(Very) basic elements on parameterization of oceanic and atmospheric turbulence

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is relativity/quantum mechanics/quantum electrodynamics, and the other is turbulent motion of fluids. About the former I am rather optimistic"
(Werner Heisenberg, 1932 Nobel Prize winner)

Context

- **Turbulence:** motions on wide range of scales from a few centimeters to thousand of kilometers that continuously interact

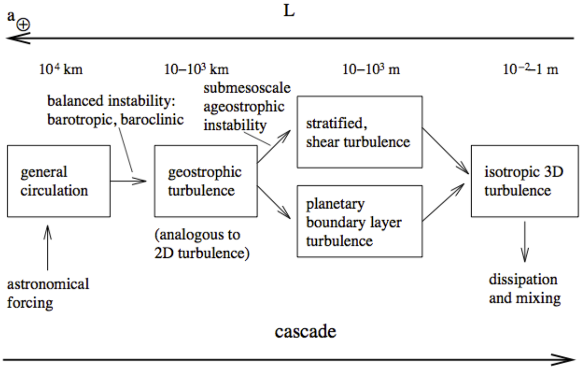
→ to develop theories and numerical models of the large-scale circulation, we need to account for these interactions,

i.e. understand how energy is transferred from the thousand of kilometers scales, to the centimeters scales, where energy is dissipated as heat.

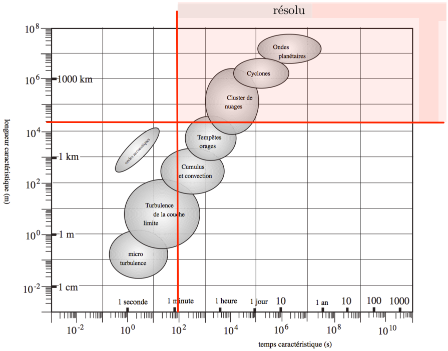
In the case of the oceanic turbulence

1. **From the large-scale currents to the mesoscale eddies**. The large-scale oceanic currents are unstable which generate eddies with scales of 10 to 100 kilometers (the mesoscales).
2. **The mesoscale eddies then interact and generate submesoscale turbulent filaments** on scales from 10 kilometers to 100 meters.
3. Only at scales below approximately 10 meters, **the turbulence becomes three-dimensional** and it is described as stratified microscale turbulence

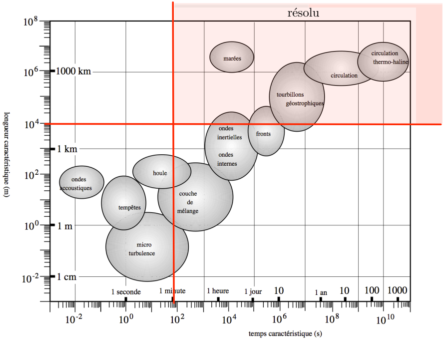
Context: different turbulent regime



Context: characteristic scales of physical processes



Atmosphere



Ocean

General properties

- From a mathematical perspective, turbulent behaviors arise because the governing equations of fluid dynamics are a nonlinear partial differential equation (PDE) system
- From a physical perspective, the advection causes the generic behavior of the entanglement of neighboring material parcels; this causes chaotic evolution, transport, and mixing.

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Reynold's decomposition

Principle

- Governing equations are deterministic, not random

$$
\begin{aligned}
\frac{D\mathbf{u}}{Dt} &= -\frac{1}{\rho_0}\boldsymbol{\nabla} p + \nu \Delta \mathbf{u} + \widehat{\boldsymbol{z}} b - f \widehat{\boldsymbol{z}} \times \mathbf{u} \\
\boldsymbol{\nabla} \cdot \mathbf{u} &= 0 \\
\frac{Db}{Dt} &= \kappa \Delta b, \qquad b = g\left(1 - \rho/\rho_0\right)
\end{aligned}
$$

- However, turbulent motions are generally described using statistical methods considering $\mathbf{u}$ and b as random variables
- Reynold's decomposition for $X = u, v, w, b$:

$$
X = \underbrace{\langle X \rangle}_{\text{mean}} + \underbrace{X'}_{\text{fluctuations}}, \qquad \left\langle X' \right\rangle = 0
$$

