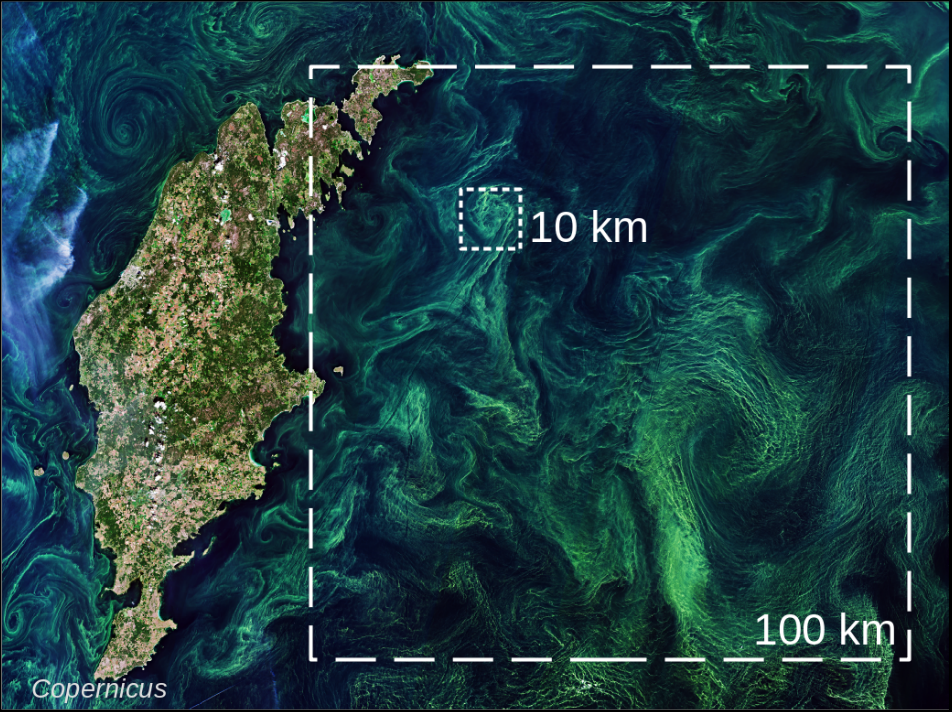
Fluids Under Location Uncertainty for surface layers

Groupe de travail AIRSEA – 11/02/2021

Manolis Perrot (ENS Paris)
with Elise Arnaud, Eric Blayo, Florian Lemarié
and Etienne Mémin (Fluminance – Inria Rennes)


10 km
100 km
Copernicus

- Model resolution is finite

- Unresolved processes: turbulence, eddies, waves...

- Model resolution is finite
- Unresolved processes: turbulence, eddies, waves...

⟶ generate uncertainty

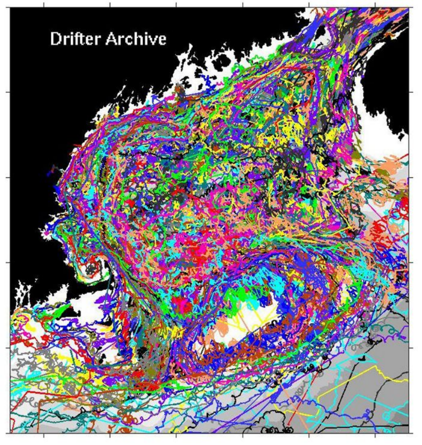
⟶ effects of unresolved processes need parameterization

- Model resolution is finite
- Unresolved processes: turbulence, eddies, waves...

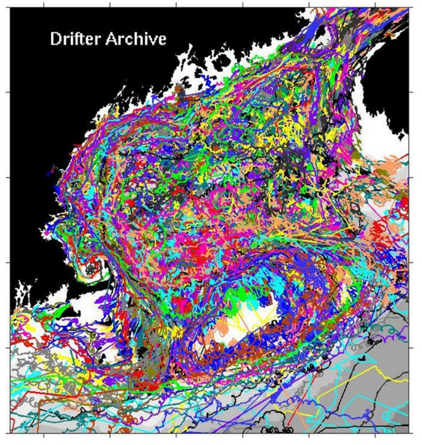
⟶ generate uncertainty

⟶ effects of unresolved processes need parameterization

How to introduce physically consistent uncertainty and parameterization ?



