Experimental observation on surface undulations over erodible bed

Annual Workshop of Advanced Modeling on Shear Shallow Flows

on Curved Topography AMoSS

(INRIA-MOST 2014-2016) Associated Team Work

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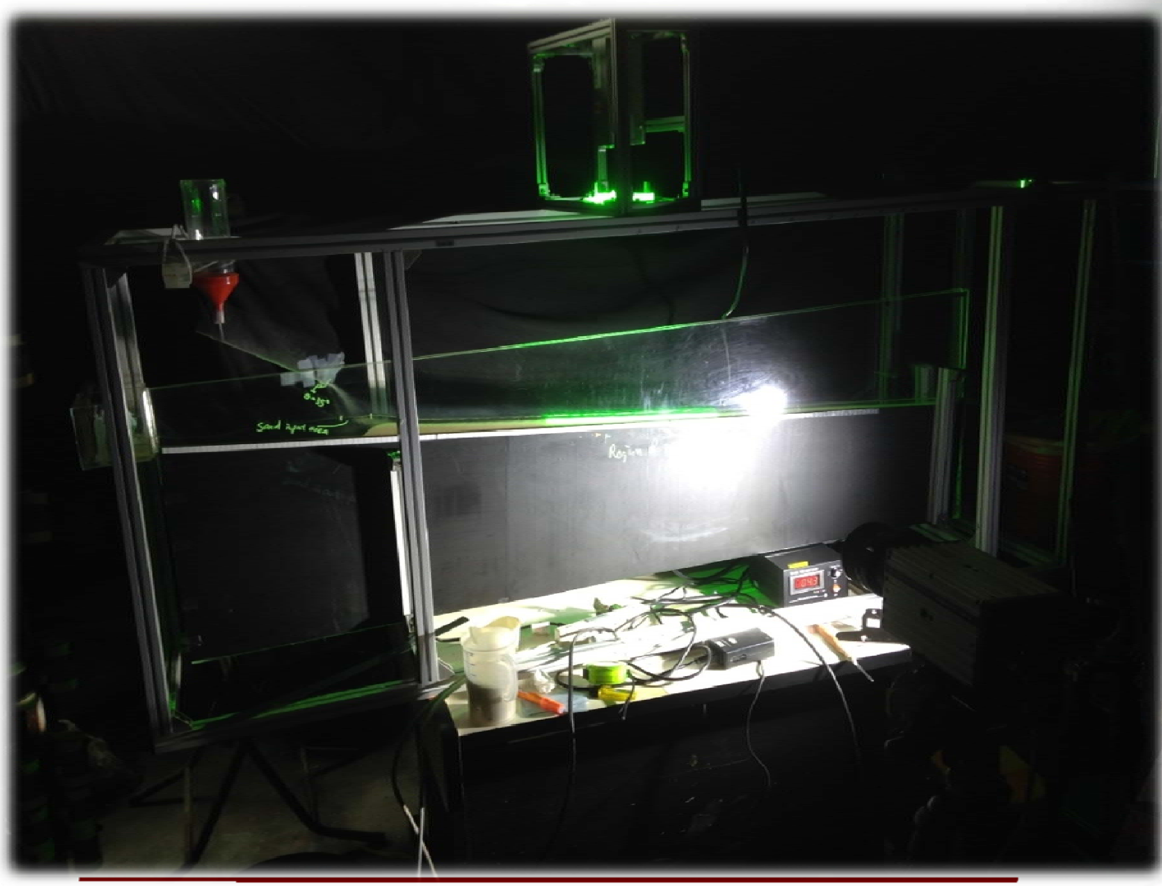
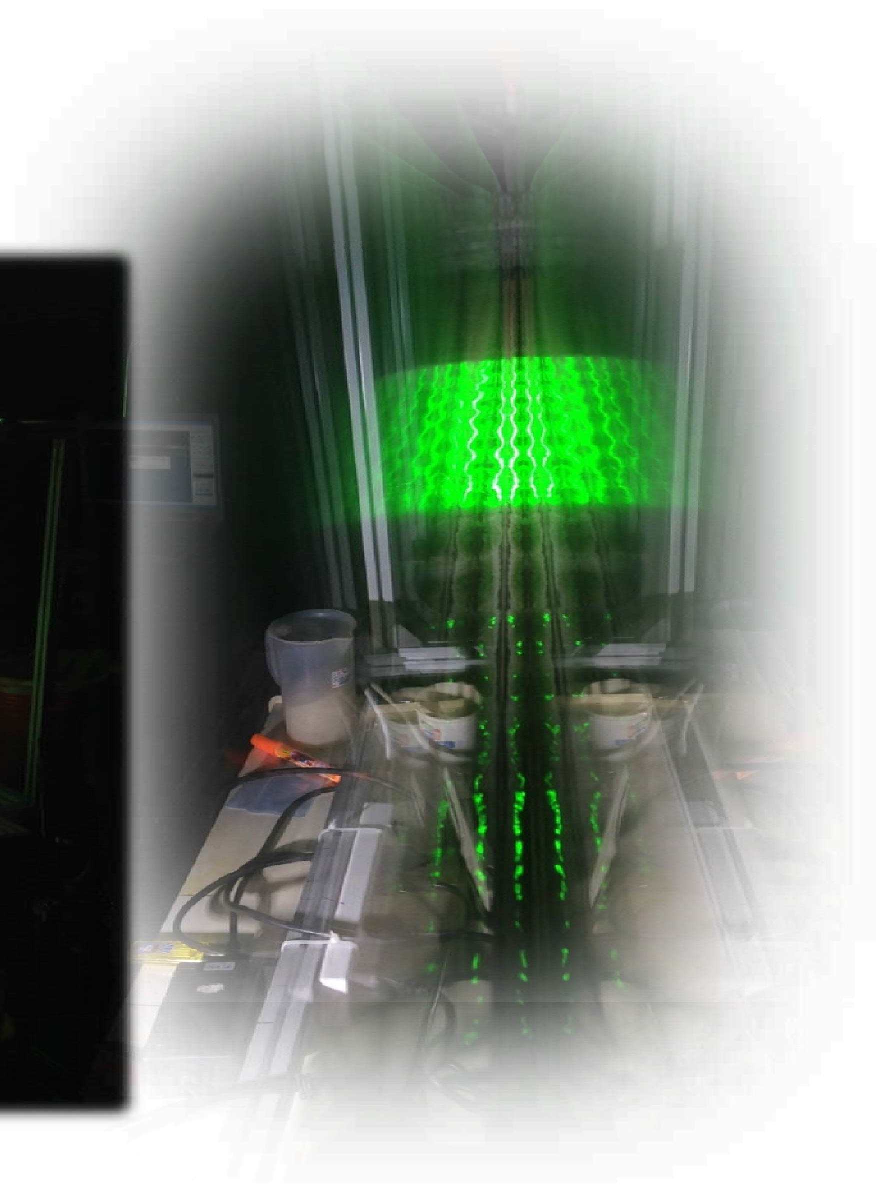
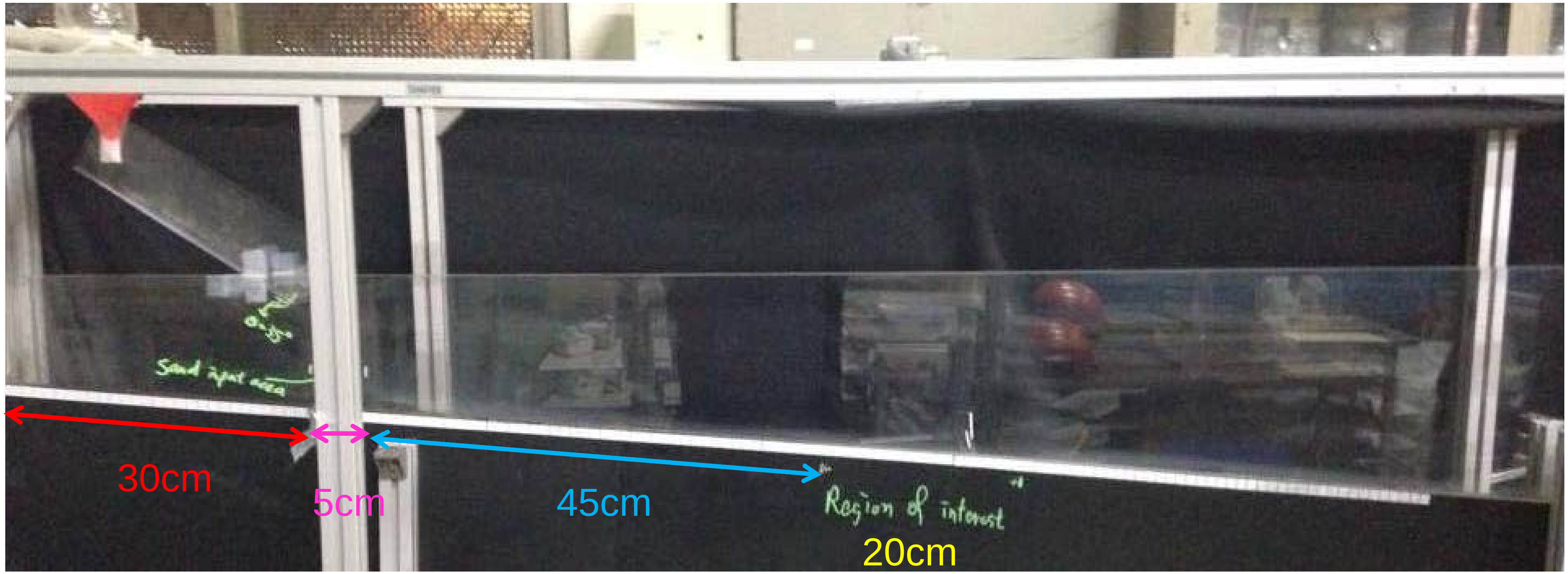
Water-sand mixture

