

# A Causal Hierarchical Markov Framework for the Classification of Multiresolution and Multisensor Remote Sensing Images

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# The Addressed Problem

## Joint availability of images at different (and high) resolutions thanks to current missions

- Optical: Pléiades, PRISMA, Sentinel-2, WorldView-3, SPOT-6/7, etc.
- SAR: COSMO-SkyMed Second Generation, Sentinel-1, TerraSAR-X, RADARSAT Constellation, etc.

## The challenge of joint multisensor and multiresolution fusion

- To develop classification methods that benefit from all available data

## Proposed approach: hierarchical latent Markov random fields (MRF)

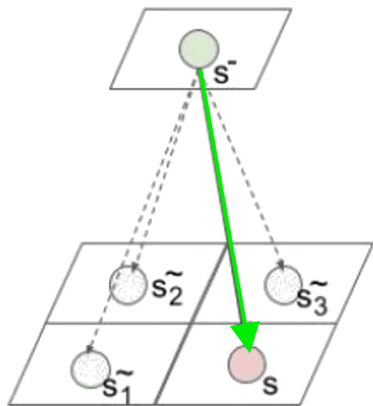
- General framework for causal hierarchical spatial-contextual Markov modeling
- Proof of causality and inference formulation
- Specific algorithm incorporating spatial Markov chains

# The Proposed Framework

# The Rationale

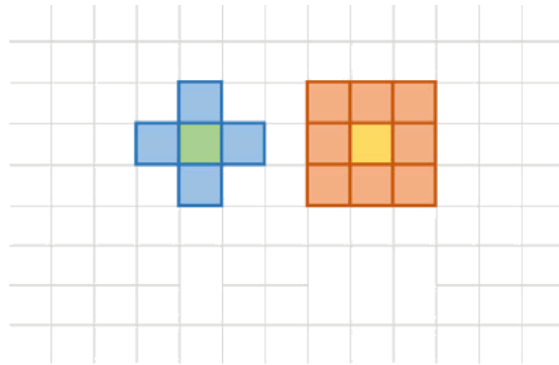
## Hierarchical MRF on quadtrees

Causal  
Efficient non-iterative inference  
Does not model spatial information within each scale



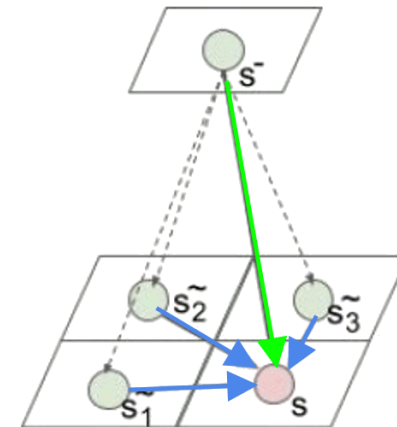
## Planar MRF

Models spatial information  
Generally non-causal  
Usually iterative inference



## Proposed framework

Markovianity on scale and within each layer  
Multiresolution fusion through tree topology  
Multisensor fusion at each scale through ensemble learning