[Oceanic drifters, *NOAA*]



[Oceanic drifters, *NOAA*]

trajectory = coherent + random

Fluid Under Location Uncertainty (ULU)

Assumptions: *[Resseguier, Mémin, Chapron 2017]*

- slow/fast decomposition
- fast component modeled by Brownian motion

Fluid particle trajectory

$$\boldsymbol{X}_t = \boldsymbol{X}_0 + \underbrace{\int_0^t \boldsymbol{u}(\boldsymbol{X}_s, s)ds}_{\text{slow}} + \underbrace{\int_0^t \boldsymbol{\sigma}(\boldsymbol{X}_s, s)d\boldsymbol{B}_s}_{\text{fast (Brownian)}} \quad \text{Itô integral}$$

$$d\boldsymbol{X}_t = \boldsymbol{u}dt + \boldsymbol{\sigma}d\boldsymbol{B}_t$$

Fluid Under Location Uncertainty (ULU)

Assumptions: *[Resseguier, Mémin, Chapron 2017]*

- slow/fast decomposition
- fast component modeled by Brownian motion

Fluid particle trajectory

$$\boldsymbol{X}_t = \boldsymbol{X}_0 + \underbrace{\int_0^t \boldsymbol{u}(\boldsymbol{X}_s, s)ds}_{\text{slow}} + \underbrace{\int_0^t \boldsymbol{\sigma}(\boldsymbol{X}_s, s)d\boldsymbol{B}_s}_{\text{fast (Brownian)}} \quad \text{Itô integral}$$

$$d\boldsymbol{X}_t = \boldsymbol{u}dt + \boldsymbol{\sigma}d\boldsymbol{B}_t$$

$\boldsymbol{\sigma}$ spatial diffusion operator :

$$\boldsymbol{\sigma}(\boldsymbol{x}, t)d\boldsymbol{B}_t := \int_{\mathbb{R}^3} \breve{\boldsymbol{\sigma}}(\boldsymbol{x}, \boldsymbol{y}, t)d\boldsymbol{B}_t(\boldsymbol{y})d\boldsymbol{y}$$

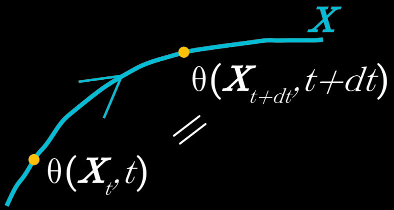
Correlations of the noise

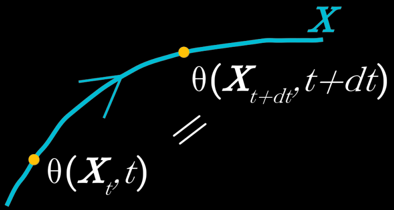
$$\mathbb{E}\left[(\boldsymbol{\sigma}(\boldsymbol{x},t)d\boldsymbol{B}_t)_i(\boldsymbol{\sigma}(\boldsymbol{x},s)d\boldsymbol{B}_s)_j\right] = (\boldsymbol{\sigma}\boldsymbol{\sigma}^t)_{ij}(\boldsymbol{x},t)\delta(t-s)dt$$

Spatial correlations: variance tensor $\boldsymbol{a}(\boldsymbol{x},t) := \boldsymbol{\sigma}\boldsymbol{\sigma}^t(\boldsymbol{x},t)$, generalized eddy-diffusivity

No time correlation

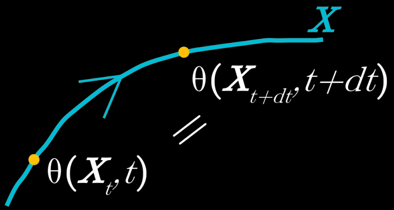
Transport of scalar field ULU





Deterministic transport: $d\boldsymbol{X}_t = \boldsymbol{u} dt$

$$D_t\theta := \partial_t\theta + \boldsymbol{u} \cdot \nabla\theta = 0 \text{ at } \boldsymbol{x}, t$$



