PASSIFICATION BASED CONTROLLED SYNCHRONIZATION OF COMPLEX NETWORKS

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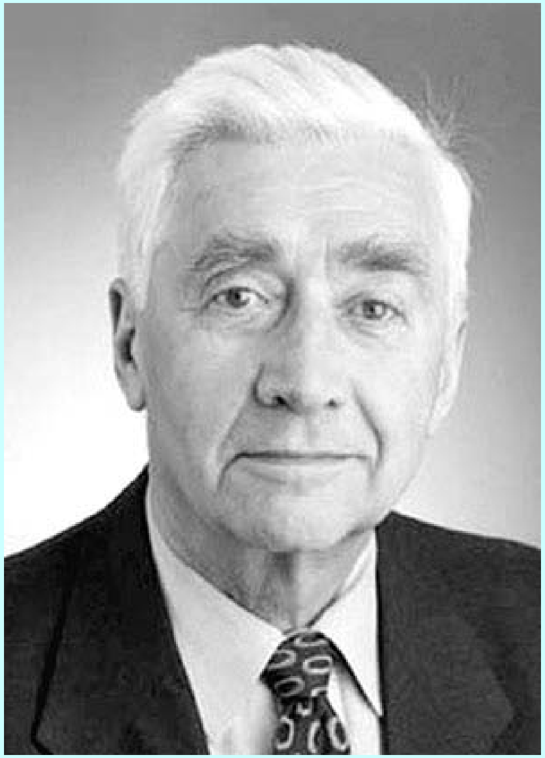
Outline

1. Introduction.
2. Control of dynamical networks: problems
3. Control of dynamical networks: models
4. Controlled synchronization of dynamical networks: results
5. Conclusions

It is convenient to split history of automatic control into three most relevant periods: 'deterministic', 'stochastic' and 'adaptive'.

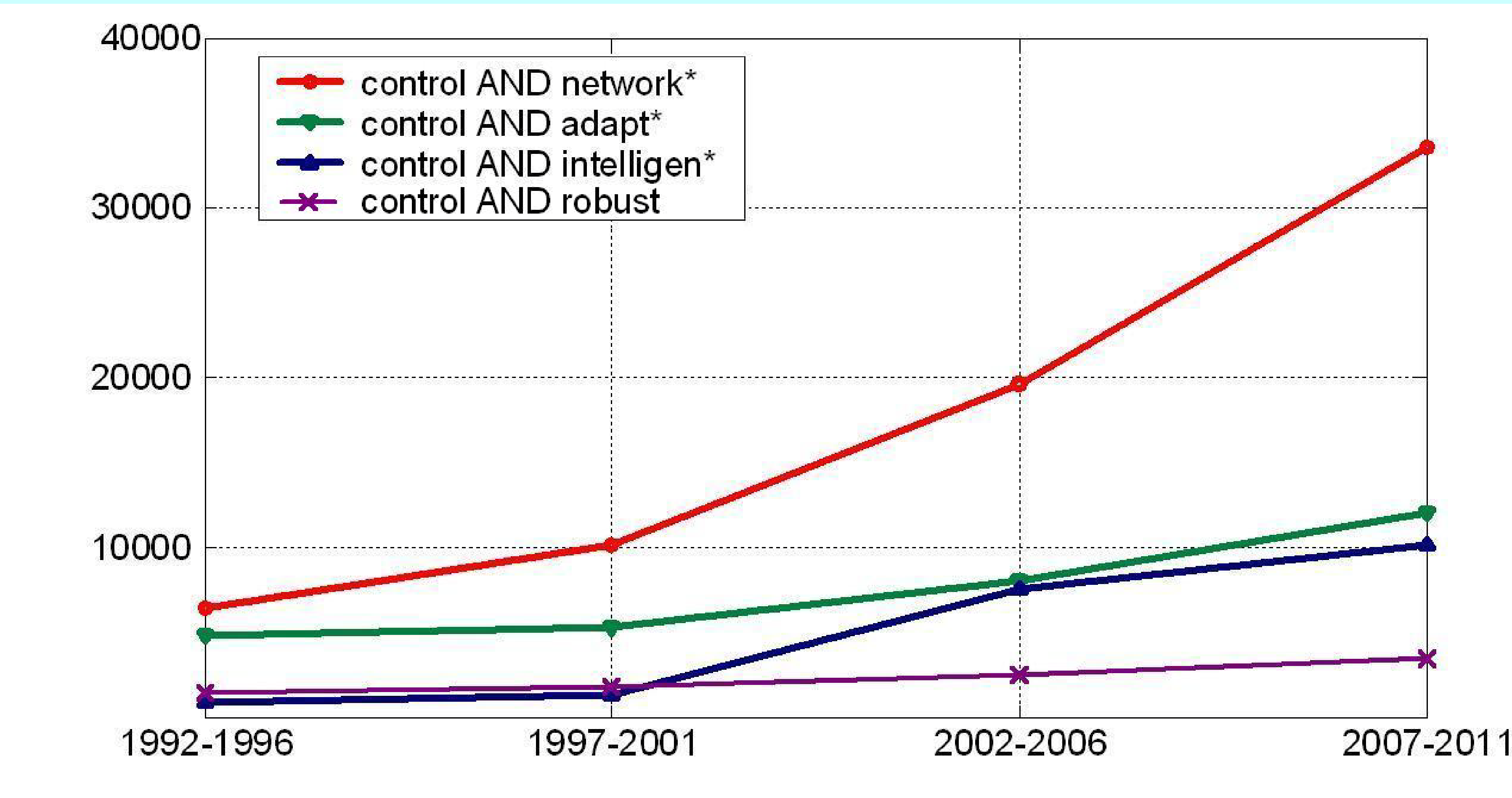


Ya.Z.Tsypkin. Adaptation and learning in automatic systems. Moscow: Nauka, 1968 (New York: Academic Press, 1971)