# Model Assumptions

Quadtree topology

Total order relation  $\prec$  on each layer of the quadtree

Neighborhood relation  $\triangleleft$  consistent with  $\prec$  on each layer

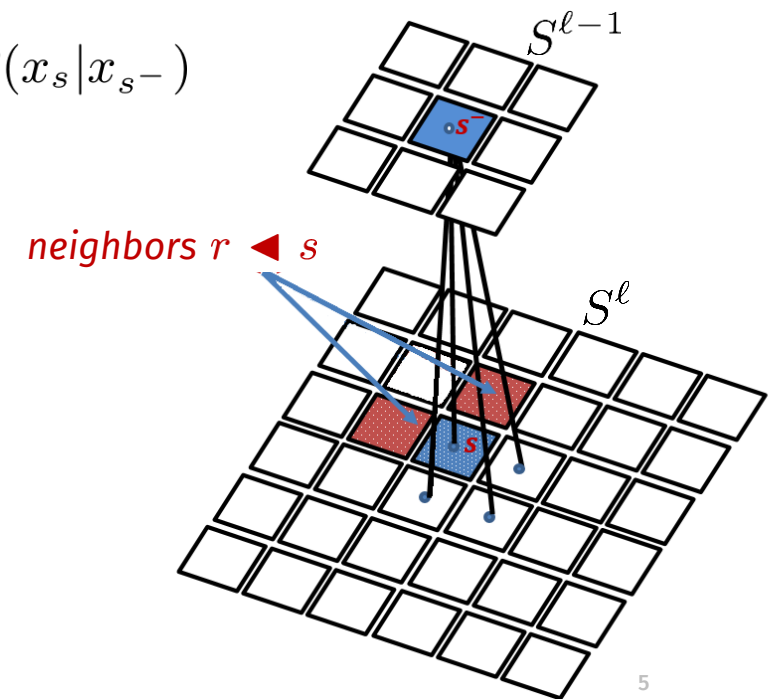
Markovianity of labels across scales and in each layer (with respect to  $\triangleleft$ )

$$P(\mathcal{X}^\ell | \mathcal{X}^{\ell-1}, \mathcal{X}^{\ell-2}, \dots, \mathcal{X}^0) = P(\mathcal{X}^\ell | \mathcal{X}^{\ell-1}) \propto \prod_{s \in S^\ell} P(x_s | x_r, r \triangleleft s) P(x_s | x_{s^-})$$

$$P(\mathcal{X}^0) = \prod_{s \in S^0} P(x_s | x_r, r \triangleleft s)$$

Conditional independence of feature vectors given labels

$$P(\mathcal{Y} | \mathcal{X}) = \prod_{s \in S} P(y_s | x_s) = \prod_{\ell=0}^L \prod_{s \in S^\ell} P(y_s | x_s)$$



# Methodological Properties

## Model causality

**Theorem.** The joint distribution of all labels and feature vectors in the quad-tree is entirely defined by the parent-child transition probabilities, the past neighbor transition probabilities, and the pixelwise data conditional likelihoods.

## Marginal posterior mode (MPM)

**Theorem.** Under mild assumptions:

$$P(x_s) = \sum_{x_{s-}} P(x_s|x_{s-})P(x_{s-}),$$

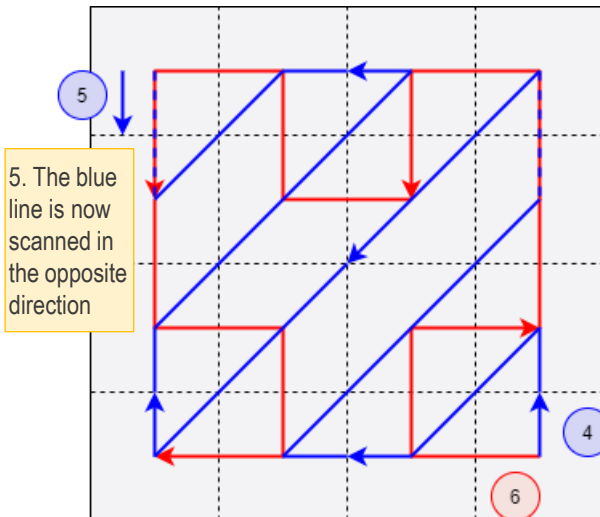
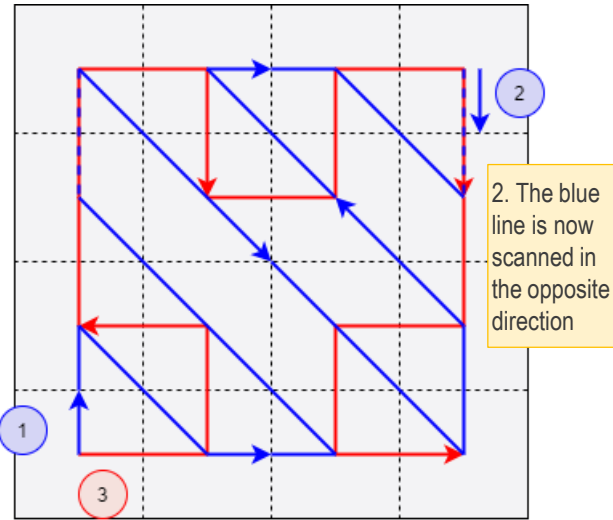
$$P(x_s|\mathcal{C}_s, \mathcal{D}_s) \propto P(x_s|\mathcal{D}_s)P(x_s|x_{s-})P(x_{s-})P(x_s)^{-|\mathcal{C}_s|} \\ \cdot \prod_{r \blacktriangleleft s} P(x_s|x_r)P(x_r),$$