Sand material	:	Ottawa sand (C-778, diameter: 06 – 0.8 mm)
Plexi-glass channel	:	1.6 m long, 1 cm wide
Channel slope	:	3 – 5 degree
Water flux	:	0.00898 kg/s
Sand supply	:	0.3 kg/s



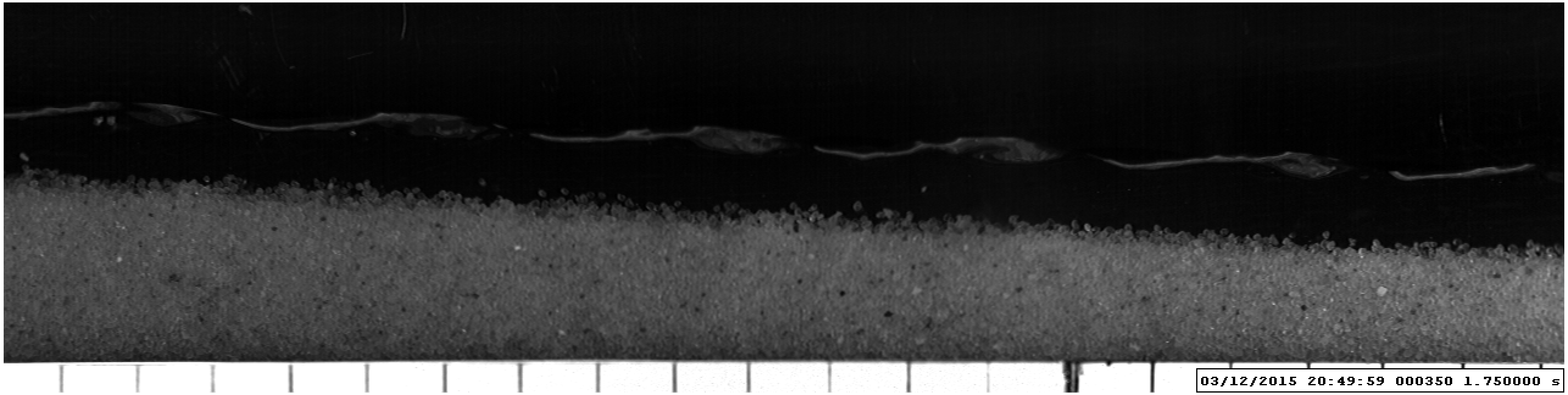




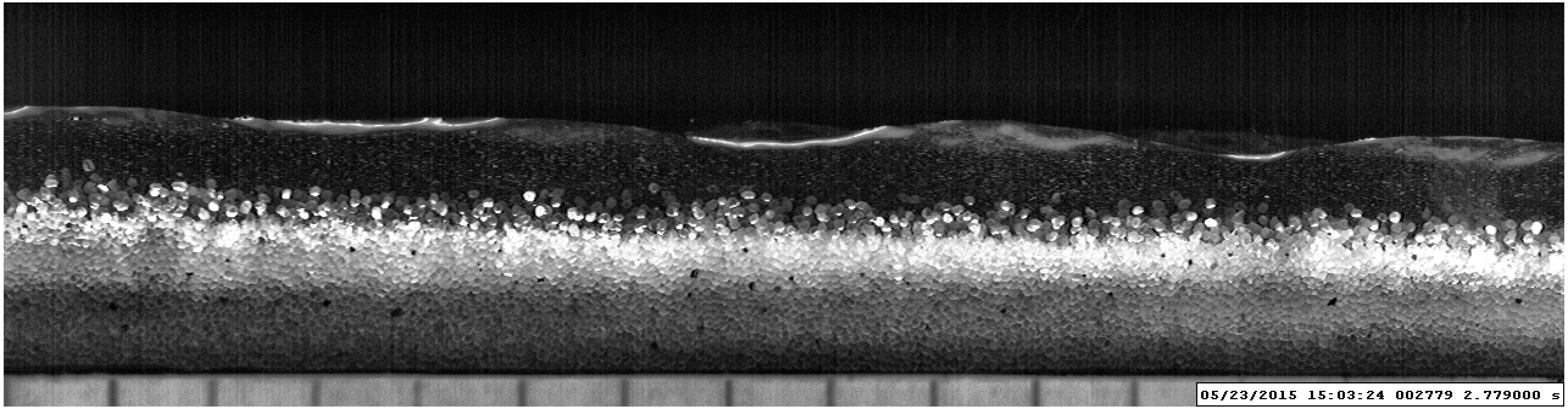