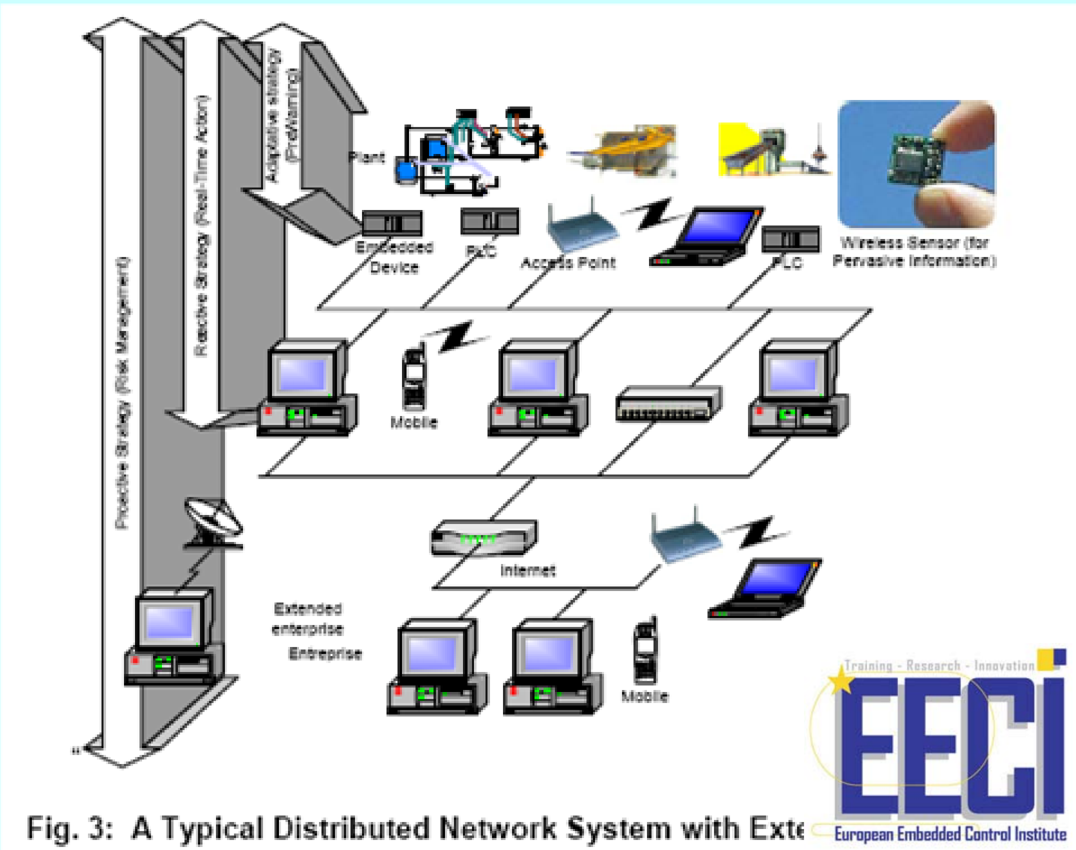
XXI century: ‘networked’ period

Search over Web of Science: number of papers with ‘control’ AND ‘network’ is doubled every 5-6 years

B T:



Examples of controlled networks



Distributed factories

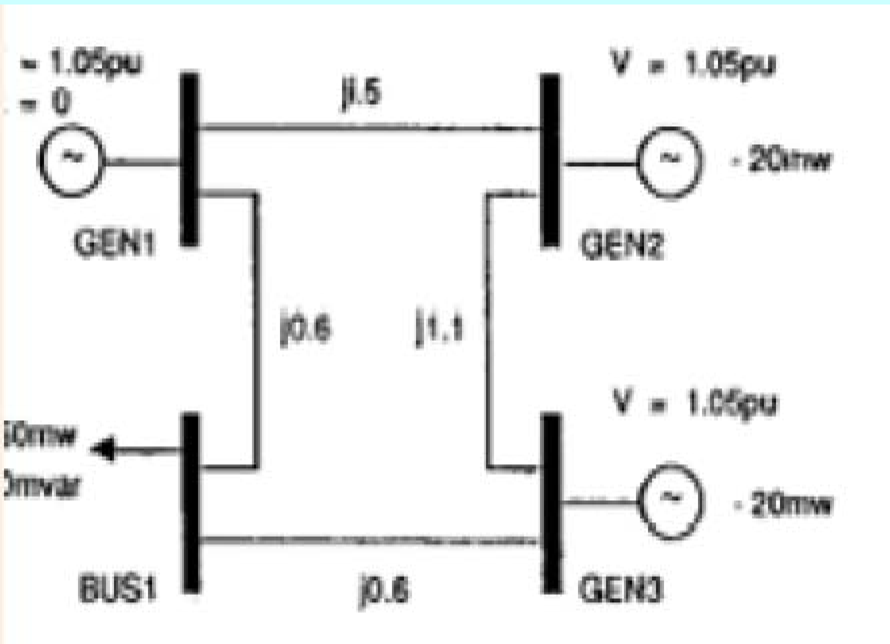
Networks of mobile robots

(production, transportation, finance networks)

Electric power networks

Pogromsky, A.Yu., A.L. Fradkov and D.J. Hill, Passivity based damping of power system oscillations. 35th IEEE Conf. Decision and Control, Kobe, 1996, pp.3876-3881.

D. J. Hill and G. Chen, “Power systems as dynamic networks,” IEEE Int. Symp. On Circuits and Systems, Kos, Greece, 2006, 722–725.



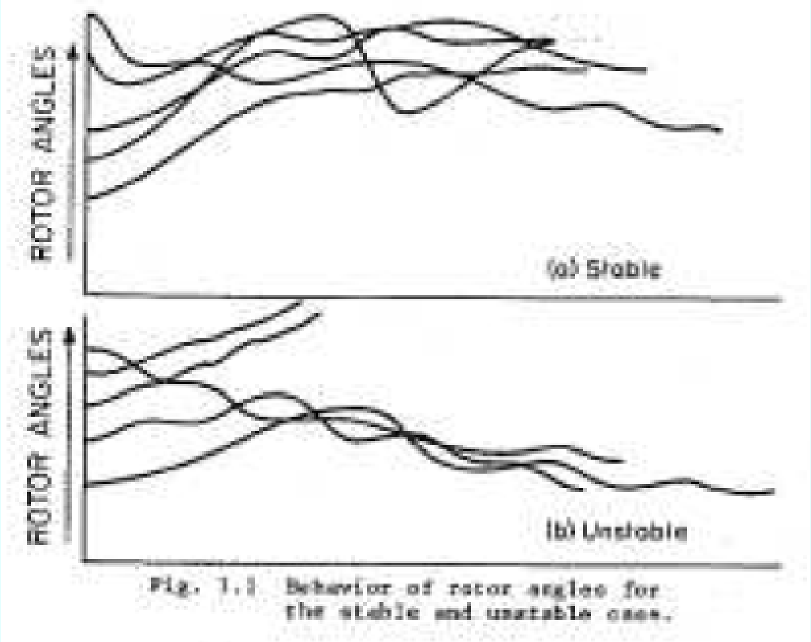
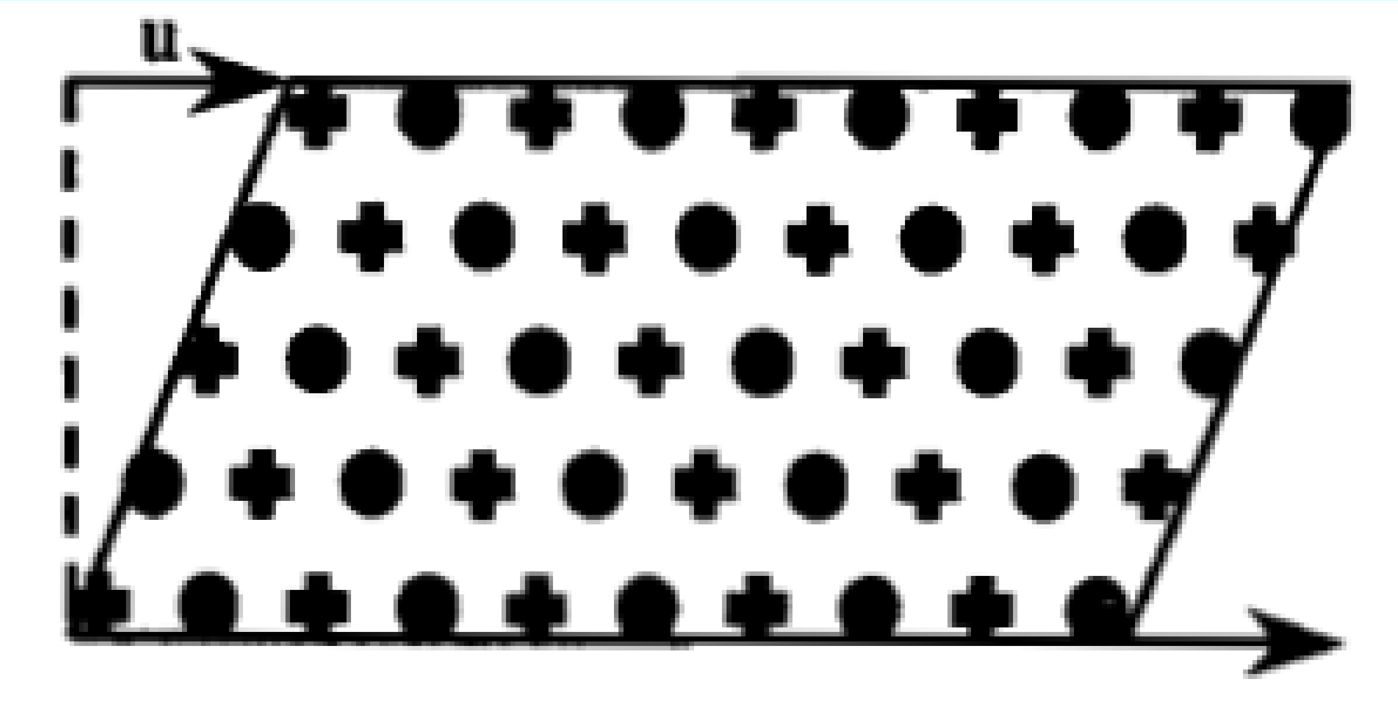