Which filter $\langle\cdot\rangle$ is adequate

- Ensemble average

$$\langle u\rangle_{\mathrm{E}} = \int q(\mathbf{x}, t, \omega)\, d\omega$$

- Low-pass filter

$$\langle u\rangle_{F} = \sum_{|\mathbf{k}|\leq k_e} F(q) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Space averaging

$$\langle u\rangle_{B} = \frac{1}{|B|}\int_{B(x,r)} u(y,z,t)\, dy$$

- Time averaging

$$\overline{u} = \frac{1}{T}\int_{t-T}^{t} u(\mathbf{x},\tau)\, d\tau$$

- Space-Time averaging

$$\langle \overline{u}\rangle_{B} = \frac{1}{T}\int_{t-T}^{t} \langle u\rangle_{B}\, d\tau$$

Filter properties

	$\langle\cdot\rangle_E$	$\langle\cdot\rangle_F$	$\langle\cdot\rangle_B$	$\bar{\cdot}$	$\langle\bar{\cdot}\rangle_B$
Linearity ($\langle u+v\rangle = \langle u\rangle + \langle v\rangle$)	X	X	X	X	X
Derivatives and averages commute	X	X	X	X	X
Double averages ($\langle\langle u\rangle\rangle = \langle u\rangle$)	X	X			
Product average ($\langle v\,\langle u\rangle\rangle = \langle v\rangle\,\langle u\rangle$)	X				

Only the ensemble averaging satisfies all properties

Link with observations and experiments: turbulence is assumed ergodic

$\Rightarrow$ *The ergodic theorem of probability theory (Neveu[1967]) says that, under certain conditions, statistical means can be replace by time averages or spatial averages in the case of multidimensional processes.*

Reynolds averaged equations

$$
\begin{aligned}
\partial_t \langle \mathbf{u} \rangle + \langle \mathbf{u} \rangle \cdot \boldsymbol{\nabla} \langle \mathbf{u} \rangle &= -\frac{1}{\rho_0} \boldsymbol{\nabla} \langle p \rangle + \nu \Delta \langle \mathbf{u} \rangle + \widehat{\boldsymbol{z}} \langle b \rangle - f \widehat{\boldsymbol{z}} \times \langle \mathbf{u} \rangle - \boldsymbol{\nabla} \cdot \left\langle \mathbf{u}' \mathbf{u}' \right\rangle \\
\boldsymbol{\nabla} \cdot \langle \mathbf{u} \rangle &= 0 \\
\partial_t \langle b \rangle + \langle \mathbf{u} \rangle \cdot \boldsymbol{\nabla} \langle b \rangle &= \kappa \Delta \langle b \rangle - \boldsymbol{\nabla} \cdot \left\langle \mathbf{u}' b' \right\rangle
\end{aligned}
$$

- eddy momentum flux (a.k.a. Reynolds stress)

$$
\boldsymbol{R} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle \end{pmatrix}
$$

- eddy tracer flux $\boldsymbol{R}_{\mathrm{b}} = \left(\langle u'b' \rangle , \langle v'b' \rangle , \langle w'b' \rangle \right)^T$

$\Rightarrow$ how to close this system ?

The turbulence closure problem

- **Second-moment closure models**
  e.g. under the assumption $\partial_x \cdot = \partial_y \cdot = 0$

$$\partial_t \left\langle w'b' \right\rangle = - \left\langle w'w' \right\rangle \partial_z \left\langle b \right\rangle - \frac{1}{\rho_0} \left\langle b' \partial_z p' \right\rangle - \partial_z \left\langle w'w'b' \right\rangle + \dots$$

  Second-moments are always function of higher order moments

- **Eddy viscosity/diffusivity models (a.k.a. 1^{st}-moment models, K-theory)**

$$\boldsymbol{R} = \boldsymbol{R}(\nabla \langle \mathbf{u} \rangle), \qquad \boldsymbol{R}_b = \boldsymbol{R}_b(\nabla \langle b \rangle)$$

Boussinesq assumption : $- \left\langle \mathbf{u}' \phi' \right\rangle = K_\phi \nabla \langle \phi \rangle$

  - analogy with molecular viscosity
  - Down-gradient fluxes
  - Turbulence acts as “mixing”

How to determine K_ϕ ?

