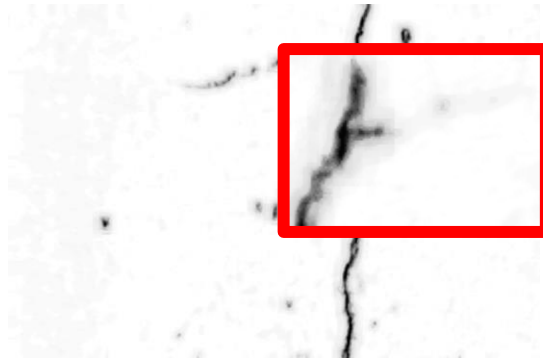
Ground truth



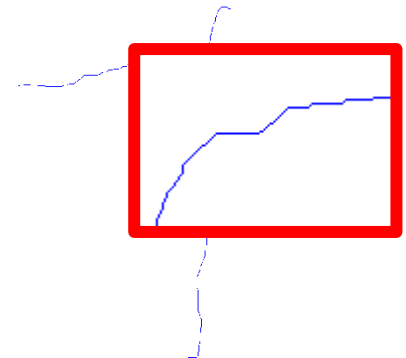
[Law 08]



[Becker 13]



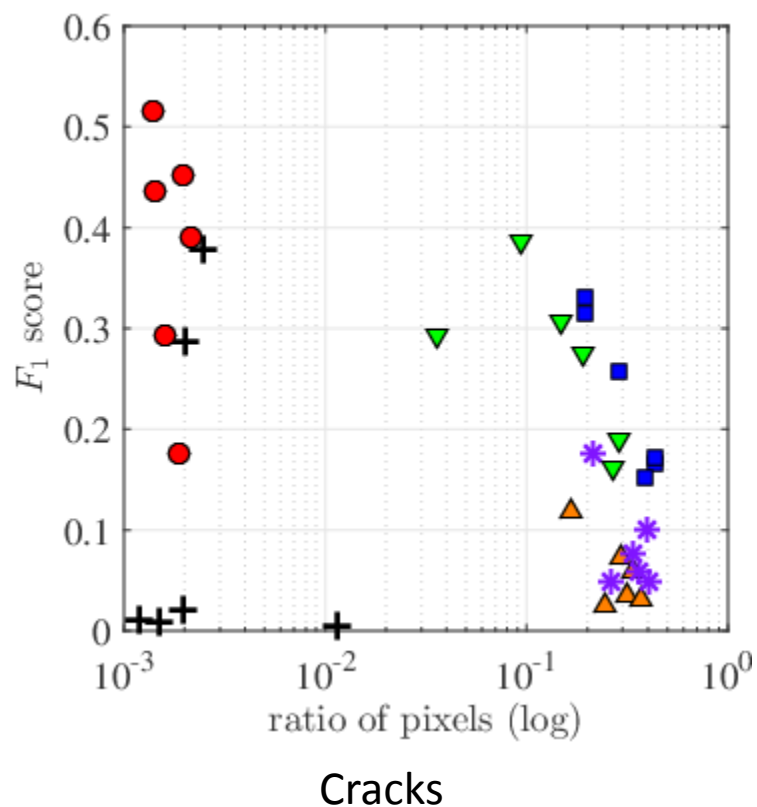
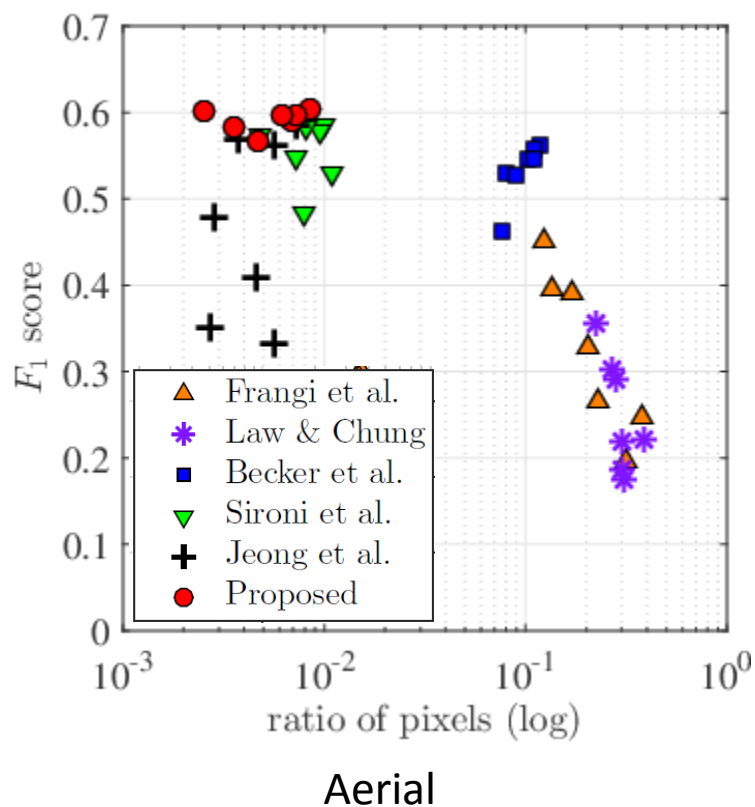
[Sironi 14]