Sand input area
30cm
5cm
45cm
Region of interest
20cm

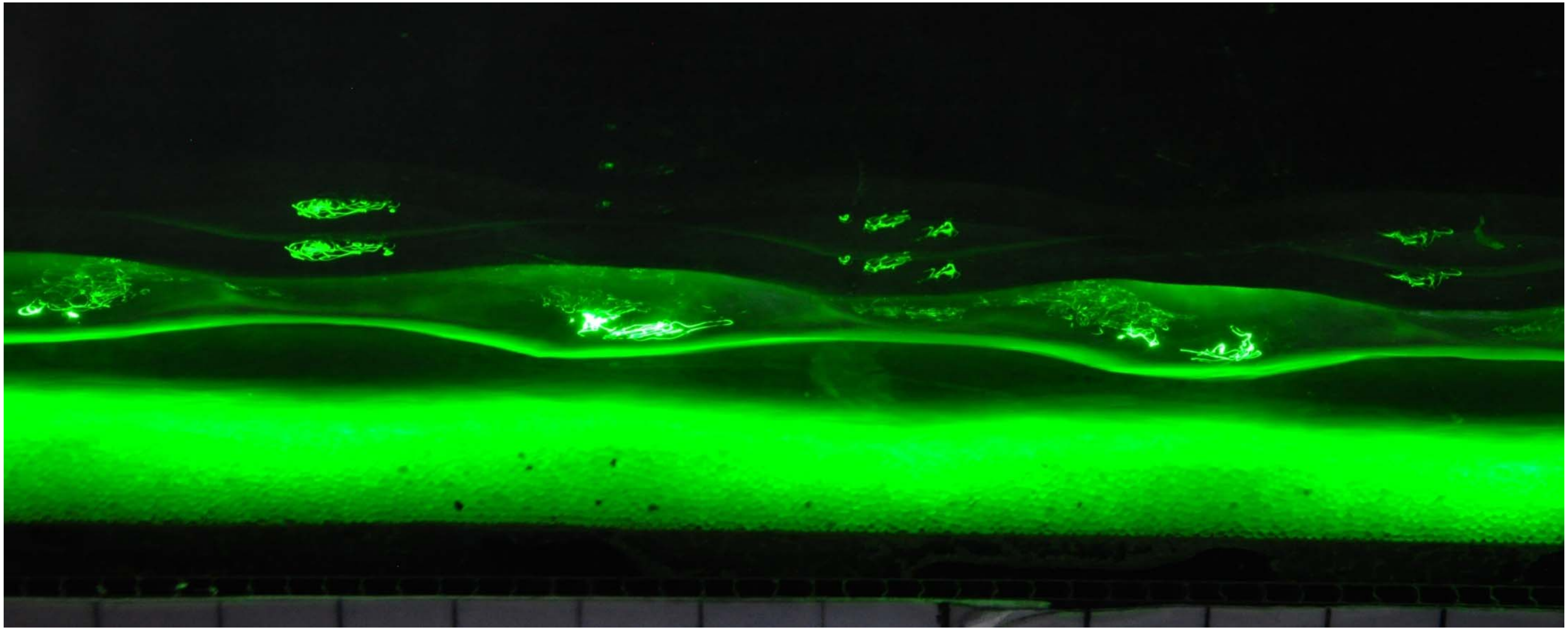
Experimental observation

- Steady state
- high-speed camera (fps = 200)
- Channel slope: 0 degree
- Bed slope: ca. 2.5 degree
- The surface undulation is exhibited by the Styrofoam ball

