## Efficient optimization for robotics applications

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#### **Research team**

ACENTAURI is a robotic team that studies and develop intelligent, autonomous and mobile robots that can help humans in their day-to-day lives at home, at work or during their displacements. The team focuses on perception, decision and control problems for multi-robot collaboration by proposing an original hybrid model-driven / data driven approach to artificial intelligence and by proposing efficient algorithms. The team focuses on robotic applications in smart territories, smart cities and smart factories. In these applications several collaborating robots will help humans by using multi-sensor information eventually coming from infrastructure. The team demonstrates the effectiveness of the proposed approaches on real robotic systems like cars AGVs and UAVs together with industrial partners. Innovation and the transfer of the research work towards industrial partners are a concern of ACENTAURI.

### **Motivations and general objectives**

Many perception and control problems in robotics are solved as the optimization of a non-linear cost function [1]. In general, a "closed form" solution (a solution that can be expressed analytically in terms of a finite number of certain simple operations) for such problems does not exists and we need to use iterative method like Gradient Descent, Newton or Gauss-Newton [2] or more efficient second order methods [3]. These methods need an initial starting point to compute a solution and may fall in a local minimum. Unfortunately, such knowledge of a good starting point close to the global minimum is usually unavailable a priori.

In this internship we will focus on the optimization of polynomial cost functions since the number of equilibrium points (global minimum, local minima, saddle points) can be deterministically numbered. Indeed, when the cost function is a polynomial function, the optimal solution can be obtained by solving a polynomial system of equations [4]. For such problems, we can find a "closed form" using only standard linear algebra operations: matrix decomposition (LU, QR, SVD, ...), eigenvalues and eigenvector computations, etc. Many of the state-of-the-art specific polynomial systems solvers are based on Gröbner bases and the action-matrix method [4]. There are now powerful tools available for the automatic generation of efficient Gröbner basis solvers for [5]. Such methods work extremely well for problems of reasonably low degree, involving a few variables. Currently, the limiting factor in using these methods for larger and more demanding problems are numerical difficulties. When considering larger problems numerical stability is still an issue [6]. Another approach to polynomial system solution based on generalization of the homogeneous resultant introduced by Macaulay to multivariate homogeneous polynomials. Resultant based [7] can be divided into u-resultant where an additional equation is added to the system of h-resultant where one of the unknowns is hided in the coefficients. In both cases the solution is found by computing the eigenvalues (u-resultant) or the generalized eigenvalues (h-resultant) of matrices which size (and computation time) depends on the number of unknowns and the degree of the polynomials. For sparse polynomial systems it is possible to obtain more compact resultants using Sparse Resultants [8].

The main objective is to study and design efficient solvers for large scale polynomial system of equations that have the property of being computed in real-time (typically few milliseconds) while providing sufficiently accurate solution closed to the global minimum that may be refined by non-linear iterative methods. Indeed, extremely precise (but costly to compute) solution are often not needed in real-time applications and somehow unnecessary since the cost functions we need to optimize are generated with imperfect models. Finding the solver which yields that smallest template, or the best numeric result is a difficult problem. A complete study and analysis of computational requirements of different algorithms will be carried out building new benchmarks. We will consider estimation problems using data acquired by lidar and vision sensors. The algorithms will be designed in Matlab and then implemented in  $C/C++$  for real-time experiments with data acquired by the ACENTAURI robots.

# **Work-plan**

The work will be decomposed with incremental steps as follows:

- 1. Bibliography on polynomial systems solvers
- 2. Mathematical background in Gröbner bases and Macaulay resultants
- 3. Design of new efficient algorithms and implementation
- 4. Comparison with the state-of-the-art techniques
- 5. Experimental results on real data
- 6. Writing master Thesis and potential papers

### **Skills**

The candidate is expected to follow a Master in Control, or Computer Sciences, more generally in robot-ics, as well as solid skills in software development (MATLAB/Simulink, LINUX, ROS, Git, PYTHON,  $C/C +$ ), and mathematics. A good level of written/spoken English is also important.

# **How to apply**

Interested candidates must send a detailed CV, transcripts bachelor, M1 and M2, a motivation letter and at least one recommendation letter to ezio.malis@inria.fr

## **Financial support**

Financial support offered to the student: around  $580 \in \text{per month during 6 months.}$ 

### **References**

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- 5 V. Larsson, K. Astrom, and M. Oskarsson. Efficient solvers for minimal problems by syzygy-based reduction. In IEEE Conference on Computer Vision and Pattern Recognition, pages 2383–2392, 2017
- 6 M. Byrod, K. Josephson, and K. Astrom. Fast and stable polynomial equation solving and its applica-tion to computer vision. International Journal of Computer Vision, 84(3):237–256, 2009.
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