Proposed

# Experimental results

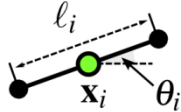
- Proposed curvilinear path reconstruction shows the highest  $F_1$  performance with the minimum number of pixels



# Summary

- **Structured ranking learning** for curvilinear structures
  - ***Learn a ranking function:** to evaluate correspondence between line segments and feature maps*
  - ***Orientation-aware feature vector:** permute elements in feature vector according to the given orientation*
  - ***Structured SVM:** employed to obtain model parameter*
- **Progressive curvilinear path reconstruction** shows topological features in different levels of detail
- **Limitation**
  - Ambiguous stopping criterion
  - Fails to interpret varying thicknesses
  - Few parameters are manually chosen, *e.g., pre-defined length*

# Conclusions

- Generic curvilinear structure reconstruction models
  - *Retina, DNA filament, Road, Cracks, ...*
- Curvilinear features
  - Image gradients
  - Morphological filtering
- **Line segment**  $s_i = (\mathbf{x}_i, \ell_i, \theta_i)$  
- Find an optimal set of line segments
- **Stochastic modeling**
  - Maximize a posterior probability of lines segments for given image
- **Structured ranking learning**
  - Learn a function to evaluate the correspondence between lines and feature maps
- Progressive curvilinear path reconstruction algorithm
  - Provides topological features in different levels of detail

# Perspectives

- Multiscale approach
  - To take into account varying thickness
- Stochastic model
  - Hierarchical modeling
  - Bezier curve [[Bama 15](#)]
- Structured ranking learning
  - Employing non-line shape templates (bifurcation patterns [[Azzopardi 11](#)]) to evaluate rankings
- Speed up
  - Applying parallel MCMC sampler [[Verdie 14](#)]

**Thank you!**



# References

- [Frangi 98] A. F. Frangi, W. J. Niessen, K. L. Vincken & M. A. Viergever. Multiscale vessel enhancement filtering. In MICCAI, pages 130–137, 1998.
- [Staal 04] J. J. Staal, M. D. Abramoff, M. Niemeijer, M. A. Viergever & B. van Ginneken. Ridge based vessel segmentation in color images of the retina. IEEE TMI, vol. 23, no. 4, pages 501–509, 2004.
- [Law 08] M. W. Law & A. Chung. Three dimensional curvilinear structure detection using optimally oriented flux. In ECCV, pages 368–382, 2008.
- [Zhao 11] T. Zhao, J. Xie, F. Amat, N. Clack, P. Ahammad, H. Peng, F. Long & E. Myers. Automated reconstruction of neuronal morphology based on local geometrical and global structural models. Neuroinformatics, vol. 9, no. 2–3, pages 247–261, 2011.
- [Wang 11] Y. Wang, A. Narayanaswamy & B. Roysam. Novel 4D open-curve active contour and curve completion approach for automated tree structure extraction. In CVPR, pages 1105–1112, 2011.
- [Peng 11] H. Peng, F. Long & G. Myers. Automatic 3D neuron tracing using all-path pruning. Bioinformatics, vol. 27, no. 13, pages 239–247, 2011.
- [Turetken 13] E. Türetken, C. Becker, P. Glowacki, F. Benmansour & P. Fua. Detecting irregular curvilinear structures in gray scale and color imagery using multi-directional oriented flux. In ICCV, pages 1553–1560, 2013.