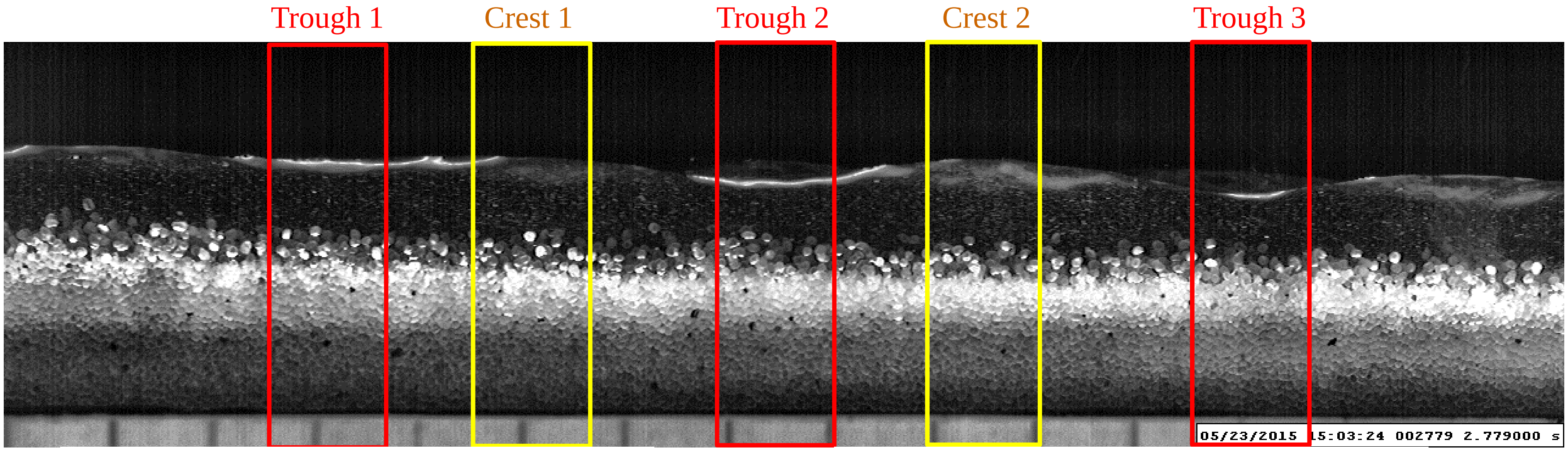


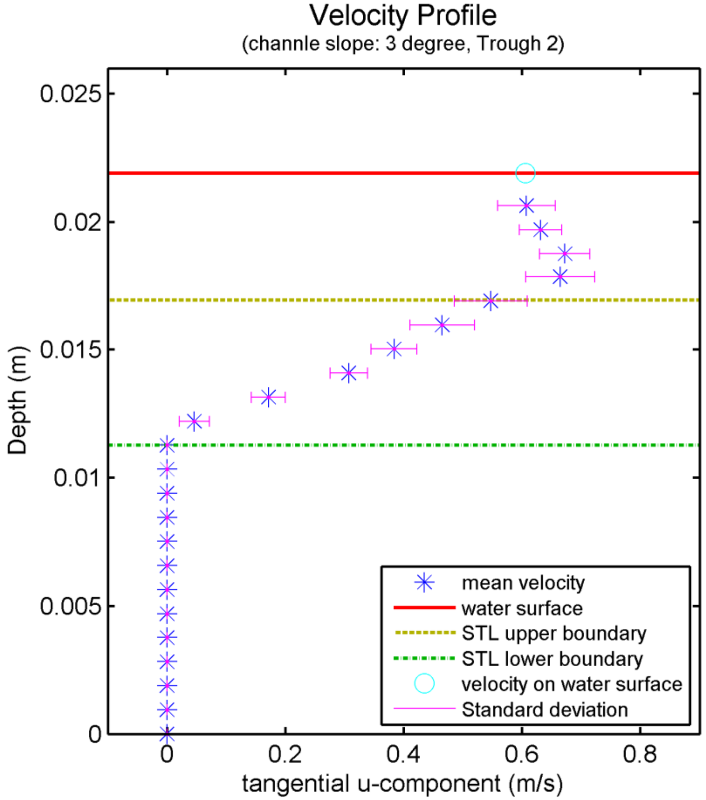
• Channel slope: 3 degree,   Bed slope: ca. 1 degree,   fps = 1,000