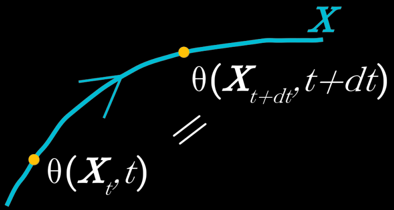
Deterministic transport: $d\boldsymbol{X}_t = \boldsymbol{u} dt$

$$D_t\theta := \partial_t\theta + \boldsymbol{u}\cdot\nabla\theta = 0 \text{ at } \boldsymbol{x}, t$$

Stochastic transport: $d\boldsymbol{X}_t = \boldsymbol{u} dt + \boldsymbol{\sigma} d\boldsymbol{B}_t$

$$\mathbb{D}_t\theta := d_t\theta + (\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a})\cdot\nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t)\cdot\nabla\theta - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta) dt = 0$$

Transport of scalar field ULU



Deterministic transport: $d\boldsymbol{X}_t = \boldsymbol{u}dt$

$$D_t\theta := \partial_t\theta + \boldsymbol{u}\cdot\nabla\theta = 0 \text{ at } \boldsymbol{x}, t$$

Stochastic transport: $d\boldsymbol{X}_t = \boldsymbol{u}dt + \boldsymbol{\sigma}d\boldsymbol{B}_t$

$$\mathbb{D}_t\theta := d_t\theta + (\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a})\cdot\nabla\theta dt + (\boldsymbol{\sigma}d\boldsymbol{B}_t)\cdot\nabla\theta - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta)dt = 0$$

Rigorous derivation of known parameterizations!

Physical interpretation

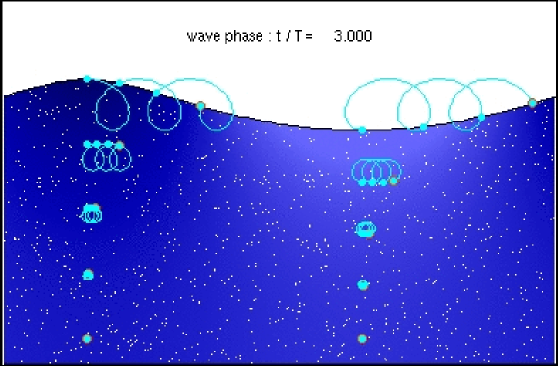
$$d_t\theta + \underbrace{\left(\boldsymbol{u} - \frac{1}{2}\nabla \cdot \boldsymbol{a}\right)}_{\boldsymbol{u}^*} \cdot \nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t) \cdot \nabla\theta - \frac{1}{2}\nabla \cdot (\boldsymbol{a}\nabla\theta)dt = 0$$

Modified advection due to statistical inhomogeneity: Stokes drift (surface waves), eddy-induced velocity (ocean), turbophoresis

Physical interpretation

$$d_t\theta + \underbrace{\left(\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a}\right)}_{\boldsymbol{u}^*}\cdot\nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t)\cdot\nabla\theta - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta)dt = 0$$

Modified advection due to statistical inhomogeneity: Stokes drift (surface waves), eddy-induced velocity (ocean), turbophoresis



Physical interpretation

$$d_t\theta + \underbrace{(\boldsymbol{u} - \frac{1}{2}\nabla \cdot \boldsymbol{a})}_{\boldsymbol{u}^*} \cdot \nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t) \cdot \nabla\theta - \frac{1}{2}\nabla \cdot (\boldsymbol{a}\nabla\theta) dt = 0$$

Modified advection due to statistical inhomogeneity: Stokes drift (surface waves), eddy-induced velocity (ocean), turbophoresis

Advection by the noise: backscattering of energy

Physical interpretation

$$d_t\theta + \underbrace{(\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a})}_{\boldsymbol{u}^*}\cdot\nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t)\cdot\nabla\theta - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta)dt = 0$$

