**Internship**

# Simulation of time-dependent wave propagation problems in complex media with adaptive finite elements

Laboratory: Inria Lille (France)

Supervisors: Dr. Théophile Chaumont-Frelet (Inria Lille) and Dr. Martin Vohralík (Inria Paris)
https://team.inria.fr/rapsodi/ https://team.inria.fr/serena/

Duration: 6 months
Start: Ideally from April to July 2024
Funding: ANR JCJC grant APOWA

Key words: Numerical simulation, Finite element method, Wave propagation, Numerical analysis, Error estimation, Adaptive computations

## General context:

Accurate simulation of **time-dependent wave propagation** phenomena is of central interest in many areas of physics and engineering. **Finite element** and **discontinuous Galerkin** methods have become very popular to perform such simulations, due to their ability to handle complex propagation media. The ANR project **APOWA** aims at improving the reliability and efficiency of these discretization methods through the use of a posteriori **error estimators** and **adaptive mesh refinements**.

The internship will take place in the context of the APOWA project, funded by the French national research agency ANR. The goal of the internship is to discover the topics at the heart of the project, before the intern continues with a **PhD thesis**, also funded by the APOWA project.

## Description of the internship subject:

A posteriori error estimation for wave propagation is very involved; [1,3,4] are some pioneering contributions paving the way to a sound numerical analysis. A **novel approach** to a posteriori error estimation of finite element discretizations of time-dependent wave propagation problems has been recently introduced in [2]. Currently, it applies to the **second-order formulation** of wave propagation problems, where, given $f$, the unknown solution $u$ satisfies

$$\ddot{u} - \Delta u = f. \tag{1}$$

A numerical illustration is provided in Figure 1. There are, however, many advantages in working with the (equivalent) **first-order formulation** of the problem whereby the pair of unknowns $(u, \boldsymbol{\sigma})$ satisfies

$$\begin{cases} \dot{u} - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} &= F, \\ \dot{\boldsymbol{\sigma}} - \boldsymbol{\nabla} u &= 0, \end{cases} \tag{2}$$

with $F$ a time anti-derivative of $f$. This last formulation will be the center of interest in the internship.

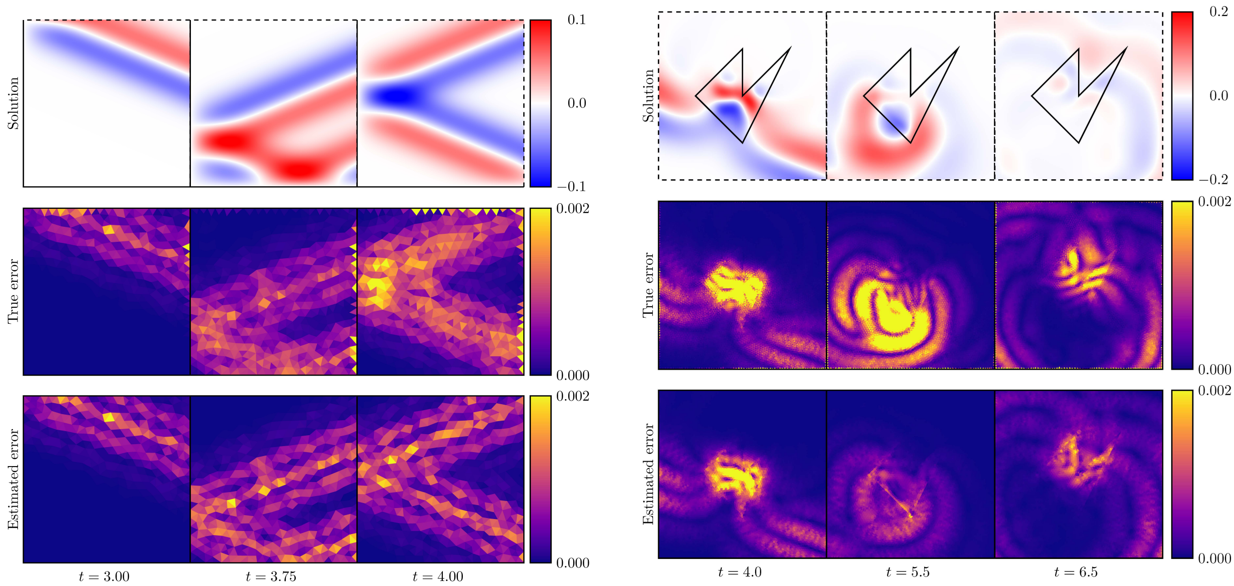


FIGURE 1. Numerical simulation of a wave reflected by two walls (left) and propagating through a penetrable obstacle (right). Numerical approximation (top), actual error (known for these model cases, middle), and a posteriori error estimate from [2] (bottom)

Specifically, **the objectives of the internship are threefold**. First (i), the intern will become familiar with the numerical analysis of wave propagation problems and a posteriori error estimation, and in particular become familiar with [2]. Then (ii), he/she will adapt the techniques developed in [2] for the second-order formulation (1) of the wave equation to the first-order formulation (2). Finally (iii), the intern will develop a computer code to perform one-dimensional simulations, thereby implementing a finite element discretization of (2) and the newly developed a posteriori error estimator.

## Profile of the candidate:

We are looking for a candidate with a Master or engineering degree in applied mathematics eager to pursue his/her study with a PhD thesis. He/She is expected to be familiar with PDEs, numerical analysis and finite element methods. A priori knowledge in wave propagation problems and/or a posteriori error estimation as well as programming skills are also appreciated.

## Practical information for application:

Interested candidates should send an application letter along with a CV, copies of their grades and reference letters to

- Dr. Théophile Chaumont-Frelet `theophile.chaumont@inria.fr`,
- Dr. Martin Vohralík `martin.vohralik@inria.fr`.

## References:

[1] C. Bernardi, E. Süli, *Time and space adaptivity for the second-order wave equation*. Math. Models Methods Appl. Sci. **15** (2005), 199–225.

[2] T. Chaumont-Frelet, *Asymptotically constant-free and polynomial-degree-robust a posteriori estimates for space discretizations of the wave equation*. SIAM J. Sci. Comput. **45** (2023), A1591–A1620.

[3] T. Chaumont-Frelet, A. Ern, M. Vohralík, *On the derivation of guaranteed and p-robust a posteriori error estimates for the Helmholtz equation*, Numer. Math. **148** (2021), 525–573.

[4] W. Dörfler and S.A. Sauter, *A posteriori error estimation for highly indefinite Helmholtz problems*, Comput. Methods Appl. Math. **13** (2013), 333–347.