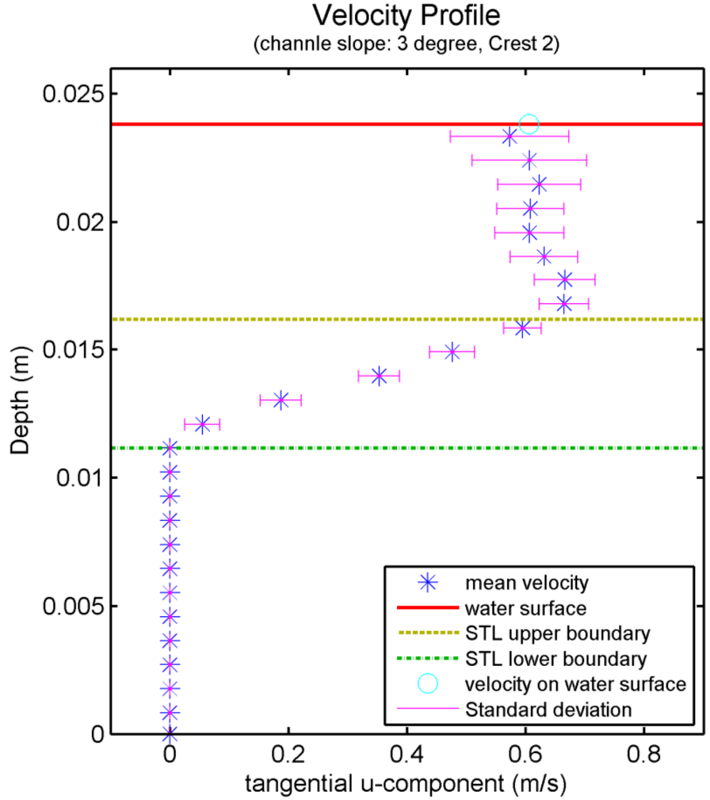




• Channel slope: 3 degree, Bed slope: ca. 1 degree, fps = 1,000







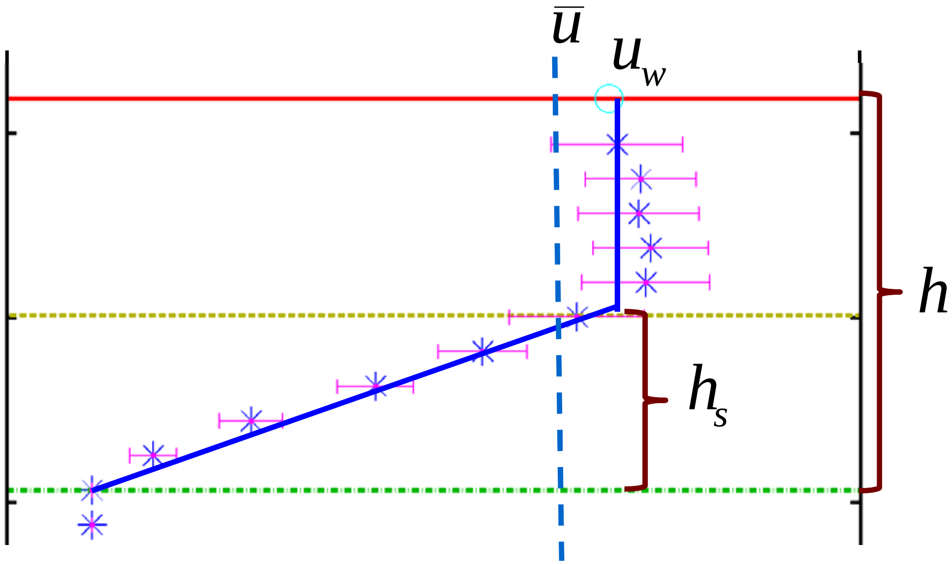
Frouhe Number (only approximation)