Fig. 1.1 Behavior of rotor angles for the stable and unstable cases.

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i + \sum_{j\in C_i} V_i V_j b_{ij} \sin(\theta_i - \theta_j) = P_i$$

Control of complex crystalline lattices

Aero E., Fradkov A.L., Andrievsky B., Vakulenko S. Dynamics and control of oscillations in a complex crystalline lattice. Physics Letters A., 2006, V. 353 (1), pp 24-29.



Ecological networks

Pchelkina I., Fradkov A.L. Control of oscillatory behavior of multi-species populations. Ecological Modelling 227 (2012) 1– 6.

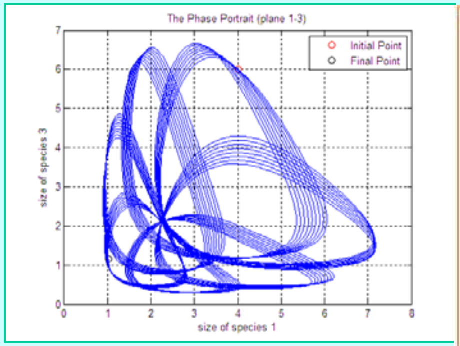
Multi-species Lotka-Volterra model:

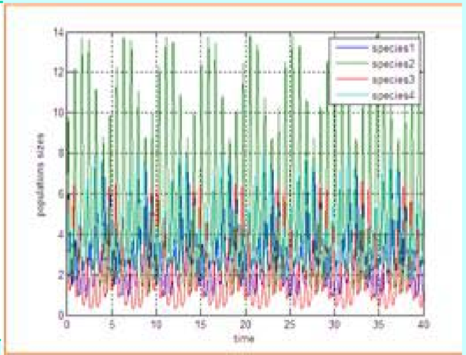
$$\begin{cases} \dfrac{dx_i}{dt} = x_i(t)\cdot\left(k_i + \beta_i^{-1}\sum_{j=1}^{N} a_{ij}\cdot x_j(t)\right), i = 1,2,..M, \\ \dfrac{dx_l}{dt} = x_l(t)\cdot\left(k_l + \beta_l^{-1}\sum_{j=1}^{N} a_{lj}\cdot x_j(t) + u_l(t)\right), l = M+1,..N. \end{cases}$$

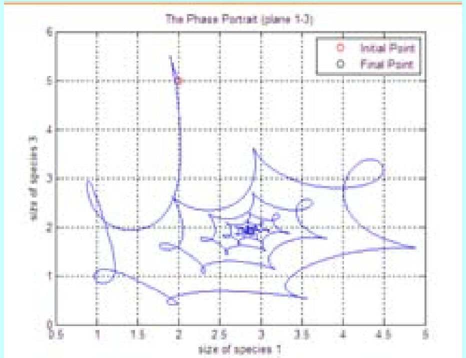
Energy/
entropy production function:

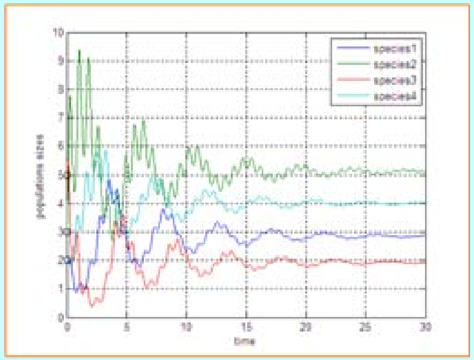
$$W(x) = \sum_{i=1}^{N} \beta_i n_i \left(\frac{x_i}{n_i} - \log\frac{x_i}{n_i}\right).$$