Modified advection due to statistical inhomogeneity: Stokes drift (surface waves), eddy-induced velocity (ocean), turbophoresis

Advection by the noise: backscattering of energy

Subgrid diffusion:

Phenomenology of turbulent diffusion

Phenomenology of turbulent diffusion

$$\underbrace{\text{Large scale energy}}_{\text{resolved}} \longrightarrow \underbrace{\text{Small scale energy}}_{\text{unresolved}}$$

Phenomenology of turbulent diffusion

$$\underbrace{\text{Large scale energy}}_{\text{resolved}} \longrightarrow \underbrace{\text{Small scale energy}}_{\text{unresolved}}$$

Modeled by diffusion of resolved scales!

ULU diffusion is natural

$$d_t\theta + \left(\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a}\right)\cdot\nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t)\cdot\nabla\theta - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta)dt = 0$$

Subgrid or turbulent diffusion:

Usual approach: assume turbulent diffusion $\nabla\cdot(\nu_{\mathrm{turb}}\nabla\bar{\theta})$

ULU diffusion is natural

$$d_t\theta + (\boldsymbol{u} - \frac{1}{2}\nabla \cdot \boldsymbol{a}) \cdot \nabla\theta dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t) \cdot \nabla\theta - \frac{1}{2}\nabla \cdot (\boldsymbol{a}\nabla\theta)dt = 0$$

Subgrid or turbulent diffusion:

Usual approach: assume turbulent diffusion $\nabla \cdot (\nu_{\mathrm{turb}}\nabla\bar{\theta})$

ULU: diffusion appears naturally
"turbulent diffusivity" tensor $\boldsymbol{a} = \boldsymbol{\sigma}\boldsymbol{\sigma}^\top$ directly related to statistics of the noise

Noise balances diffusion

Physical origin of the noise

$\implies$ Conservation of statistical energy (for $\nabla \cdot \boldsymbol{u}^* = 0$)

$$d_t \int_V \theta^2 = 0$$

Exact balance between diffusion and noise forcing!

Noise balances diffusion

Physical origin of the noise

$\implies$ Conservation of statistical energy (for $\nabla \cdot \boldsymbol{u}^* = 0$)

$$d_t \int_V \theta^2 = 0$$

Exact balance between diffusion and noise forcing!

$\implies$ Statistical balance

$$\frac{d}{dt} \int_V (\mathbb{E}[\theta])^2 = -\frac{d}{dt} \int_V \mathrm{Var}[\theta]$$

Mean energy decrease $\longrightarrow$ variance increase

Navier-Stokes ULU

Newton second law:

$$\frac{d}{dt}\text{momentum} = \text{forces}$$

Navier-Stokes ULU

Newton second law:

$$\frac{d}{dt}\text{momentum} = \text{forces}$$

Newton second law for deterministic fluids = Navier-Stokes

$$\underbrace{\frac{D}{Dt}}_{\partial_t + \boldsymbol{u}\cdot\nabla} \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \boldsymbol{F}(\boldsymbol{u})$$

Navier-Stokes ULU

Newton second law:

$$\frac{d}{dt}\text{momentum} = \text{forces}$$

Newton second law for deterministic fluids = Navier-Stokes

$$\underbrace{\frac{D}{Dt}}_{\partial_t + \boldsymbol{u}\cdot\nabla} \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \boldsymbol{F}(\boldsymbol{u})$$

Newton second law for fluids Under Location Uncertainty

$$\mathbb{D}_t \boldsymbol{u} = -\frac{1}{\rho}\nabla(p dt + dp_t) + \boldsymbol{F}(\boldsymbol{u} + \boldsymbol{\sigma} d\boldsymbol{B}_t)$$

Newton second law:

$$\frac{d}{dt}\text{momentum} = \text{forces}$$

Newton second law for deterministic fluids = Navier-Stokes

$$\underbrace{\frac{D}{Dt}}_{\partial_t + \boldsymbol{u}\cdot\nabla} \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \boldsymbol{F}(\boldsymbol{u})$$

