Numerical analysis of the Navier-Stokes equations with noise

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We start with the construction of martingale solutions of SDE's with discontinuous coefficients by iterates of the Euler scheme. The main part is then to show convergence of (LBB-stable) finite-element based space-time discretizations of the 3D stochastic Navier-Stokes equations with multiplicative noise. The convergence proof rests on a discrete energy law, and uniform control of higher moments for increments of approximates. In 2D, strong solutions with improved regularity properties are approximated with certain rates: a main step in this convergence analysis is to properly address the interplay of multiplicative noise and nonlinear drift. Finally, we propose more efficient time-splitting schemes, where optimal convergence of standard schemes hinges on regularity properties of the related pressure which in turn couple back with the type of noise (solenoidal or not); we propose new variants thereof which recover optimal convergence behavior for general noise.