N. N. Osipov. *Littlewood–Paley–Rubio de Francia inequality for the Walsh system*

Let $\{I_m\}_{m\in\mathbb{N}}$ be mutually disjoint intervals in $\mathbb{Z}$. In 1983, Rubio de Francia proved that for $f \in L^p([0,1])$, we have

$$\Big\|\Big(\sum_m \big|(\widehat{f}\, \mathbb{1}_{I_m})^\vee\big|^2\Big)^{1/2}\Big\|_{L^p} \le C_p \|f\|_{L^p}, \quad 2 \le p \le \infty, \tag{1}$$

where the constant C_p does not depend on the collection $\{I_m\}$ and the function f.

The Walsh system consists of various products of the Rademacher functions. We recall that the Rademacher functions are defined by the formula $r_k(x) = \operatorname{sign}\sin 2^k\pi x$. The Walsh system is an orthonormal basis in $L^2([0,1])$ and can be considered as some discretization of the trigonometric basis $\{e^{2\pi i\, nt}\}$. We will prove Rubio de Francia estimate for the Walsh basis. For this purpose, we will use the connection of the Walsh system with the notion of martingale as well as some interesting combinatorial arguments.