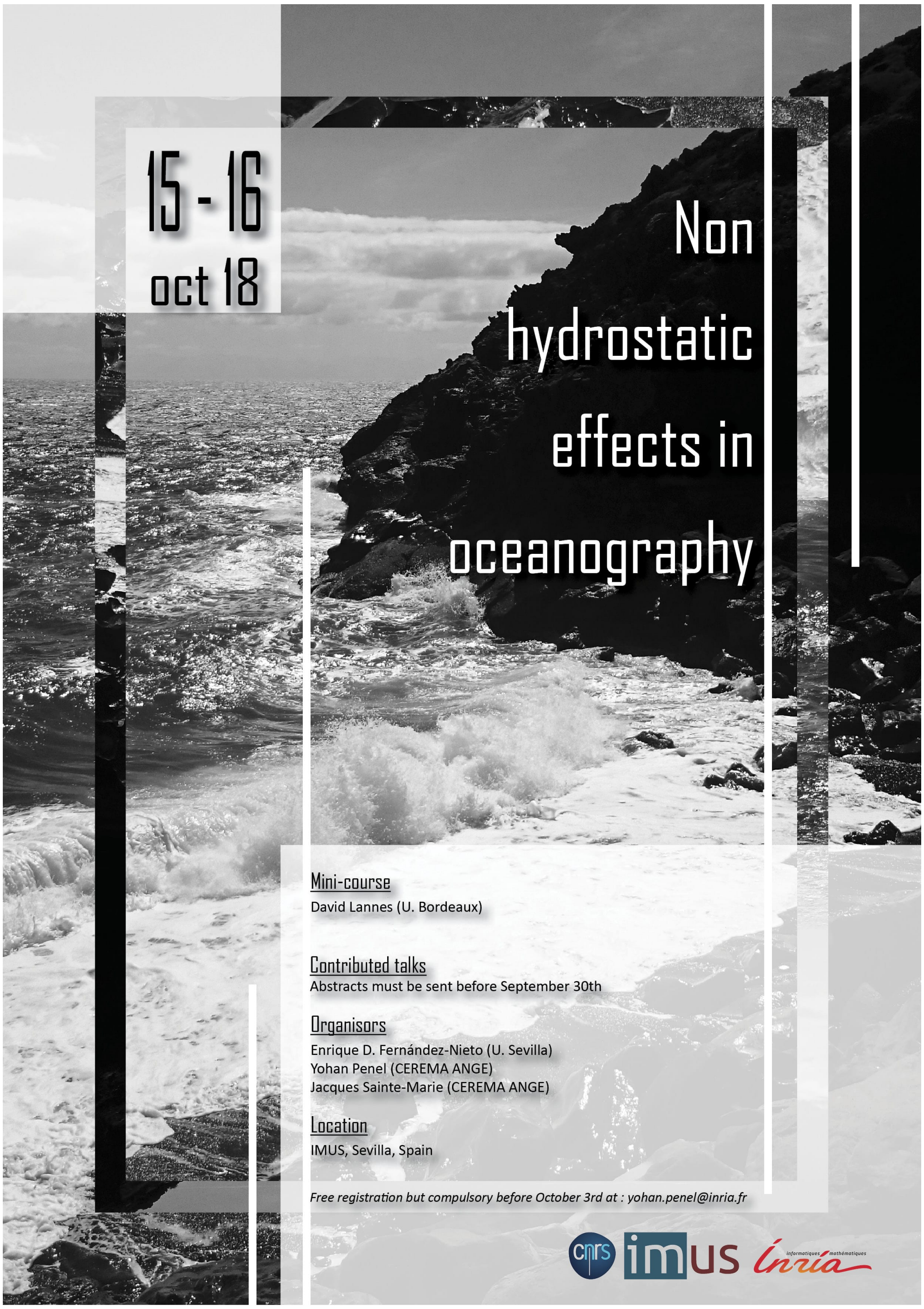
Workshop

Non-hydrostatic effects in oceanography

October 15-16, 2018

IMUS, Sevilla

PROGRAMME



Practical information

Organising committee

- E.D. Fernández-Nieto, Dpto. Matemática Aplicada I, E.T.S Arquitectura, Universidad de Sevilla, `edofer@us.es`
- Y. Penel, CEREMA team ANGE, Inria Paris, `yohan.penel@inria.fr`
- J. Sainte-Marie, CEREMA team ANGE, Inria Paris, `jacques.sainte-marie@inria.fr`