Control goal: $W(X(t)) \rightarrow W^*$

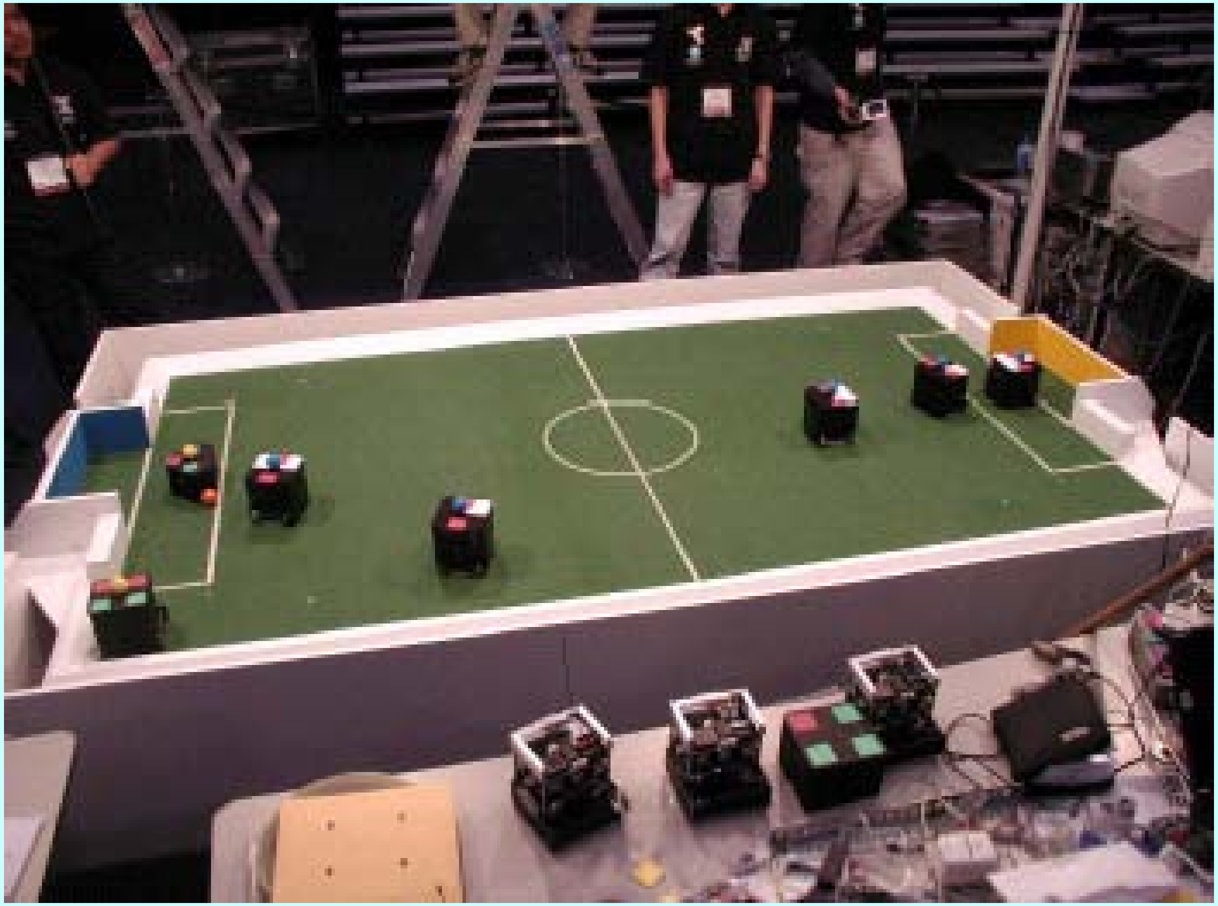






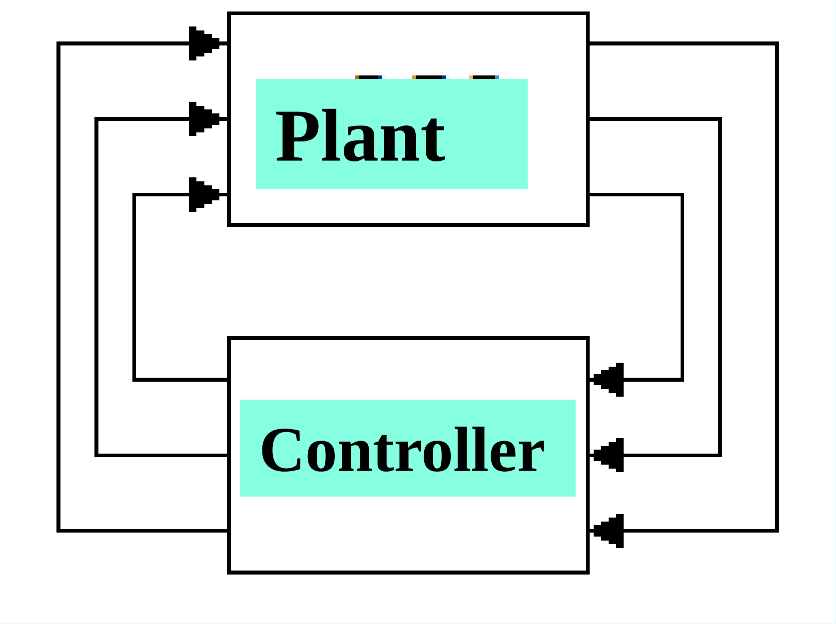


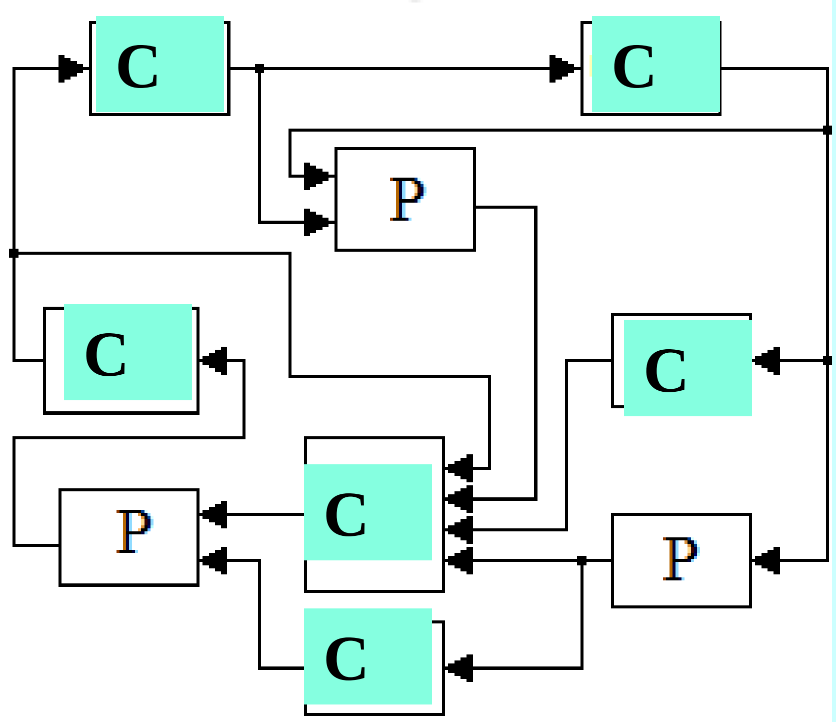
COOPERATIVE CONTROL OF ROBOTS



Football of robots

What is control of network?