- **Piecewise linear distribution**

$$\int_{z_b}^{z_s} u^2(z)\,\mathrm{d}z = \beta \bar{u}^2 h,$$

$$\beta = (1-2\alpha_h/3)(1-0.5\alpha_h)^{-2},\ \ \alpha_h = h_s/h$$

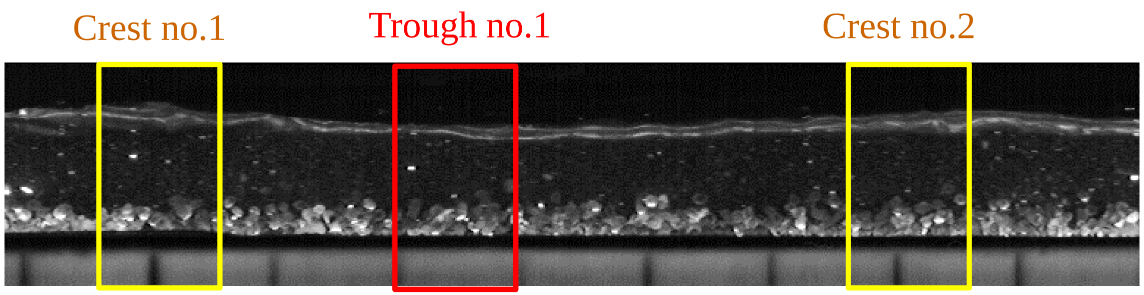
$$\mathrm{Fr} = \bar{u} \Big/ \sqrt{gh\cos\theta + 3(\beta-1)\bar{u}^2}$$

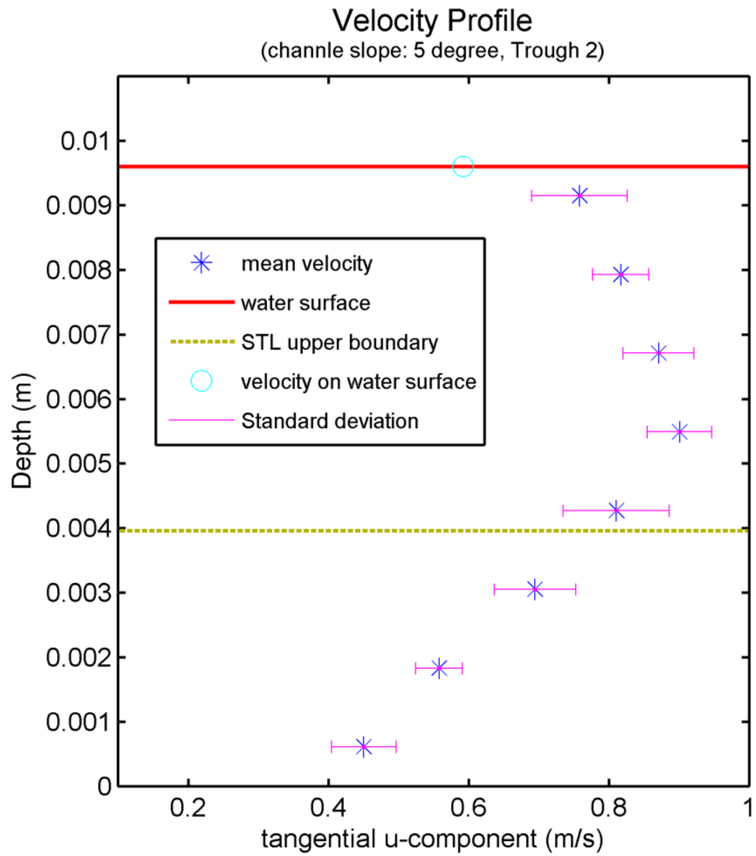


- **Channel slope: 3 degree**

	$\bar{u}$ (m/s)	h (m)	h_s (m)	β	Fr
Trough 01	0.4874	0.0111	0.0049	1.1612	1.0346
Crest 01	0.5091	0.0125	0.0052	1.1517	1.0387
Trough 02	0.4729	0.0106	0.0057	1.1977	0.9721
Crest 02	0.4986	0.0127	0.0050	1.1451	1.0352
Trough 03	0.4977	0.0106	0.0047	1.1641	1.0473

- Channel slope: 5 degree,   fps = 1,000





	$\bar{u}$ (m / s)	h (m)	h_s (m)	β	Fr
Crest 01	0.6830	0.0101	0.0040	1.1427	1.2504
Trough 01	0.6592	0.0091	0.0040	1.1588	1.211
Crest 02	0.6513	0.0101	0.0040	1.1427	1.2303
Trough 02	0.6644	0.0096	0.0040	1.1507	1.2268

Fawer, C. (1937). Étude de quelques écoulements permanents à filets courbes (Study of some steady flows with curved streamlines). *Thesis*, Université de Lausanne. La Concorde, Lausanne (in French).