Location of the workshop

The workshop will take place at **IMUS**, 35 Avenida de la Reina Mercedes in the room **Seminario I**, Building Celestino Mutis. By bus, you can use lines 03, 06 or 34 and stop at Reina Mercedes (E.S.I. Informática). The conference dinner on Monday evening will take place at the "La Raza" restaurant located in Avenida María Luisa at 9pm.

Website

https://team.inria.fr/ange/workshop-non-hydrostatic-effects-in-oceanography/

Funds

This workshop is funded by French committee for scientific research (CNRS) through an international project (PICS 07701) called "accurate numerical methods for free surface flows" by IMUS (institute of Mathematics of Sevilla) and the University of Sevilla (Dpto. Matemática Aplicada I). It is the second edition after the first workshop organised in Paris in November 2017 ("an overview on free surface flows").

Program At A Glance

Time	Monday 15/10	Tuesday 16/10
9h00		
9h30		**David Lannes** *Modelling non-hydrostatic effects in free surface flows* – Part 2 Chairman: M. Parisot
10h00	**Welcome**	
10h30	**David Lannes** *Modelling non-hydrostatic effects in free surface flows* – Part 1 Chairman: E. Fernández-Nieto	
11h00		**Coffee Break**
11h30		**Yohan Penel** – see page 7 Chairman: T. Morales de Luna
12h00	**Coffee Break**	**Virgile Dubos** – see page 5 Chairman: T. Morales de Luna
12h30	**Rafael Rodríguez Galván** – see page 7 Chairman: J. Macias	**Martin Parisot** – see page 6 Chairman: T. Morales de Luna
13h00	**Krisztián Benyó** – see page 4 Chairman: J. Macias	**Lunch Break**
13h30	**Boris Haspot** – see page 6 Chairman: J. Macias	
14h00	**Lunch Break**	
14h30		**Work sessions**
15h30	**Arnaud Duran** – see page 4 Chairman: E. Audusse	
16h30	**Edoardo Bocchi** – see page 5 Chairman: E. Audusse	
17h00	**Coffee Break**	
17h30	**Cipriano Escalante Sanchez** – see page 6 Chairman: J. Sainte-Marie	
18h00	**Tomas Morales de Luna** – see page 6 Chairman: J. Sainte-Marie	
18h30–19h00		

Lectures

Invited talk

A new model of shoaling and breaking waves – Numerical aspects, one and two-dimensional applications

Arnaud Duran
Lyon 1, France

Wave propagation in coastal zones involves very complex mechanisms, channelling considerable modelling and numerical issues. If most of nonlinear processes are generally captured by Boussinesq-type models, these equations are energy-conservative and hence intrinsically unable to describe dissipative mechanisms such as wave breaking for instance. For that purpose, we introduce in this work a new two-dimensional fully nonlinear dispersive model able to account for the underlying turbulent effects. The approach is characterized by the presence of a new variable relying on the vertical velocity variations, referred to as enstrophy, accounting for the turbulent energy. Once considered the transport of the associated symmetric tensor, embedded in the hyperbolic part of the equations, the proposed model shares a strucutre similar to the Green-Naghdi equations and can therefore be integrated on the basis of any existing numerical model. Following recent works on the classical Green-Naghdi equations, the system is reformulated in an asymptotically equivalent form, allowing to enhance its dispersive properties and bring computational efficiency, revoking the time-dependency of the elliptic operator acting on the velocity field. As an extension of recent works, we consider a discontinuous Galerkin discretization of the system, based on a decoupling between hyperbolic and non-hydrostatic parts. One and two-dimensional benchmarks involving breaking wave propagation over non-trivial topographies are proposed to validate the model. Comparisons with experimental data confirm the efficiency of the present strategy, highlighting the enstrophy as a robust and reliable tool for the detection and description of wave breaking, even in the two dimensional frame.

