Imaging the Solar Interior with Seismic Holography

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Abstract

Despite 20 years of high-quality observations of Doppler velocities at the solar surface, the internal structure of the Sun still presents some mysteries. As it is impossible to treat all the data at once, some averages have to be performed a priori. These averages are based on some physical understanding or on the analysis of wave propagation in a reference solar-like background. We analyze and improve a classical method called seismic holography which aims at propagating the surface data to any location in the solar interior.

Keywords: helioseismology, holography, inverse problem

1 Introduction

Time-distance helioseismology [1] aims at recovering subsurface properties of the Sun from the observation of the Doppler velocities at the surface. Due to convection, the data are stochastic and one generally analyzes the cross-covariances of the wavefield ψ between any two points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ at the solar surface

$$C^\omega(\boldsymbol{r}_1, \boldsymbol{r}_2) = \psi^*(\boldsymbol{r}_1, \omega)\psi(\boldsymbol{r}_2, \omega). \quad (1)$$

Here ω denotes the frequency and the product of the wavefield in frequency space corresponds to a convolution in the time domain. Every 45 s, a map of size 4k×4k pixels of Doppler velocities on the solar surface is available which lead to 10^{13} possible cross-covariance measurements at each time. The number of available frequencies is linked to the observation time (e.g. 5000 frequencies for 4 days of observations). It is thus impossible to save all measurements and some averagings in space and/or frequency have to be performed a priori. One possibility is called seismic holography which aims at propagating the wavefield from the surface to any target point in the interior based on the knowledge of the wave equation satisfied by the wavefield. Here, we analyze this method and propose some possible improvements.

2 Forward model

We suppose that the wavefield satisfies an acoustic wave equation [2]

$$-(\Delta + k^2)\psi - \frac{2\mathrm{i}\omega}{\rho^{1/2}c}\rho\boldsymbol{u}\cdot\nabla\left(\frac{\psi}{\rho^{1/2}c}\right) = s, \quad (2)$$

where ρ and c are the density and sound speed taken from a standard solar model, γ is the attenuation, $\boldsymbol{u}$ a flow term, and s a stochastic source of excitation. The local wavenumber k is given by

$$k^2 = \frac{(\omega^2 + 2\mathrm{i}\omega\gamma) - \omega_\mathrm{c}^2}{c^2}, \quad \omega_\mathrm{c}^2 = \rho^{1/2}c^2\Delta(\rho^{-1/2}).$$

A radiation boundary condition that takes into account the exponential decay of the density close to the solar surface is used to complement Eq. 2 [3]. The wave equation is solved using high-order finite elements on a mesh adapted to the strong stratification of the Sun [4].

At first order, the perturbation to the cross-covariance with respect to a reference model is linked to perturbations in the background medium via a sensitivity kernel [4]

$$\delta C^\omega(\boldsymbol{r}_1, \boldsymbol{r}_2) = \int_V K^\omega(\boldsymbol{x}; \boldsymbol{r}_1, \boldsymbol{r}_2)\delta q(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \quad (3)$$

where $q \in \{\rho, c, \gamma, \boldsymbol{u}\}$. The kernels K can be computed from the knowledge of the Green's function of the wave equation.

Aim: *Averaging the data δC in space and frequency in order to be as sensitive as possible to a scatterer δq at a given location $\boldsymbol{x}$ in the solar interior.*

3 Seismic holography

Seismic holography is one possibility to average the data. From the knowledge of the wavefield everywhere on the observed surface A of the Sun, one can generate the hologram Φ_α^ω

$$\Phi_\alpha^\omega(\boldsymbol{x}) = \int_A H_\alpha^\omega(\boldsymbol{x}, \boldsymbol{r}')\psi(\boldsymbol{r}', \omega)\mathrm{d}\boldsymbol{r}', \quad (4)$$

