Modeling, analysis and efficient numerical resolution for erosion processes

The modeling of sediment transport is a question of major interest in terms of applications and presents interesting difficulties in terms of numerical simulations and analysis of the partial differential equations. On the one hand, sediment transport processes involve a very large amount of material and have a major impact on human populations through, among others, the erosion of farmland, the alterations of coastal areas, the necessary protection of harbours and industrial facilities… On the other hand the model needs to couple different kinds of processes (the leading effect results from an interaction between a fluid flow and the soil material) that have different time scales and distinct underlying physical principles and for which the interactions are strongly nonlinear. Two major sediment processes are usually separated, namely the suspended load, where low concentrated sediments are transported into the water column at the water velocity, and the bed load where concentrated sediments remain close to the bottom and travel at their own velocity. This research project is mostly concerned with bed load phenomena, for which the coupling aspect seems to raise more difficult problems in the realistic applications.

Up till now, it seems that all the industrial softwares that deal with realistic sediment transport processes at a large scale are based on Saint-Venant – Exner type models. This model couples a classical fluid model (the Saint-Venant system) with a conservation equation for the mass of sediment. The closure laws (friction coefficient in the fluid model and sediment flux formula in the Exner equation) are mostly empirical but are the heart of the problem since they almost entirely characterize the interaction between both phases (a third term, the derivative of the bottom topography that appears in the fluid model, also characterizes the action of the solid phase on the fluid flow). This system has been widely validated on a relatively large class of test cases and remains mostly valid, at least for qualitative results, as long as the interaction remains smooth but is not so easy to use as a predictive model since it needs the tuning of a relatively large number of empirical parameters. Moreover the classical numerical methods that are used in the solvers and that are based on simple splitting techniques between a fluid solver and a specific solver for the Exner equation are not always stable. Lately, the research team ANGE from Inria, in collaboration with the French Electricity Group EDF-LNHE, has investigated alternative numerical methods for this system and has proposed relatively simple modifications for the industrial codes. The team also performed tests to quantify the uncertainties, in order to estimate the dependency of the model on the different parameters.

The purpose of my research project is to go further, by proposing alternative sediment transport models. In this perspective, a study of the underlying physics and of the partial differential equations modeling them is needed. The first step of the work would be the analysis of different kinds of three-dimensional energetically consistent couplings (e.g. Navier-Stokes type coupling), paying a particular attention to the coupling conditions and to the rheology of the materials. These couplings will be investigated analytically and numerically. The second step would be to use this knowledge to derive simplified but always energetically consistent models, through a rigorous derivation. (Note that the classic Saint-Venant – Exner model does not have any associated energy balance.) These models should be designed for use at large scales for realistic applications in river hydraulics. The final step would be the derivation and implementation of numerical methods for these models, paying attention, among others aspects, to the efficiency of the methods. Academic tests and realistic applications would then be carried out. Finally, after having fully investigated this class of models, an entirely different approach could even be studied. So far, models are concerned with only one of the two modes of sediment transport – none describes both modes at the same time. The aim of this new approach would be to overcome the traditional and arbitrary dichotomy between bed load and suspended load, i.e. to be able to simulate all types of sediment transports simultaneously without making distinctions between them. For this, an in-depth mathematical analysis of the equations is needed. "Checks" would be performed: asymptotic simplifications of such a model should allow to find again known models for bed load transport and suspended load transport.