$$P(x_s|\mathcal{Y}) = \sum_{\mathcal{C}_s} P(x_s|\mathcal{C}_s, \mathcal{D}_s)P(x_{s-}|\mathcal{Y}) \prod_{r \blacktriangleleft s} P(x_r|\mathcal{Y})$$

$$P(x_s|\mathcal{D}_s) \propto P(x_s|y_s) \prod_{t \in s^+} \sum_{x_t} \frac{P(x_t|\mathcal{D}_s)P(x_t|x_s)}{P(x_t)}$$

# The Proposed Algorithm

# Spatial Markov Chain



Relations  $\square$  and  $\square$  are defined by a 1D scan of each layer of the quadtree • **Markov chain**

Four zig-zag scans and two Hilbert curve scans

Symmetric visit on each pixel, no directional artifacts

$$P(x_s) = \sum_{x_{s-}} P(x_s | x_{s-}) P(x_{s-}),$$

$$P(x_s | \mathcal{C}_s, \mathcal{D}_s) \propto P(x_s | \mathcal{D}_s) P(x_s | x_{s-}) P(x_{s-}) P(x_s)^{-|\mathcal{C}_s|} \cdot \prod_{r \triangleleft s} P(x_s | x_r) P(x_r),$$

$$P(x_s | \mathcal{Y}) = \sum_{\mathcal{C}_s} P(x_s | \mathcal{C}_s, \mathcal{D}_s) P(x_{s-} | \mathcal{Y}) \prod_{r \triangleleft s} P(x_r | \mathcal{Y})$$

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# Role of Decision Tree Ensembles

Feature vectors from input multisensor images at the same spatial resolution are integrated in the model through the pixelwise posteriors.

Modeled through ensemble learning • **Gradient Boosted Regression Trees (GBRT)**

Random ensemble of decision trees

Trained using a boosting approach

$$P(x_s) = \sum_{x_{s-}} P(x_s|x_{s-})P(x_{s-}),$$

$$P(x_s|\mathcal{C}_s, \mathcal{D}_s) \propto P(x_s|\mathcal{D}_s)P(x_s|x_{s-})P(x_{s-})P(x_s)^{-|\mathcal{C}_s|} \\ \cdot \prod_{r \blacktriangleleft s} P(x_s|x_r)P(x_r),$$

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# Experimental Results

# Data Sets for Experiments





# Classification Accuracies

resolution 1 m	urban	agri	rangeland	forest	water	wet soil	bare soil	OA
Single-res. MRF after resampling	98.58	99.12	92.23	36.98	100	98.30	96.82	96.03
Method in (Moser et al., 2016)	99.70	99.21	97.34	64.92	100	100	99.66	98.60
Method in (Montaldo et al., 2019b)	100	99.07	99.65	100	99.65	100	100	99.12
<b>Proposed method</b>	<b>99.54</b>	<b>98.26</b>	<b>99.91</b>	<b>86.40</b>	<b>99.80</b>	<b>100</b>	<b>100</b>	<b>98.58</b>
resolution 2 m	urban	agri	rangeland	forest	water	wet soil	bare soil	OA
Method in (Montaldo et al., 2019b)	99.54	98.06	99.91	78.74	99.53	100	100	98.01
<b>Proposed method</b>	<b>99.49</b>	<b>98.08</b>	<b>99.91</b>	<b>79.52</b>	<b>99.84</b>	<b>100</b>	<b>100</b>	<b>98.03</b>
resolution 4 m	urban	agri	rangeland	forest	water	wet soil	bare soil	OA
Method in (Montaldo et al., 2019b)	99.09	96.81	99.82	64.86	98.72	100	100	96.36
<b>Proposed method</b>	<b>99.09</b>	<b>96.81</b>	<b>99.82</b>	<b>64.86</b>	<b>98.72</b>	<b>100</b>	<b>100</b>	<b>96.36</b>