In collaboration with Gaël Richard (University Savoie Mont-Blanc), Maria Kazakova (University of Toulouse) and Benoît Fabrèges (University Claude Bernard Lyon 1).

Contributed talks

Wave-structure interaction for long wave models in the presence of a freely moving object on the bottom

Krisztián Benyó
Bordeaux, France

The Cauchy problem for the water waves equations is consid- ered in a fluid domain which has a free surface on the upper vertical limit and a flat bottom on which a solid object moves horizontally, its motion determined by the pressure forces exerted by the fluid. The shallow water asymptotic regime for the weakly nonlinear Boussinesq system is presented, coupled with Newtons equation characterizing the solid motion. Using the particular structure of the coupling terms, one is able to go beyond the standard scale for the existence time of solutions to the Boussinesq system with a moving bottom. An extended numerical study has also been carried out for the system. We observed qualitative differences for the transformation of a passing wave over a moving solid object as compared to an immobile one. The movement of the solid not only influences wave attenuation but it affects the shoaling process as well as the wave breaking. The importance of the coefficient of friction is also highlighted, influencing qualitative and quantitative properties of the coupled system. Furthermore, we showed the hydrodynamic damping effects of the waves on the solid motion, reminiscent of the so-called dead water phenomenon.

Floating structures in axisymmetric shallow water: local well-posedness and decay test

Edoardo Bocchi
Bordeaux, France

The floating structure problem describes the interaction between surface water waves and a floating body, generally a boat or a wave energy converter. As shown by Lannes the equations for the fluid motion can be reduced to a set of two evolution equations on the surface elevation and the horizontal discharge. The presence of the object is accounted for by a constraint on the discharge under the object; the pressure exerted by the fluid on this object is then the Lagrange multiplier associated with this constraint. Our goal is to prove the well-posedness of this fluid-structure interaction problem in the shallow water approximation under the assumption that the flow is axisymmetric without swirl. We write the fluid equations as a quasilinear hyperbolic mixed initial boundary value problem and the solid equation as a second order ODE coupled to the fluid equations.

We prove the local in time well-posedness for this coupled problem, provided some compatibility conditions on the initial data are satisfied. Finally we consider the decay test.

Non-hydrostatic shallow water model and Gradient Discretization Method

Virgile Dubos
Sorbonne University, France

We propose to use the Gradient Discretisation Method (GDM) to analyse numerical schemes for the elliptic part of a dispersive Shallow Water system which comprises the Green-Naghdi model. GDM is a framework which contains classic discretisation schemes for diffusion problems of different kinds: linear or non-linear, steady-state or time-dependent. The schemes may be conforming or non-conforming, low or high order, and may be built on very general meshes. We present the key properties that are required to prove the convergence of a GDM, and the method is applied on the studied problem.

In collaboration with C. Guichard (Sorbonne University, France), Y. Penel (CEREMA, France) and J. Sainte-Marie (CEREMA, France).

An efficient hyperbolic relaxation system for dispersive water waves

Cipriano Escalante Sánchez
Málaga, Spain

When modelling and simulating geophysical flows, the Shallow-Water equations (SWE), is often a good choice as an approximation of the Navier-Stokes equations. Nevertheless, SWE do not take into account effects associated with dispersive water waves. The incompressible depthintegrated non-hydrostatic system derived by Sainte-Marie et al in [3] for dispersive water waves is considered in this work.