XX century: MIMO systems

XXI century: networks

Control of network: problem statement

Consider N interacting subsystems (agents)

$$\dot{x}_i = F(x_i, u_i) + \sum_{j=1}^{n} a_{ij}\varphi_{ij}(x_i, x_j) \qquad y_i = h_i(x_i)$$

$x_i(t)$ - state vectors of agents, $u_i(t)$ - inputs (controls),

$y_i(t)$ - measurable outputs

Control goal: synchronization (consensus) for outputs:

$$\lim_{x\to\infty} \left| y_i(t) - y_j(t) \right| = 0, \;\; i, j = 1,\ldots, N$$

P-controller (consensus protocol):

$$u_i = \sum_{j\in N_i} K(y_j - y_i), i = 1,\ldots, N$$

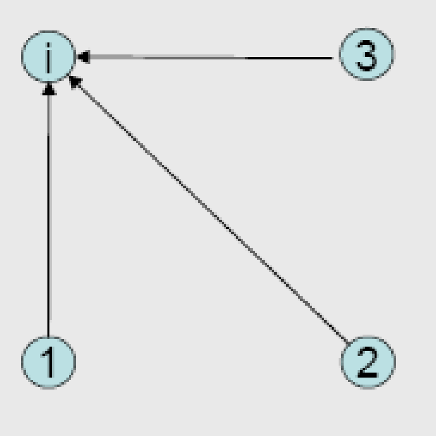
N_i is set of neighbors for i th agent

Control of network: special case

$$\dot{x}_i = f(x_i) + c \sum_{j=1,\, j\neq i}^{N} \mathrm{a}_{ij}\Gamma(x_j - x_i) + u_i, i = 1,\dots,N$$

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} \mathrm{a}_{ij}\Gamma x_j + u_i, i = 1,\dots,N$$

$$A = \begin{pmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} & \cdots & \mathrm{a}_{1N} \\ \mathrm{a}_{21} & \mathrm{a}_{22} & \cdots & \mathrm{a}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{a}_{N1} & \mathrm{a}_{N2} & \cdots & \mathrm{a}_{NN} \end{pmatrix} \qquad \mathrm{a}_{ii} = -\sum_{j=1,\, j\neq i}^{N} \mathrm{a}_{ij}$$



Neighbors of agent i

$$L = -A$$ – Laplace matrix.

Information graph G: $\mathrm{a}_{ij} > 0 \Leftrightarrow \quad \exists\, arc : j \to i$

Networked controller

$$u_i = \sum_{j=1,\, j\neq i}^{N} b_{ij}\Lambda(x_j - x_i), i = 1,\ldots,N$$

- B - matrix of interactions in controller,
- $L = -B$ - Laplace matrix of controller.
- Control goal – consensus:

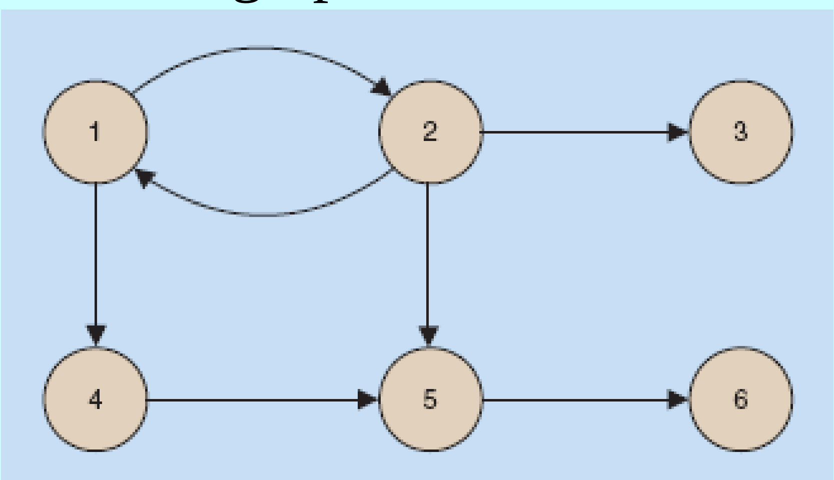
$$\lim_{x\to\infty}\left\|x_i(t) - x_j(t)\right\| = 0,\ i,\ j = 1,\ldots,N$$

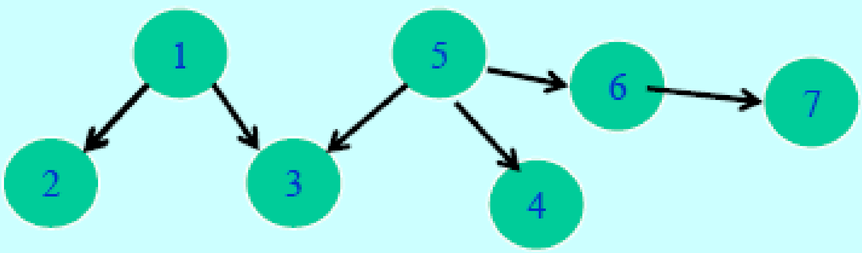
Network control of network:

$L = -(A + B)$ - closed loop Laplace matrix

Laplace matrix properties:

- Defect of *L* = number of graph connected components (λ_1=0)
- *L is* symmetric ⇔ graph is undirected (λ_2 is algebraic connectivity)
- Spectrum of *L* is in the closed right hand part half-plane
- For digraph defect *L* =1 ⇔ directed spanning tree exists





Agaev-Chebotarev theorem (Aut.Rem.Control, 9, 2000, LAA, 2005):
Defect of *L = min. number of trees in spanning forest*

Existing results

Consensus conditions - I

$$\dot{x}_i = u_i, \quad y_i = x_i, \quad i = 1, \ldots, N, \; x_i \in \boldsymbol{R}^1$$

Consensus exists ⇔ directed spanning tree exists

The states of the agents tend to the average in initial conditions

(W. Ren and R.W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Automat. Contr., vol. 50, no. 5, pp. 655–661, 2005.)*

$$X = (x_1, \ldots, x_N)^T, \qquad dX/dt = -KLX$$

Consensus conditions - II

Consensus in the networks of 2nd order agents

(W. Yu, G.Chen, M.Cao. On second-order consensus in multi-agent dynamical systems with directed topologies and time delays IEEE Conf. Dec.Contr. (CDC-2009), Shanghai Dec.2009.)

$$\dot{x}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = -\alpha \sum_{j=1}^{N} L_{ij} x_j(t) - \beta \sum_{j=1}^{N} L_{ij} v_j(t), i = 1, 2, \ldots, N$$

Consensus ⇔ Directed spanning tree exists and the inequality holds:

$$\frac{\beta^2}{\alpha} > \max_{2 \le i \le N} \frac{\Im^2(\mu_i)}{\Re(\mu_i\left[\Re^2(\mu_i) + \Im^2(\mu_i)\right])}$$

μ_i **- eigenvalues of Laplace matrix** $\mu_i = \Re(\mu_i) + j\Im(\mu_i)$

Consensus conditions - III

Consensus in the networks of 2nd order agents with delay

(W. Yu, G.Chen, M.Cao. IEEE CDC-2009, Shanghai Dec.2009)

$$\dot{x}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = -\alpha \sum_{j=1}^{N} L_{ij} x_j(t-\tau) - \beta \sum_{j=1}^{N} L_{ij} v_j(t-\tau)$$