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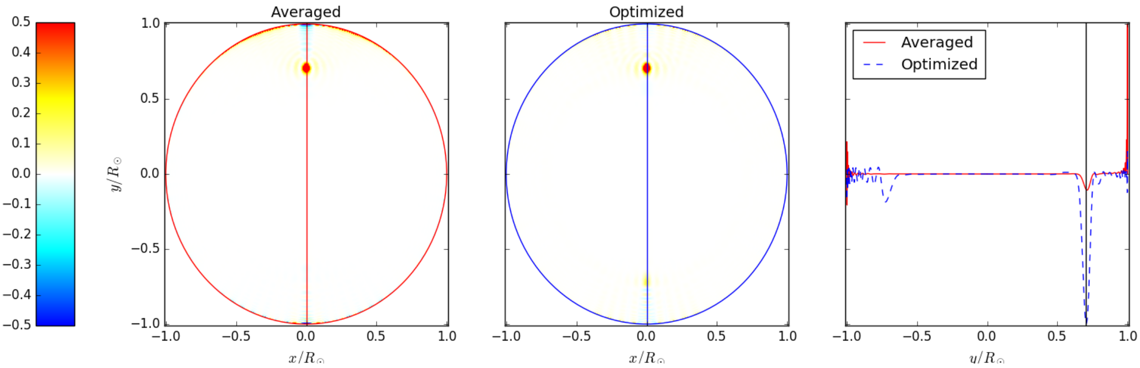

Figure 1: Sound speed sensitivity kernel $\langle \mathcal{K}^{\omega}_{\alpha\beta}(\boldsymbol{x}; \boldsymbol{x}_s, \boldsymbol{x}_s)\rangle$ for a scatterer located at a depth of $z_s = 0.7R_\odot$ along the polar axis. Left: the frequencies are directly averaged ($W = 1$), middle: optimal averaging obtained by principal component analysis, right: cut of the two kernels along the polar axis.

which aims at estimating the wavefield ψ at a target location $\boldsymbol{x}$. H^{ω}_{α} is a wave propagator, for example the Green's function associated to a wave equation, but it can be seen as a general weighting of the observations.

Similarly to the cross-covariance of the wavefield, the cross-covariance between two holograms is defined as

$$I^{\omega}_{\alpha\beta}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \Phi^{\omega *}_{\alpha}(\boldsymbol{x}_1)\Phi^{\omega}_{\beta}(\boldsymbol{x}_2) = \int_A \int_A H^{\omega *}_{\alpha}(\boldsymbol{x}_1, \boldsymbol{r}) H^{\omega}_{\beta}(\boldsymbol{x}_2, \boldsymbol{r}') C^{\omega}(\boldsymbol{r}, \boldsymbol{r}') \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}'. \quad (5)$$

The measurement $I_{\alpha\beta}$ is thus an averaging of all the cross-covariances at the solar surface. Combining Eqs. 3 and 5, one can link the measurement $I^{\omega}_{\alpha\beta}$ to perturbations δq

$$\delta I^{\omega}_{\alpha\beta}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \int_V \mathcal{K}^{\omega}_{\alpha\beta}(\boldsymbol{x}; \boldsymbol{x}_1, \boldsymbol{x}_2)\delta q(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}. \quad (6)$$

By choosing properly the wave propagators, one can get sensitive to a physical parameter at a given target location in the solar interior. Ideally the averaging should lead to a kernel $\mathcal{K}$ that is localized close to the target location $\boldsymbol{x}$.

4 Optimizing the averaging

Due to convection, the data at one frequency are extremely noisy. One needs to average frequencies using a given weighting $w(\omega)$ such that the averaged measurement is given by

$$\langle \delta I^{\omega}_{\alpha\beta}(\boldsymbol{x}_1, \boldsymbol{x}_2)\rangle = \sum_{\omega} W(\omega)\delta I^{\omega}_{\alpha\beta}(\boldsymbol{x}_1, \boldsymbol{x}_2). \quad (7)$$

Usually, a constant weighting in some frequency bands is used. This is not optimal as the kernel is oscillating extremely rapidly as a function of frequency [2]. We use a principal component analysis in order to obtain the most significant averaging in the frequency domain. A representation of a sound speed kernel for constant and optimized weightings is shown in Fig. 1. One can see that the optimized version is more localized close to the target location with a way smaller contribution from the surface. This optimization is promising and will be used to improve the current capabilities of imaging the interior and farside of the Sun. Morever, it is unclear if the holography weightings given by Eq. 5 are optimal. We will perform a similar analysis to determine the most significant averaging of the data in space depending on the type of scatterers.

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