The incompressibility equation of the original system proposed in [3] is replaced by an artificial compressibility equation, where errors are transported to the domain boundaries with the maximal admissible speed c following the so-called *Hyperbolic Divergence Cleaning* ideas applied in magneto-hydrodynamic (see [2]). The resulting first-order system formally tends to the proposed in [3] when $c \to \infty$. Moreover, it is shown that the system is hyperbolic, and it satisfies an extra energy conservation law. Some linear dispersive properties are also discussed.

We then use arbitrary high order accurate (ADER) discontinuous Galerkin finite element schemes with an a posteriori subcell finite volume limiter in order to solve the proposed PDE system numerically. The stability of the scheme as well as the positivity of the water height is achieved by the a posteriori Multi-dimensional Optimal Order Detection (MOOD [4]), combine with a robust TVD finite volume PVM method (see [1]). Thus, the resulting scheme is high-order accurate, well-balanced and positive preserving for the water height.

Exacts travelling wave solutions are computed, and the order of the scheme is validated. The method has been applied to idealized and challenging physical situations in bidimensional domains that involve nearshore breaking, and an efficient breaking mechanism is also proposed. Agreement with laboratory data is excellent. This technique adapts well to GPU-architectures, resulting in an accurate and efficient method.

In collaboration with M. Dumbser (Trento, Italy) and M.J. Castro Díaz (Málaga, Spain).

Bibliography

[1] M.J. Castro, E. Fernández-Nieto, *A class of computationally fast first order finite volume solvers: PVM methods*, **SIAM Journal on Scientific Computing** 34(4) (2012) 173–196.

[2] A. Dedner, F. Kemm, D. Kröner, C.-D. Munz, T. Schnitzer, M. Wesenberg, *Hyperbolic Divergence Cleaning for the MHD Equations*, **Journal of Computational Physics** 175 (2002) 645–673.

[3] M.-O. Bristeau, A. Mangeney, J. Sainte-Marie, N. Seguin, *An energy-consistent depth-averaged Euler system: Derivation and properties*, **Discrete and Continuous Dynamical Systems Series B** 20(4) (2015) 961–988.

[4] M. Dumbser, O. Zanotti, R. Loubère, S. Diot, *A posteriori subcell limiting of the discontinuous Galerkin finite element method for hyperbolic conservation laws*, **Journal of Computational Physics** 278 (2014) 47–75.

Global weak solution of a compressible viscous model

Boris Haspot

University Paris Dauphine & Inria ANGE, France

We consider Navier-Stokes equations for compressible viscous fluids in the one-dimensional case with general viscosity coefficients. We prove the existence of global weak solution when the initial momentum $\rho_0 u_0$ belongs to the set of the finite measure $\mathcal{M}(\mathbb{R})$ and when the initial density ρ_0 is in the set of bounded variation functions $BV(\mathbb{R})$. In particular it includes at the same time the case of initial momentum which are Dirac masses and initial density which admit shocks. We can observe in particular that this type of initial data have infinite energy. Furthermore we show that if the coupling between the density and the velocity is sufficiently strong then the initial density which admits initially shocks is instantaneously regularizing and becomes continuous. This coupling is expressed via the regularity of the so called effective velocity $v = u + \frac{\mu(\rho)}{\rho^2}\partial_x \rho$ with $\mu(\rho)$ the viscosity coefficient. Inversely if the coupling between the initial density and the initial velocity is too weak (it means $\rho_0 v_0 \in \mathcal{M}(\mathbb{R})$) then we prove the existence of weak energy in finite time but the density remains a priori discontinuous on the time interval of existence.

An efficient two-layer non-hydrostatic approach for dispersive water waves

Tomas Morales de Luna

Córdoba, Spain

We propose to study non-hydrostatic shallow water type systems. Usually, a layer averaged model is introduced that includes dispersive effects via a non-hydrostatic pressure term. This type of models may be generalized by using a multilayer approach. Here we propose a two-layer depth-integrated non-hydrostatic system with improved dispersion relations. This improvement is obtained by means of three free parameters: two of them related to the representation of the pressure at the interface and a third one that controls the relative position of the interface with respect to the total height. These parameters are then optimized in order to improve the dispersive properties of the resulting system. The optimized model shows good linear wave characteristics up to $kH \approx 10$, that can be improved for long waves.