Let directed spanning tree exists and the following inequalities hold:

$$\frac{\beta^2}{\alpha} > \max_{2 \le i \le N} \frac{\Im^2(\mu_i)}{\Re(\mu_i \left[\Re^2(\mu_i) + \Im^2(\mu_i)\right])}$$

$$\tau < \tau_0 = \min_{2 \le i \le N} \left\{ \frac{\theta_{i1}}{\omega_{i1}} \right\}, where \quad 0 < \theta_{i1} < 2\pi$$

$$\cos\theta_{i1} = \frac{\left[\Re(\mu_i)\alpha - \Im(\mu_i)\omega_{i1}\beta\right]}{\omega_{i1}^2} \qquad \omega_{i1} = \sqrt{\frac{\|\mu_i\|^2 \beta^2 + \sqrt{\|\mu_i\|^4 \beta^4 + 4\|\mu_i\|^2 \alpha^2}}{2}}$$

Controlled synchronization of networks

$$\dot{x}_i = f(x_i) + c\sum_{j=1,\, j\neq i}^{N} \mathrm{a}_{ij}\Gamma(x_j - x_i) + u_i, i = 1,\ldots,N$$

Most existing results deal with fully controlled agents, especially in adaptive control…

Challenge:

To design decentralized adaptive output feedback control ensuring synchronization under conditions of uncertainty and incomplete control

Controlled network of linear agents:

$$dx_i/dt=Ax_i+Bu_i\ ,\ y_i=C^Tx_i$$

$W(s)=C^T(sI-A)^{-1}B$ – **transfer matrix**

Output feedback (Diffusion coupling, Consensus protocol):

$$u_i=\sum_{j\in N_i}K(y_j-y_i),i=1,\ldots,N$$

Control goal: $\lim\limits_{x\to\infty}\left\|x_i(t)-x_j(t)\right\|=0,\ i,\ j=1,\ldots,N$

Passivity (Willems, 1972)

$$dx/dt=f(x)+g(x)u\ ,\ y=h(x)$$

There is a function V(x) ≥ 0 (storage function):

$$V(x(t)) \le V(x(0)) + \int_0^t y(\tau)^T u(\tau) d\tau$$

For linear systems storage function can be quadratic:

$V(x)=x^T P x,\ P=P^T>0$ $\quad$ $(\ PA+A^T P<0,\ \ PB=C^T\ )$

Passification theorem

(Fradkov: Aut.Rem.Contr.,1974(12), Siberian Math.J. 1976, Europ. J.Contr. 2003; Andrievsky, Fradkov, Aut.Rem.Contr., 2006 (11))

Let $W(s)=C^T(sI-A)^{-1}B$ – transfer matrix of a linear system. The following statements are equivalent:

A) There exist matrix $P=P^T>0$ and row vector K, such that $PA_K+A_K^T P<0$, $PB=(CG^T)$, $A_K=A+BKC^T$

B) Matrix GW(s) is hyper-minimum-phase (det[GW(s)] has Hurwitz numerator, $GC^TB=(GC^TB)^T>0$)

C) There exists K such that feedback u=Ky+v renders the system strictly passive «from input v to output Gy»

Theorem (Junussov, Fradkov, Aut. Rem.Contr., 2011, 8).
A1. Network graph is balanced and has directed spanning tree.
A2. *gW(s)* is hyper-minimum-phase for some *g*=(g_1,…g_n).
(det[*gW(s)*] has Hurwitz numerator, $gC^TB=(gC^TB)^T>0$)

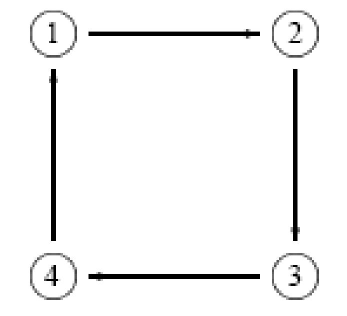
Let *K*=*μg*, where $\mu > \kappa / \lambda_2$ $\quad \kappa = \inf_{\omega \in R} \Re e(gW(i\omega))$

Then synchronization is achieved and

$$\lim_{t\to\infty}(x_i(t) - d^{-1}e^{At}(1_d^T \otimes I_n)x(0)) = 0, \; i = 1,\dots,d.$$

Example. Network of double integrators, N=4.

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

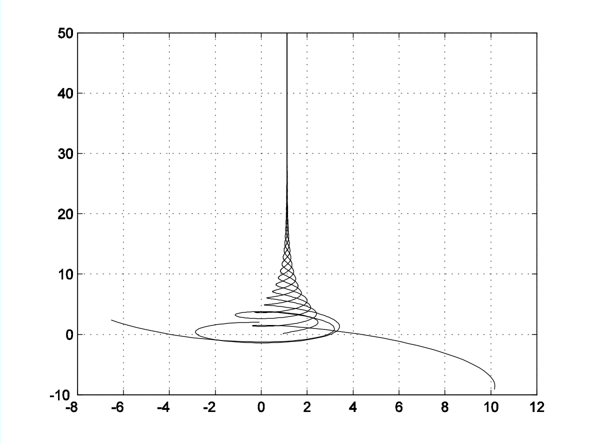


$$\chi(s) = C^{\mathrm{T}}(sI_2 - A)^{-1}B = \frac{s+1}{s^2}$$

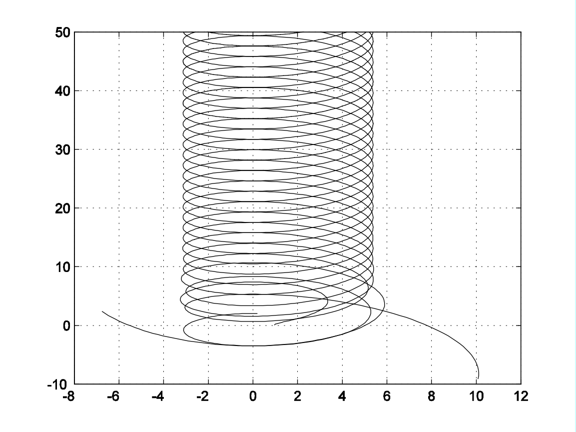
Example. Network of double integrators, N=4.

Simulation results.

