Équipe Projet ANGE

Inria-CEREMA-UPMC-CNRS

Sujet de thèse de M. Virgile Dubos

Titre

Numerical methods for the elliptic/parabolic parts of non-hydrostatic fluid models

Encadrants

Cindy Guichard
Maître de conférence UPMC
✉ cindy.guichard@upmc.fr

Yohan Penel
Chargé de recherche CEREMA
✉ yohan.penel@inria.fr

Jacques Sainte-Marie
Directeur de recherche CEREMA
✉ jacques.sainte-marie@inria.fr
(directeur de thèse)

Détails

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- Laboratoire Jacques-Louis Lions, UMR CNRS 7598, Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris cedex 05
- Inria centre de Paris, 2 rue Simone Iff, CS 42112, 75589 Paris cedex 12

Contenu scientifique

A major drawback of hydrostatic Shallow-Water models is that it leads to poor accuracy for crucial applications such as the arrival of tsunamis on coasts or the evolution of stratified flows. This is due to the simplifying assumptions under which shallow water models are derived. In order to better represent dispersive effects, a new class of models – called non-hydrostatic – was introduced in the literature.

Among them, let us mention the recent PhD doctoral thesis by Nora Aïssiouenne, defended in 2016 ("Numerical analysis and discrete approximation of a dispersive shallow water model", https ://tel.archives-ouvertes.fr/tel-01418676) : a new approach is proposed for the numerical resolution of the depth-averaged Euler system developed in "An energy-consistent depth-averaged Euler system : Derivation and properties", by M.-O. Bristeau *et al* (Discrete and Continuous Dynamical Systems - Series B, 20(4) :961–988, 2015, https ://hal.archives-ouvertes.fr/hal-01011691v2). The numerical results obtained therein show a very good agreement with experimental data and expected physical behaviour. This model involves the resolution of a coupled problem : a model, mathematically similar to the Shallow water model, is coupled to additional equations whose unknowns are the vertical velocity and the pressure correction with respect to the hydrostatic pressure. The equation for the pressure correction leads, after time discretisation, to the approximation of an elliptic equation with nonstandard operators (Sturm-Liouville). From the numerical point of view, different methods can be considered for the discretisation of this equation. In particular mixed finite element methods seem to be appropriate. The variety of the possible discrete spaces is high, provided that they respect mathematical criteria needed for the convergence of the method (inf-sup condition and interpolation properties), and the first results have shown the need of research improvements in this direction.

To go further and deal with a wider range of applications like stratified flows with an improved accuracy, multilayer models were introduced and rely on a vertical semi-discretisation of the unknowns (https ://hal.archives-ouvertes.fr/hal-01324012v1). Numerical methods were assessed under the hydrostatic assumption. A potential application of the aforementioned directions could be to adapt the numerical strategy from non-hydrostatic models to multilayer models with a non-hydrostatic component.

The research work will be focused on the numerical approximation of the elliptic part of non-hydrostatic models coupled with numerical schemes dedicated to the hyperbolic part.

First, simplified cases of the coupling between an elliptic equation and a hyperbolic equation will be addressed including the derivation of analytical solutions. Then the proof of convergence for some numerical approximations will be studied, and numerical evidence of this convergence property will be obtained. The next step is then the application of a variety of methods for the elliptic part to realistic models : mixed finite elements, conforming finite elements, discontinuous Galerkin methods. Finally, the strategy will be extended to the case of multilayer models.

This research work will provide the PhD student with an opportunity of improving his/her knowledge on both elliptic and hyperbolic problems, from the theoretical and scientific computing points of view. This will include the study and the implementation of numerical methods in the framework of complex and realistic problems of fluid mechanics. Applications to environmental issues will be considered as there is a strong need of scientific expertise in this field.