Newton second law for fluids Under Location Uncertainty

$$\mathbb{D}_t \boldsymbol{u} = -\frac{1}{\rho}\nabla(p dt + dp_t) + \boldsymbol{F}(\boldsymbol{u} + \boldsymbol{\sigma} d\boldsymbol{B}_t)$$

$$\mathbb{D}_t \boldsymbol{u} := d_t \boldsymbol{u} + \left(\boldsymbol{u} - \frac{1}{2}\nabla\cdot\boldsymbol{a}\right)\cdot\nabla\boldsymbol{u} dt + (\boldsymbol{\sigma} d\boldsymbol{B}_t)\cdot\nabla\boldsymbol{u} - \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\boldsymbol{u})dt$$

LES-like model

Forced stochastic partial differential equation:

$$\mathbb{D}_t \boldsymbol{u} = -\frac{1}{\rho}\nabla(p dt + dp_t) + \boldsymbol{F}(\boldsymbol{u} + \boldsymbol{\sigma} d\boldsymbol{B}_t)$$

LES-like model

Forced stochastic partial differential equation:

$$\mathbb{D}_t \boldsymbol{u} = -\frac{1}{\rho}\nabla(p dt + dp_t) + \boldsymbol{F}(\boldsymbol{u} + \boldsymbol{\sigma} d\boldsymbol{B}_t)$$

Time regularity assumption on $\boldsymbol{u}$ $\implies$ LES-like **deterministic** model:

$$\partial_t \boldsymbol{u} + \left(\boldsymbol{u} - \frac{1}{2}\nabla \cdot \boldsymbol{a}\right) \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} + \frac{1}{2}\nabla \cdot (\boldsymbol{a}\nabla \boldsymbol{u})$$

$$(\boldsymbol{\sigma} d\boldsymbol{B}_t) \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla dp_t + \boldsymbol{F}(\boldsymbol{\sigma} d\boldsymbol{B}_t)$$

Summary on ULU

Physical uncertainty in fluid models

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Natural diffusion and modified (”eddy-induced”) advection

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Natural diffusion and modified (”eddy-induced”) advection

Exact balance between forcing and diffusion

Conservation of kinetic energy

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Natural diffusion and modified (”eddy-induced”) advection

Exact balance between forcing and diffusion

Conservation of kinetic energy

Applications:

- data assimilation
- ensemble simulation
- reduced-order models
- fluid modelling (Langmuir circulations, law-of-the-wall)

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Natural diffusion and modified ("eddy-induced") advection

Exact balance between forcing and diffusion

Conservation of kinetic energy

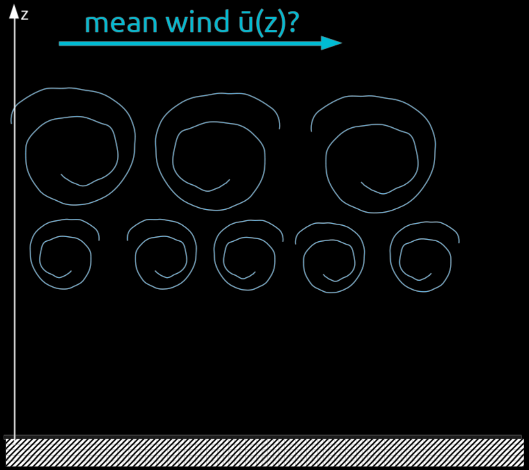
Applications:

- data assimilation
- ensemble simulation
- reduced-order models
- fluid modelling (Langmuir circulations, law-of-the-wall)

What about Air-Sea interactions?