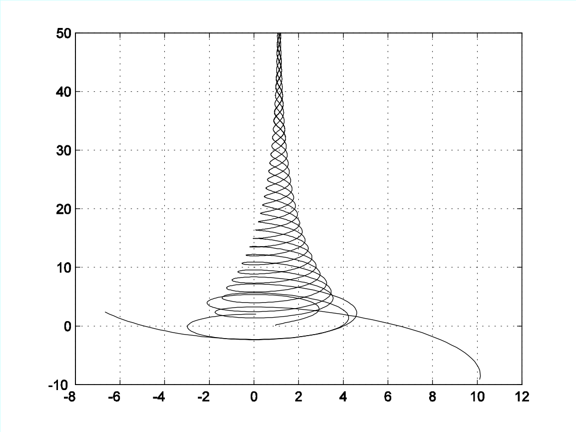
μ=1



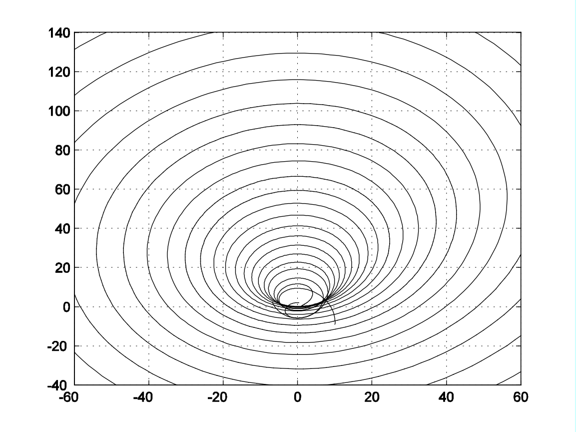
μ=0.5



μ=0.7



μ=0.3



Extension 1 (Adaptive control):

1. Network graph with a spanning tree
2. Adaptive controller:

$$u_i(t) = \theta_i(t) y_i(t),$$

$\theta_i(t) \in \mathbb{R}^{1 \times l}$ - tunable parameters

Adaptation algorithm:

$$\theta_i(t) = -g^{\mathrm{T}} \cdot k_i(t),$$
$$\dot{k}_i(t) = y_i(t)^{\mathrm{T}} g g^{\mathrm{T}} \overline{y}_i.$$

where

$$\overline{y}_i = \sum_{j \in N_i} (y_i - y_j), i = 1, \ldots, N$$

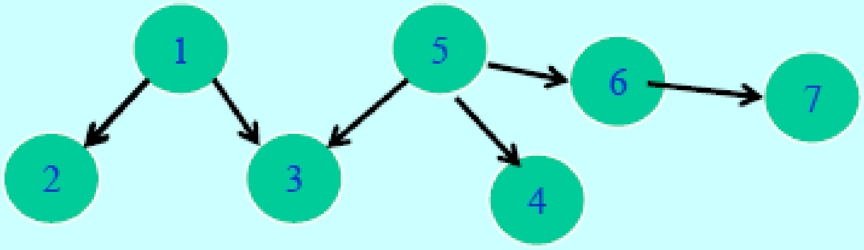
Extension 2 (Fradkov, Junussov, IEEE Conf. Dec.Contr., Orlando, Dec. 2011):

1. Network digraph with a directed spanning tree

$$\Lambda = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda_e \end{pmatrix} = P^{-1} L P,$$

$$I_{N-1} + \frac{k}{2}(\Lambda_e + \Lambda_e^*) \leq 0$$

2. Nonlinear (Lurie) models of agent dynamics
3. Clasterization: trees in the spanning forest do not overlap.



Decentralized control of Lurie networks under bounded disturbances (Fradkov, Grigoriev, Selivanov, IEEE Conf. Dec.Contr. Orlando, Dec. 2011):

$$\dot{x}_i = Ax_i + Bu_i + \varphi_0(x_i) + \sum_{j=1}^{d} \alpha_{ij}\varphi_{ij}(x_i - x_j) + f_i(t),$$

$$y_i = C^{\mathrm{T}} x_i \quad (1)$$

Control goal:

$$\overline{\lim_{t\to\infty}} |x_i(t) - \overline{x}(t)| \leqslant \Delta_i \qquad \dot{\overline{x}} = A\overline{x} + B\overline{u} + \varphi_0(\overline{x}), \overline{y} = C^{\mathrm{T}}\overline{x},$$

or

$$\overline{\lim_{t\to\infty}} Q_i(x_i(t), t) \leqslant \Delta_i,$$ ***i*=1,…*N***

$$Q(z_i) = \frac{1}{2} z_i^{\mathrm{T}} H z_i,\ H = H^{\mathrm{T}} > 0. \qquad z_i = x_i - \overline{x},$$

Decentralized control of Lurie networks under bounded disturbances (Fradkov, Grigoriev, Selivanov, IEEE Conf. Dec.Contr. Orlando, Dec. 2011):

Adaptive controller:

$$\tilde{u}_i = \theta_i^{\mathrm{T}}(t)\tilde{y}_i,\ \theta_i(t) \in \mathbb{R}^l, \quad \tilde{y}_i = C^{\mathrm{T}} z_i, \quad z_i = x_i - \overline{x},\ \tilde{u}_i = u_i - \overline{u},$$

$$\dot{\theta}_i(t) = \begin{cases} -g^{\mathrm{T}}\tilde{y}_i(t)\Gamma_i\tilde{y}_i(t), & Q_i(x_i(t),t) > \Delta_i \\ 0\,, & Q_i(x_i(t),t) \leqslant \Delta_i. \end{cases}$$

Lyapunov function: $V(x_1,\ldots x_N,t)=\sum\alpha_i Q_i(x_i,t)$,

$\alpha_i > 0$ **– some weights**

Decentralized control of Lurie networks with delayed couplings (Fradkov, Selivanov, Fridman, 18th World Congress on Aut. Control, Milan, Aug. 2011):

$$\dot{x}_i(t) = Ax_i(t) + h(x_i(t), t) + \sigma \sum_{j=1}^{N} \alpha_{ij}(x_j(t) - x_i(t)) + \sigma \sum_{j=1}^{N} \beta_{ij}(x_j(t-\tau) - x_i(t-\tau)) + Bu_i(t),$$

Control goal:

$$\lim_{t\to\infty} \|x_i(t) - \bar{x}(t)\| = 0, \qquad i = 1, 2, \ldots, N.$$

$$\dot{\bar{x}}(t) = A\bar{x}(t) + h(\bar{x}(t), t) + B\bar{u}(t),$$
$$\bar{y}(t) = C\bar{x}(t),$$

Decentralized control of Lurie networks with delayed couplings (Fradkov, Selivanov, Fridman, 18th World Congress on Aut. Control, Milan, Aug. 2011):

Adaptive controller:

$$u_i(t) = -\theta_i(t)^{\mathrm{T}}(y_i(t) - \bar{y}(t)) + \bar{u}(t),$$
$$\dot{\theta}_i = \gamma_i[(y_i(t) - \bar{y})^{\mathrm{T}} g(y_i(t) - \bar{y})],$$

Lyapunov functional:

$e_i(t) = x_i(t) - \bar{x}(t)$

$$V(e) = \sum_{i=1}^{N} e_i(t)^{\mathrm{T}} P e_i(t) + \sum_{i=1}^{N} (\theta_i(t) - \theta_*)^{\mathrm{T}} \gamma_i^{-1} (\theta_i(t) - \theta_*) +$$
$$\sum_{i=1}^{N} \int_{t-\tau}^{t} e_i(s)^{\mathrm{T}} H_i e_i(s) ds, \quad (11)$$