Alessandria

Similar accuracies for the developed algorithm and for a previous method, which is another special case of the proposed framework and which uses Markov mesh

Very different computational complexity:  $O(C^3)$  for proposed method vs  $O(C^4)$ ,  $O(C^5)$ , etc., for Markov mesh approach (where  $C$  is the number of classes)

resolution 1.25 m	containers	vegetation	asphalt	buildings	sea	OA
Single-res. MRF after resampling	63.97	74.81	98.64	99.30	76.24	94.58
Adaptation of (Moser et al., 2016)	75.06	85.32	95.89	97.75	99.36	95.11
Method in (Montaldo et al., 2019b)	87.08	33.27	95.17	99.04	97.18	96.36
<b>Proposed method</b>	<b>86.97</b>	<b>34.57</b>	<b>97.82</b>	<b>99.36</b>	<b>100</b>	<b>96.90</b>
resolution 2.5 m	containers	vegetation	asphalt	buildings	sea	OA
Method in (Montaldo et al., 2019b)	86.31	32.71	94.87	100.00	96.04	96.63
<b>Proposed method</b>	<b>86.92</b>	<b>33.08</b>	<b>97.29</b>	<b>100.00</b>	<b>100</b>	<b>97.00</b>
resolution 5 m	containers	vegetation	asphalt	buildings	sea	OA
Method in (Montaldo et al., 2019b)	87.98	25.39	96.55	100.00	88.88	96.01
<b>Proposed method</b>	<b>87.98</b>	<b>25.39</b>	<b>96.55</b>	<b>100.00</b>	<b>88.88</b>	<b>96.01</b>

Haiti

Montaldo A., Fronda L., Hedhli I., Moser G., Zerubia J., Serpico S. B., 2019b. Joint classification of multiresolution and multisensor data using a multiscale Markov mesh model. IEEE IGARSS 2019



# Classification Accuracies

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Alessandria

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resolution 4 m	urban	agri	rangeland	forest	water	wet soil	bare soil	OA
Method in (Montaldo et al., 2019b)	99.09	96.81	99.82	64.86	98.72	100	100	96.36
<b>Proposed method</b>	<b>99.09</b>	<b>96.81</b>	<b>99.82</b>	<b>64.86</b>	<b>98.72</b>	<b>100</b>	<b>100</b>	<b>96.36</b>

Lower overall and/or class-wise accuracies from previous approaches to multiresolution image classification

resolution 1.25 m	containers	vegetation	asphalt	buildings	sea	OA
Single-res. MRF after resampling	63.97	74.81	98.64	99.30	76.24	94.58
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Method in (Montaldo et al., 2019b)	86.31	32.71	94.87	100.00	96.04	96.63
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resolution 5 m	containers	vegetation	asphalt	buildings	sea	OA
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Haiti

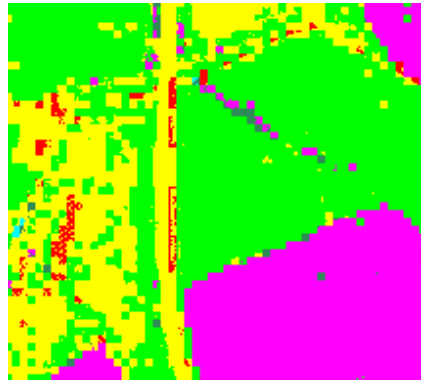
Moser G., De Giorgi A., Serpico S. B., 2016. Multiresolution supervised classification of panchromatic and multispectral images by Markov random fields and graph cuts. IEEE TGRS., 54: 5054-5070