# References

- [[Batool 12](#)] N. Batool & R. Chellappa. *Modeling and detection of wrinkles in aging human faces using marked point processes*. In ECCV Ws/Demos, pages 178–188, 2012.
- [[Jeong 14](#)] S.-G. Jeong, Y. Tarabalka & J. Zerubia. Marked point process model for facial wrinkle detection. In ICIP, pages 1391–1394, 2014.
- [[Lacoste 05](#)] C. Lacoste, X. Descombes & J. Zerubia. Point processes for unsupervised line network extraction in remote sensing. IEEE TPAMI, vol. 27, no. 10, pages 1568–1579, 2005.
- [[Hu 07](#)] J. Hu, A. Razdan, J. C. Femiani, M. Cui & P. Wonka. Road network extraction and intersection detection from aerial images by tracking road footprints. IEEE TGRS, vol. 45, no. 12, pages 4144–4157, 2007.
- [[Valero 10](#)] S. Valero, J. Chanussot, J. A. Bendiktsson, H. Talbot & B. Waske. Advanced directional mathematical morphology for the detection of the road network in very high resolution remote sensing images. Pattern Recognition Lett., vol. 31, no. 10, pages 1120–1127, 2010.
- [[Iyer 05](#)] S. Iyer & S. Sinha. A robust approach for automatic detection and segmentation of cracks in underground pipeline images. Image and Vision Computing, vol. 23, no. 10, pages 921–933, 2005.
- [[Azzopardi 11](#)] G. Azzopardi & N. Petkov, Detection of retinal vascular bifurcations by trainable V4-like filters, In CAIP, pages 451–459, 2011

# References

- [Chambon 10] S. Chambon, C. Gourraud, J.-M. Moliard & P. Nicolle. Road crack extraction with adapted filtering and Markov model-based segmentation. In VISAPP(2), pages 81–90, 2010.
- [Green 95] P. J. Green. Reversible jump Markov chain Monte Carlo computation and Bayesian model determination. *Biometrika*, vol. 82, no. 4, pages 771–732, 1995.
- [Green 01] P. J. Green & A. Mira. Delayed rejection in reversible jump Metropolis-Hastings. *Biometrika*, vol. 88, no. 4, pages 1035–1053, 2001.
- [Kirkpatrick 83] S. Kirkpatrick, C. D. Gelatt & M. P. Vecchi. Optimization by Simulated Annealing. *Science*, vol. 220, no. 4598, pages 671–680, 1983.
- [Joachims 09] T. Joachims, T. Finley & C.-N. Yu. Cutting-plane training of structural SVMs. *Machine Learning*, vol. 77, no. 1, pages 27–59, 2009.
- [Corneil 01] D. G. Corneil, F. F. Dragan, M. Habib & C. Paul. Diameter determination on restricted graph families. *Discrete Applied Mathematics*, vol. 113, no. 2–3, pages 143–166, 2001.
- [Verdié 14] Y. Verdié & F. Lafarge. Detecting parametric objects in large scenes by Monte Carlo sampling. *IJCV*, vol. 106, no. 1, pages 57–75, 2014.
- [Bama 15] K. Bama, T. Sziranyi, M. Borda, & O. Laviaille, Marked point processes for enhancing seismic fault patterns, *Journal of Applied Geophysics* no. 118, pages 115–123, 2015