- **Velocity and pressure measurements in undular hydraulic jump after Fawer (1937)**

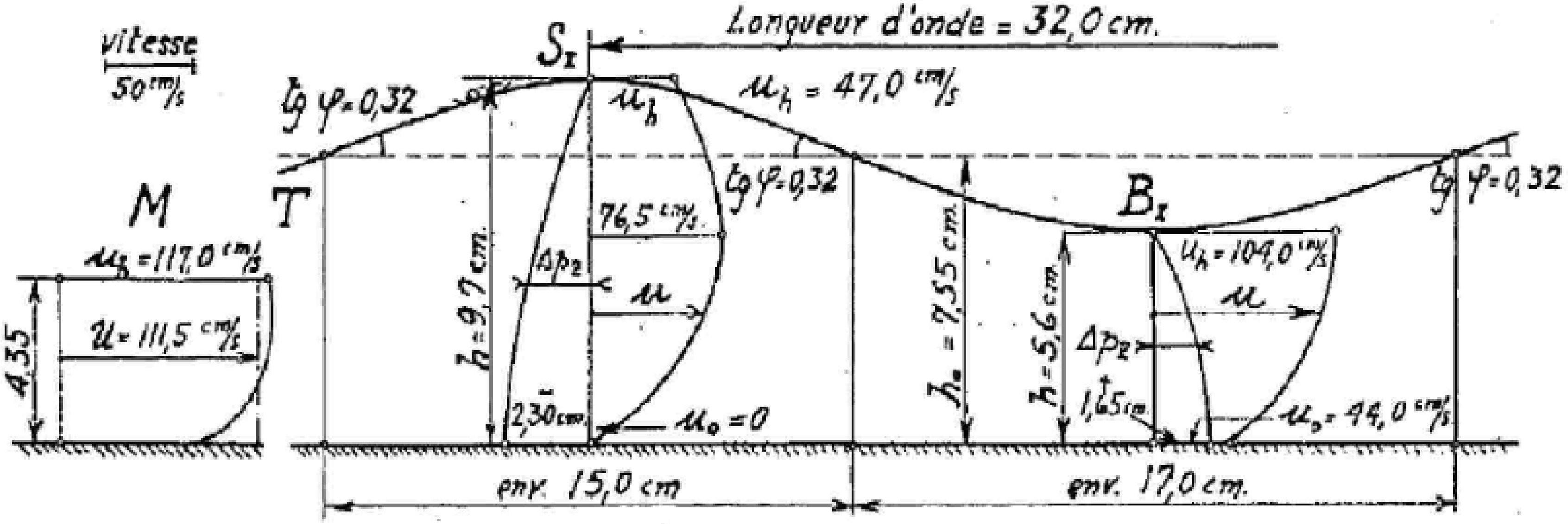


Fig. 21. — Profil observé dans l'axe (b).

Conventional method: *Boussinesq-type model*

e.g. Serre (1953), Peregrine (1966), etc.

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hU) = 0$$

$$\frac{\partial hU}{\partial t} + \frac{\partial}{\partial x}\left\{ \underbrace{hU^2 + \frac{1}{2}gh^2}_{\text{Saint-Venant term}} + \underbrace{\left(U_x^2 - U_{xt} + UU_{xx}\right)\frac{h^3}{3}}_{\text{non-hydrostatic term}} \right\} = \tau_b$$

$$w(z) = -\frac{\partial}{\partial x}\int_{z_b}^{z} u\,\mathrm{dz} \approx -\frac{\partial}{\partial x}\left\{U\left(z - z_b\right)\right\}$$

Shear Flow model: Richard and Gavrilyuk (2012, 2013)

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hU) = 0$$

$$\frac{\partial hU}{\partial t} + \frac{\partial}{\partial x}\left\{ \underbrace{hU^2 + \frac{1}{2}gh^2} + \underbrace{(\phi + \Phi)h^3} \right\} = \tau_b$$

+

$$\frac{\partial h\Phi}{\partial t} + \frac{\partial}{\partial x}(h\Phi U) = \ldots$$

$$\frac{\partial h\phi}{\partial t} + \frac{\partial}{\partial x}(h\phi U) = 0$$

Open questions for discussion

- *Is the observation a three-dimensional phenomena?*
- *Looking for optimal model*
- *The wavy formation at the bottom*
- *The impacts of the sediment-water layer.*
- *Rigid bed and erodible bottom.*