In collaboration with C. Escalante Sánchez (Málaga, Spain), E.D. Fernández Nieto (Sevilla, Spain) and M.J. Castro Díaz (Málaga, Spain).

Entropy-satisfying scheme for a hierarchy of dispersive layerwise models

Martin Parisot

Inria ANGE, France

This work is devoted to the numerical resolution in multidimensional framework of a hierarchy of reduced models of the water wave equations, such as the Serre-Green-Naghdi model. A particular attention is paid to the dissipation of mechanical energy at the discrete level, that act as a stability argument of the scheme, even with source terms such space and time variation of the bathymetry. In addition, the analysis leads to a natural way to deal with dry areas without leakage of energy. To illustrate the accuracy and the robustness of the strategy, several numerical experiments are carried out. In particular, we illustrate the convergence of the model to the solitary wave of the water wave equations.

A hierarchy of non-hydrostatic models for free-surface flows

Yohan PENEL
CEREMA ANGE, France

In the purpose of approximating the incompressible Euler system with free surface, we present a general framework to construct models comprising non-hydrostatic effects. Hence, a hierarchy of new models is derived by means of a layerwise discretisation approach. To assess these models, we use a rigorous derivation process based on a Galerkin-type approximation along the vertical axis of the velocity field and of the pressure. It is also proven that all of them satisfy an energy equality. In addition, we analyse the linear dispersion relation of these models and prove that the latter relations converge to the dispersion relation for the Euler equations when the number of layers goes to infinity.

On Stable Schemes for Large-Scale Non-Hydrostatic Boussinesq Equations

Rafael RODRÍGUEZ GALVÁN
Cádiz, Spain

This work is focused on the numerical approximation of the Boussinesq Equations, with density depending on temperature and salinity, in large-scale oceanic domains, where the ratio ε between vertical and horizontal scales is small. We present some recent approaches for the space discretization by means of the Finite Elements or Discontinuous Galerkin methods which guarantee well-posedness of the equations, regardless of the fact that $\varepsilon \to 0$, avoiding therefore the hydrostatic approximation $\varepsilon = 0$. Some numerical tests are shown using first order semi-implicit scheme for time discretization.

List of participants

Name	First name	Institution
Audusse	Emmanuel	Université Paris 13
Benyo	Krisztian	Institut de Mathématiques de Bordeaux
Bocchi	Edoardo	Institut de Mathématiques de Bordeaux
Caravaca García	Cristina	Instituto de Matemáticas de la Universidad de Sevilla
Delgado-Sánchez	Juan Manuel	University of Seville
Dubos	Virgile	Sorbonne University
Duran	Arnaud	Université Lyon 1
Escalante Sánchez	Cipriano	Universidad de Málaga
Fernández-García	Soledad	Universidad de Sevilla
Fernández-Nieto	Enrique	Universidad de Sevilla
Garres Díaz	José	Universidad de Cádiz
Gómez Mármol	Macarena	Universidad de Sevilla
Guerrero Fernández	Ernesto	University of Málaga
Haspot	Boris	Inria
Lannes	David	Université de Bordeaux
Macias	Jorge	Universidad de Málaga
Marín Gayte	Irene	Universidad de Sevilla
Mendivil Cabot	Javier	Universidad de Sevilla
Morales de Luna	Tomás	Universidad de Córdoba
Narbona Reina	Gladys	Universidad de Sevilla
Parisot	Martin	Inria
Penel	Yohan	CEREMA-Inria
Rodriguez-Bellido	Maria Angeles	Universidad de Sevilla
Sainte-Marie	Jacques	Inria
Sánchez Muñoz	Isabel	Universidad de Sevilla