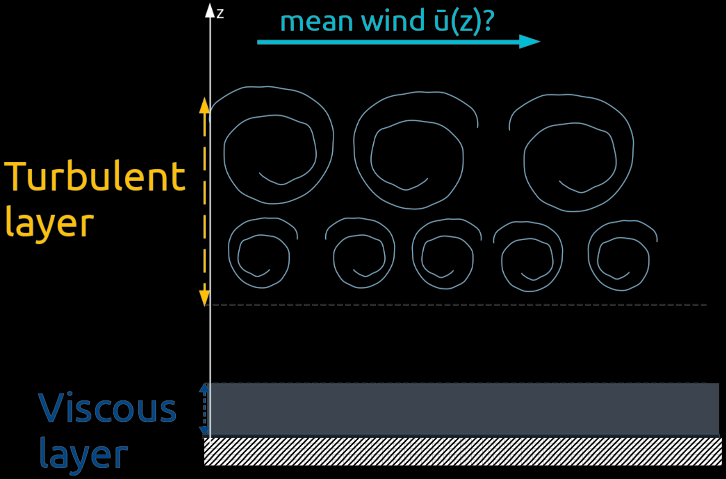
Law of the wall

Constant density fluid $\rho = \mathrm{cte}$. Mean horizontal wind profile $\overline{u}(z)$?

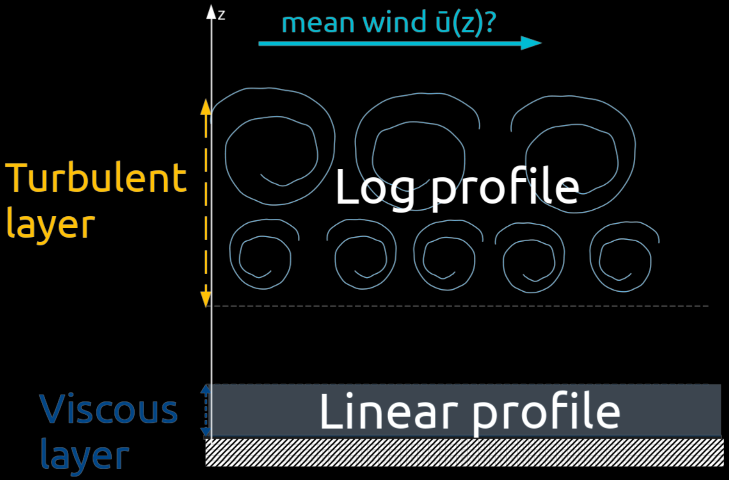


Law of the wall

Constant density fluid $\rho = \text{cte}$. Mean horizontal wind profile $\overline{u}(z)$?

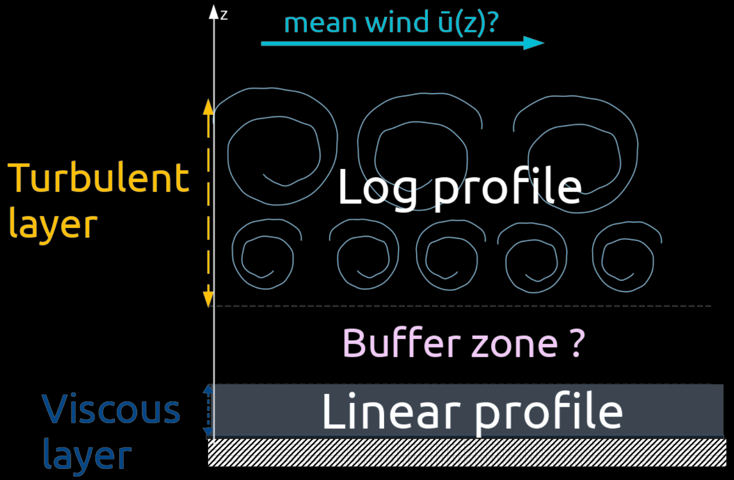


Constant density fluid $\rho = \text{cte.}$ Mean horizontal wind profile $\overline{u}(z)$?



Law of the wall

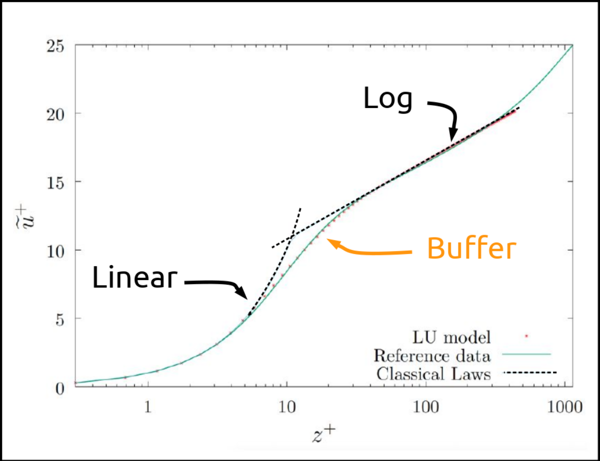
Constant density fluid $\rho = \mathrm{cte}$. Mean horizontal wind profile $\overline{u}(z)$?