Turbulent kinetic energy

Turbulent kinetic energy (TKE) is a measure of the intensity of turbulence defined as

$$\langle e \rangle = \frac{1}{2}\left(\langle u'u' \rangle + \langle v'v' \rangle + \langle w'w' \rangle\right)$$

Prognostic equation :

$$\partial_t \langle e \rangle + \boldsymbol{\nabla} \cdot \left(\langle \mathbf{u} \rangle \langle e \rangle + \langle \mathbf{u}'e \rangle + \frac{1}{\rho_0} \langle \mathbf{u}'p' \rangle - \nu \nabla \langle e \rangle\right)$$
$$= \underbrace{-\langle u'\mathbf{u}' \rangle \cdot \nabla \langle u \rangle - \langle v'\mathbf{u}' \rangle \cdot \nabla \langle v \rangle - \langle w'\mathbf{u}' \rangle \cdot \nabla \langle w \rangle}_{\text{Shear production}} + \underbrace{\langle w'b' \rangle}_{\text{buoyancy}} - \underbrace{\frac{\nu}{2} \|\nabla \mathbf{u}'\|^2}_{\text{dissipation}}$$

Closure assumptions :

- $-\left(\langle \mathbf{u} \rangle \langle e \rangle + \langle \mathbf{u}'e \rangle + \frac{1}{\rho_0} \langle \mathbf{u}'p' \rangle - \nu \nabla \langle e \rangle\right) = K_e \nabla \langle e \rangle$
- $-\langle u'\mathbf{u}' \rangle = K_u \nabla \langle u \rangle$
- $-\langle w'b' \rangle = K_b \partial_z \langle b \rangle$
- ε: ultimate dissipation of KE of all motions
  - Kolmogorov (1941) : $\varepsilon = \langle e \rangle^{3/2} / \Lambda_\varepsilon$
  - Evolution equation for ε (k-ε closure scheme)

1-equation TKE closure

- In ocean/atmosphere models
  $\rightarrow$ one-point closures assuming horizontal homogeneity

$$\partial_t \langle e \rangle = \partial_z \left(K_e \partial_z \langle e \rangle \right) + K_m \left\{ \left(\partial_z \langle u \rangle \right)^2 + \left(\partial_z \langle v \rangle \right)^2 \right\} - K_b \partial_z b - \langle e \rangle^{3/2} / \Lambda_\varepsilon$$

- It is expected that K_ϕ is related to $\langle e \rangle$

$$K_m = \Lambda_m \sqrt{\langle e \rangle}, \qquad K_b = \Lambda_b \sqrt{\langle e \rangle}, \qquad K_e = \Lambda_e \sqrt{\langle e \rangle}$$

where Λ_ϕ are turbulent length scales usually derived from a "master length scale"

$$(\Lambda_\varepsilon, \Lambda_m, \Lambda_b, \Lambda_e) = \left(\frac{1}{c_\varepsilon}, C_m, S_b(\partial_z \langle b \rangle), \mathrm{Sc}_e \right) \Lambda$$

The master length scale is diagnosed one way or another

Talk by O. Audouin (AMA): parameter control for C_m, $\frac{1}{c_\varepsilon}$, S_b, Sc_e and $\Lambda_{\min}$

2-equations turbulence models

- Algebraization of 2-equations closures : Generic Length Scale models

$$\begin{aligned} \partial_t \langle e \rangle &= \partial_z \left(K_e \partial_z \langle e \rangle \right) + K_m \left\{ \left(\partial_z \langle u \rangle \right)^2 + \left(\partial_z \langle v \rangle \right)^2 \right\} - K_b \partial_z b - \varepsilon \\ \partial_t \psi &= \partial_z \left(K_\psi \partial_z \psi \right) + \frac{\psi}{\langle e \rangle} \left(c_{\psi 1} K_m \left\{ \left(\partial_z \langle u \rangle \right)^2 + \left(\partial_z \langle v \rangle \right)^2 \right\} - c_{\psi 3} K_b \partial_z b - c_{\psi 2} \varepsilon \right) \end{aligned}$$

$c_{\psi 1}$, $c_{\psi 2}$, $c_{\psi 3}$, and Sc_ψ determined empirically

- "Generic" aspect: the quantity ψ is related to the dissipation rate as

$$\varepsilon = (c_0)^{3+p/n} \langle e \rangle^{3/2+m/n} \psi^{-1/n}, \qquad \psi = (c_0)^p \langle e \rangle^m \Lambda^n, \qquad \Lambda = (c_0)^3 \langle e \rangle^{3/2} \varepsilon^{-1}$$

Depending on the parameter values for the triplet (m, n, p) the GLS scheme will either correspond to a $k - \varepsilon$, a $k - \omega$, a $k - kl$ or the so-called generic turbulence scheme.