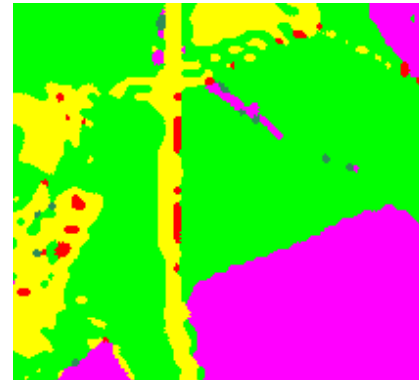
# Classification Maps (details)



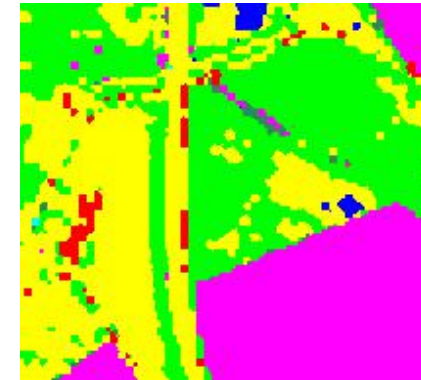
Image at the finest observed resolution



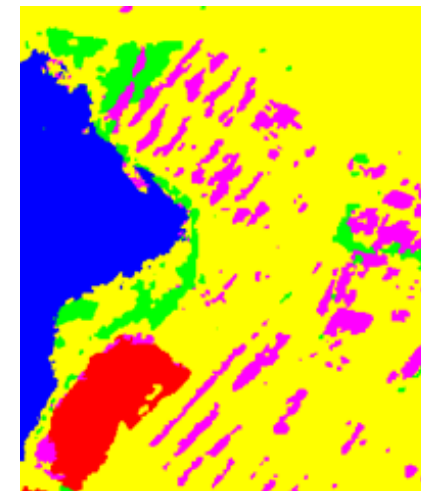
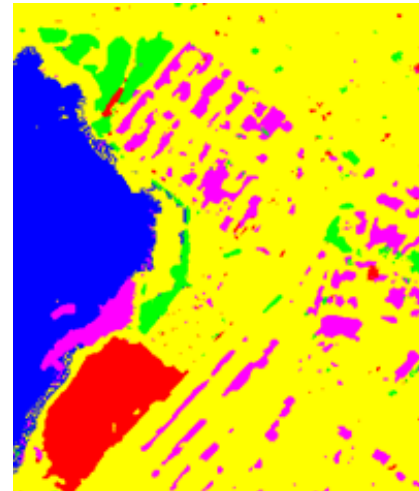
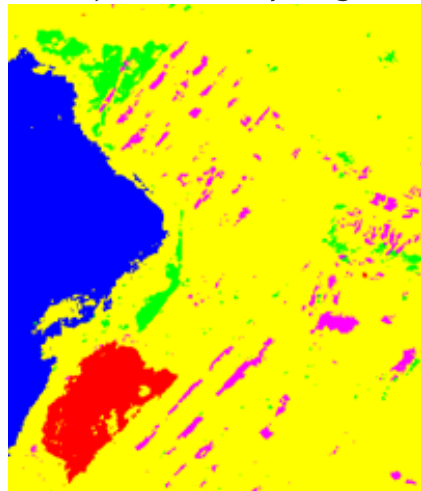
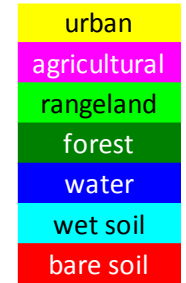
Single-resolution MRF after resampling



Adaptation of (Moser et al., 2016)



Proposed method



# Conclusion

- General hierarchical causal Markov framework for joint multiresolution and multisensor classification
- Within this framework, a specific algorithm based on spatial Markov chains
- Experimental results suggest effectiveness in the application to multiresolution fusion with optical-SAR and panchromatic-multispectral imagery
- Advantages in terms of accuracy or computational complexity as compared to previous multiresolution classification methods
- Allows classifying at all scales in the quadtree
- Future extensions include integrating with CNNs or adaptive multiresolution topologies



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