ULU adresses buffer zone problem

[Pinier et al. 2019]

Assuming log profile in turbulent layer
ULU gives a model for buffer zone!



More realistic atmosphere

Small density variations (Boussinesq): $\rho \propto -$ temperature θ (hot air is lighter)

More realistic atmosphere

Small density variations (Boussinesq): $\rho \propto -$ temperature θ (hot air is lighter)

Usual approach for turbulent layer: *[Monin-Obukhov, 54]* predicts quasi-log profiles for $\overline{u}(z), \overline{\theta}(z)$
data $\longrightarrow$ different turbulent diffusion

$$\partial_z(\nu_{\text{turb}}^u \partial_z u), \partial_z(\nu_{\text{turb}}^\theta \partial_z \theta)$$

More realistic atmosphere

Small density variations (Boussinesq): $\rho \propto -$ temperature θ (hot air is lighter)

Usual approach for turbulent layer: *[Monin-Obukhov, 54]* predicts quasi-log profiles for $\overline{u}(z), \overline{\theta}(z)$
data $\longrightarrow$ different turbulent diffusion

$$\partial_z(\nu^u_{\mathrm{turb}} \partial_z u), \partial_z(\nu^\theta_{\mathrm{turb}} \partial_z \theta)$$

ULU: same turbulent diffusion

$$\partial_z(a_{zz} \partial_z u), \partial_z(a_{zz} \partial_z \theta)$$

contradiction with data!

Why?

Why turbulent diffusion of momentum and heat are different?

$$\nu^{u}_{\mathrm{turb}} \neq \nu^{\theta}_{\mathrm{turb}}$$

Why?

Why turbulent diffusion of momentum and heat are different?

$$\nu^{u}_{\mathrm{turb}} \neq \nu^{\theta}_{\mathrm{turb}}$$

Temperature fluctuations contribute to heat transport, but not to momentum transport *[Ertel, 1942]*

Why?

Why turbulent diffusion of momentum and heat are different?

$$\nu^{u}_{\mathrm{turb}} \neq \nu^{\theta}_{\mathrm{turb}}$$

Temperature fluctuations contribute to heat transport, but not to momentum transport *[Ertel, 1942]*

$\longrightarrow$ ULU: add thermodynamical uncertainty in the model?

Why?

Why turbulent diffusion of momentum and heat are different?

$$\nu_{\text{turb}}^{u} \neq \nu_{\text{turb}}^{\theta}$$

Temperature fluctuations contribute to heat transport, but not to momentum transport *[Ertel, 1942]*

$\longrightarrow$ ULU: add thermodynamical uncertainty in the model?

Possible scale resonance between temperature and vertical speed fluctuations? *[Li et al., 2012]*

Why?

Why turbulent diffusion of momentum and heat are different?

$$\nu^{u}_{\mathrm{turb}} \neq \nu^{\theta}_{\mathrm{turb}}$$

Temperature fluctuations contribute to heat transport, but not to momentum transport *[Ertel, 1942]*

$\longrightarrow$ ULU: add thermodynamical uncertainty in the model?

Possible scale resonance between temperature and vertical speed fluctuations? *[Li et al., 2012]*

$\longrightarrow$ ULU: relax scale-separation hypothesis, consider time-correlated noise?

Summary on ULU

Physical uncertainty in fluid models

Slow/fast decomposition of trajectories

Subgrid/turbulent diffusion, modified ("eddy-induced") advection appear naturally

Exact balance between forcing and diffusion

Conservation of energy

Applications:

- data assimilation
- ensemble simulation
- reduced-order models
- fluid modelling (Langmuir circulations, law-of-the-wall)