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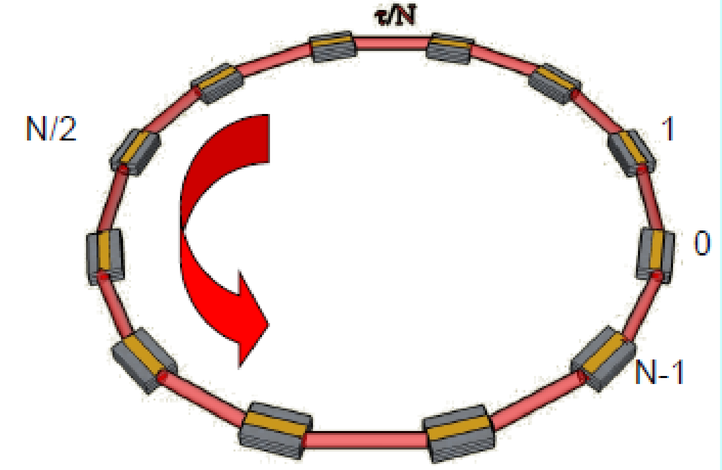
Control of Complex Systems Laboratory

Phase and claster synchronization in a network of Landau-Stuart oscillators

- **Selivanov A.A., Lehnert J, Dahms T., Hovel P., Fradkov A.L., Schoell E.**
- **Adaptive synchronization in delay-coupled networks of Stuart-Landau oscillators. Phys.Rev. E 85, 016201 (2012).**

$$\dot{z}_j(t) = f[z_j(t)] + Ke^{i\beta} \sum_{n=1}^{N} a_{jn}[z_n(t-\tau) - z_j(t)], \quad (1)$$

$$f(z_j) = [\lambda + i\omega - (1 + i\gamma)|z_j|^2]z_j, \quad (2)$$



Phase and claster synchronization in a network of Landau-Stuart oscillators (cont)

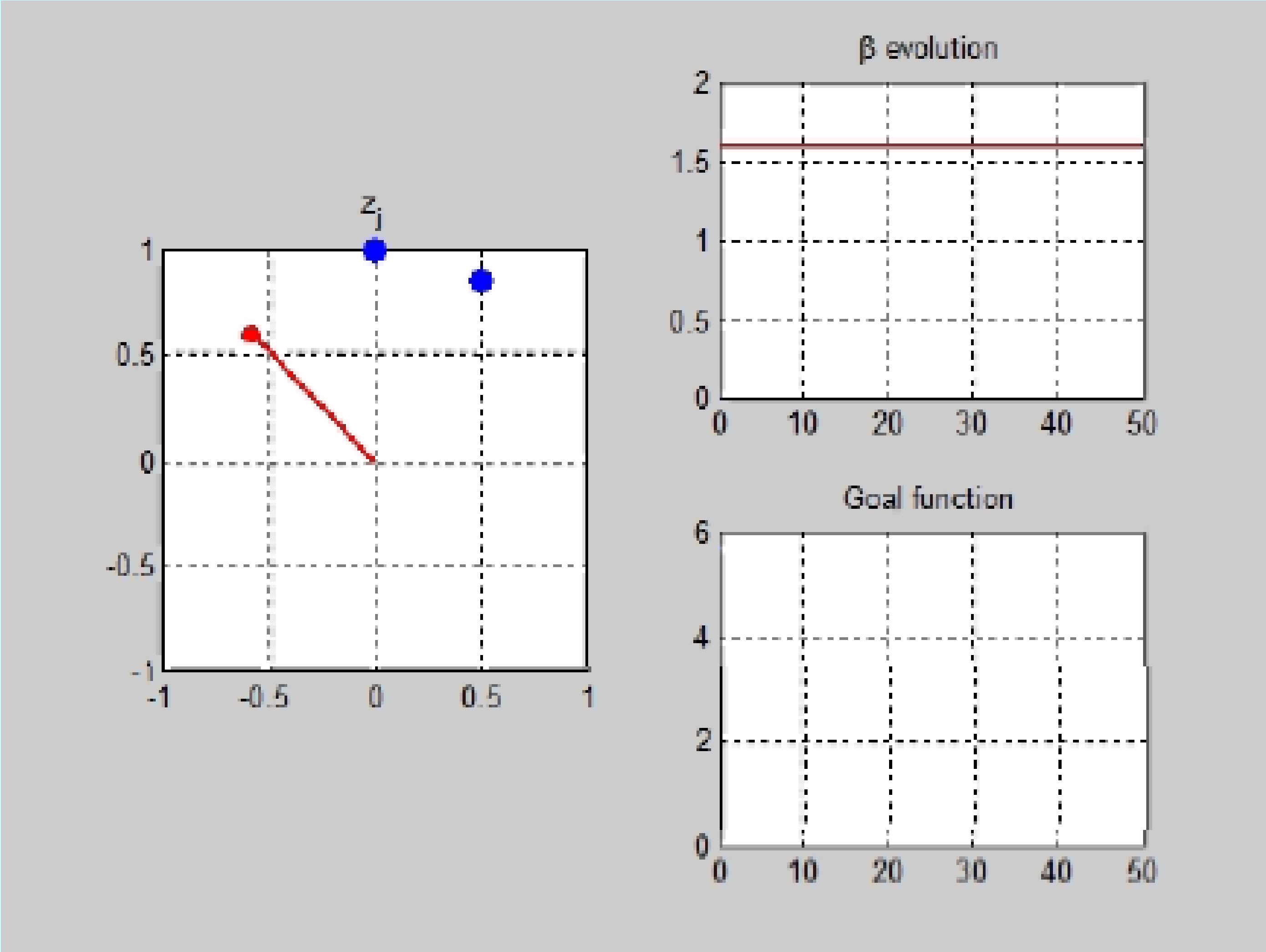
$$Q = 1 - \frac{1}{N^2} \sum_{j=1}^{N} e^{d_c i \varphi_j} \sum_{k=1}^{N} e^{-d_c i \varphi_k} + \frac{1}{2} \sum_{d_c \div s, 1 \le s < N} \sum_{k=1}^{N} e^{s i \varphi_k} \sum_{j=1}^{N} e^{-s i \varphi_j}$$

$$\nabla_\beta \dot{Q} = K \sum_{j=1}^{N} \sum_{k=1}^{N} \{ \sum_{d_c \div s, 1 \le s < N} s \sin(s(\varphi_k - \varphi_j)) - \frac{2 d_c}{N^2} \sin(d_c(\varphi_k - \varphi_j)) \}$$

$$\sum_{n=1}^{N} a_{jn} \left(\frac{r_{n,\tau}}{r_j} \cos(\beta + \varphi_{n,\tau} - \varphi_j) - \cos(\beta) \right) \tag{14}$$

$$\dot{\beta} = -\Gamma \nabla_\beta \dot{Q}$$

Phase and claster synchronization in a network of Landau-Stuart oscillators (cont.)



Robot football at Math faculty of SPbSU

Robot football at Math faculty