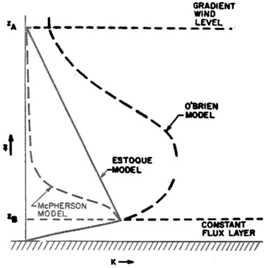
0-equation models

- O'brien (1970, JAS) : for time scales involved in climate models, an equilibrium profile for the vertical structure of the eddy viscosity K_m is

$$K(z) = P_3(z, z_A, z_B, K(z_A), K(z_B), K'(z_A), K'(z_B))$$

knowing that in the constant flux layer

$$K(z_B) = \kappa u_\star z/\phi(z), \qquad \phi(z) : \text{stability function}$$



- From TKE equation: intensity of turbulence largely determined by

$$\mathrm{Ri} = \frac{\partial_z \langle b \rangle}{(\partial_z \langle u \rangle)^2 + (\partial_z \langle v \rangle)^2}$$

From theoretical and experimental studies :
  - Convective instability for $\mathrm{Ri} < 0$
  - Non-turbulent flow becomes turbulent when $\mathrm{Ri} \lesssim \mathrm{Ri}_{\mathrm{Cr}}$
  - Turbulent flow becomes non-turbulent when $\mathrm{Ri} \gtrsim 1$

0-equation models

Integral view over the boundary layer $[z_{\rm sfc}, h_{bl}]$: $\mathrm{Ri}_B(h_{\rm bl}) = \mathrm{Ri}_{\rm cr}$ with

$$\mathrm{Ri_B}(z) = \frac{(z - z_{\rm sfc})(\langle b \rangle (z) - \langle b \rangle (z_{\rm sfc}))}{\|\mathbf{u}(z) - \mathbf{u}(z_{\rm sfc})\|^2}$$

$\rightarrow$ K-profile boundary layer model (Troen & Mahrt (1986); Large et al. (1994))

1. Extent of PBL $h_{\rm bl}$ is determined from $\mathrm{Ri}_B(h_{\rm bl}) = \mathrm{Ri}_{\rm cr}$
2. Apply the O'Brien (1970) equilibrium shape function

$$K_{m,b}(z) = w_{m,b} h_{\rm bl} G(z/h_{\rm bl})$$

where $w_{m,s} = \kappa u_\star / \phi_{m,s}(z)$ ($\phi_{m,s}(z)$ are stability functions).

Turbulent closures in realistic ocean/atmosphere models

	Ocean	Atmosphere
0-equation	Croco, MOM6, MPAS-O	IFS, AR5, LMDZ5
1-equation TKE	NEMO	LMDZ, Arpege, Arome, MesoNH
2-equations	Croco, NEMO, ROMS	MAR (k-ε)

Simulation types and turbulent closures

- **DNS (Direct Numerical Simulation)** : all scales are explicitely resolved, no turbulent closures
- **LES (Large Eddy Simulation)** : $\Delta x \approx 10\mathrm{m}$, no approximations in model equations + relax one-point closure assumption
- **MILES (Monotonic Implicit Large Eddy Simulation)** : $\Delta x \approx 10\mathrm{m}$, no approximations in model equations, no turbulent closures, "Nonlinearly stable" numerical schemes
- **CRM (Cloud Resolving Models)** : $\Delta x \approx 1-2\mathrm{km}$, no approximations in model equations + 1D or 3D closure

Parameter estimation

How to provide the reference for parameter estimation

- LES simulations (e.g. HighTunes)
- Observations (assuming ergodicity)
- Laboratory experiments + supporting theory
  (e.g. Kato & Philips, Willis & Deardorff, logarithmic layers, etc.)