Differential Algebraic Annihilators : new paradigms for estimation/observation

W. Perruquetti

Non-A INRIA - LNE,
Parc Scientifique de la Haute Borne 40,
avenue Halley Bât.A,
Park Plaza 59650 Villeneuve d'Ascq
e-mail : wilfrid.perruquetti@inria.fr

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A first simple example: parameter estimation

$$\boxed{\dot{y}(t) = ay(t) + u(t) + \gamma_0.} \qquad (1)$$

where a is an unknown parameter to be identified and γ_0 is an unknown, constant perturbation.
Using operational calculus and $y_0 = y(0)$:

$$s\widehat{y}(s) = a\widehat{y}(s) + \widehat{u}(s) + y_0 + \frac{\gamma_0}{s}.$$

☞ Eliminate the term γ_0 : use operator $D_s \times s$:

$$\frac{d}{ds}\left[s\left\{s\widehat{y}(s) = a\widehat{y}(s) + \widehat{u}(s) + y_0 + \frac{\gamma_0}{s}\right\}\right]$$

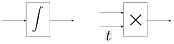
$$\Rightarrow 2s\widehat{y}(s) + s^2\widehat{y}'(s) = a\left(s\widehat{y}'(s) + \widehat{y}(s)\right) + s\widehat{u}'(s) + \widehat{u}(s) + y_0.$$

A first simple example: parameter estimation

☞ Estimation of parameter a : Assume $y_0 = 0$ (if not use D_s^2 to eliminate y_0), for any $\nu > 0$,

$$s^{-\nu}\left[2s\widehat{y}(s) + s^2\widehat{y}'(s)\right] = s^{-\nu}\left[a(s\widehat{y}'(s) + \widehat{y}(s)) + s\widehat{u}'(s) + \widehat{u}(s)\right].$$

$$a = \frac{2\int_0^T d\lambda \int_0^\lambda y(t)dt - \int_0^T ty(t)dt + \int_0^T d\lambda \int_0^\lambda tu(t)dt - \int_0^T d\lambda \int_0^\lambda d\sigma \int_0^\sigma u(t)dt}{\int_0^T d\lambda \int_0^\lambda d\sigma \int_0^\sigma y(t)dt - \int_0^T d\lambda \int_0^\lambda ty(t)dt}, (\nu = 3). \quad (2)$$

- two kind of operations 

- $T > 0$ can be very small $\Rightarrow$ fast estimation.
- ν number of iterative integrals $\Rightarrow$ filtering (mean processing) : one can also use low pass filter $s \to (1 + \tau s)$.
- including a noise (fast fluctuating signal), of zero mean $\dot{y}(t) = ay(t) + u(t) + \gamma_0 + n(t)$ (filtering)

A first simple example: parameter estimation

This example, even simple, clearly demonstrated how ALIEN's techniques proceed:

- they are algebraic: operations on s-functions;
- they are non-asymptotic: parameter a is obtained from (2) in finite time;
- they are deterministic: no knowledge of the statistical properties of the noise n is required.

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A simple example

Let us recall

$$\mathcal{L}^{-1}\left(\frac{1}{s^m}\frac{d^n X(s)}{ds^n}\right) = \frac{(-1)^n t^{m+n}}{(m-1)!}\int_0^1 w^{m-1,n}(\tau)x(t\tau)d\tau, \quad m\geq 1, \; n\in\mathbb{N} \tag{3}$$

where

$$w^{m,n}(t) = (1-t)^m t^n \tag{4}$$

☞ Normalized

☞ The noise passing through the filter is amplified by t^{m+n}

A simple example

☞ Estimate $x^{(2)}(0)$ through the truncated series of order 2:

$$\mathcal{R}: \qquad X = \frac{x(0)}{s} + \frac{x^{(1)}(0)}{s^2} + \frac{x^{(2)}(0)}{s^3}$$

Idea: kill undesired terms (blue) except the one to estimate (red)

Step 1 $\times s^2$: $s^2 X = s x(0) + x^{(1)}(0) + \dfrac{x^{(2)}(0)}{s}$

Step 2 $\dfrac{d^2}{ds^2}$: $2X + 4s\dfrac{dX}{ds} + s^2\dfrac{d^2X}{ds^2} = \dfrac{2}{s^3}x^{(2)}(0)$

Step 3 $\times \dfrac{1}{s^3}$: $\dfrac{2}{s^3}X + \dfrac{4}{s^2}\dfrac{dX}{ds} + \dfrac{1}{s}\dfrac{d^2X}{ds^2} = \dfrac{2}{s^6}x^{(2)}(0)$

Step 4 Go back to the time domain (use of $\mathcal{L}^{-1}$ (11)):

$$\frac{2t^5}{5!}x^{(2)}(0) = t^3\int_0^1 \left(2w^{2,0}(\tau) - 4w^{1,1}(\tau) + w^{0,2}(\tau)\right) y(t\tau)d\tau$$

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A simple example

Let us mention that finally we have applied to the relation $\mathcal{R}$, the operator

$$\Pi = \frac{1}{s^3} \frac{d^2}{ds^2} s^2$$

where

- $\Pi \in \mathbb{R}(s) \left[\frac{d}{ds}\right]$
- $\Pi = \frac{1}{s} \frac{d^2}{ds^2} + \frac{4}{s^2} \frac{d}{ds} + \frac{2}{s^3}$

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Preliminary remarks

Let $f(s)$ be a polynomial in the variable s. By Leibniz's rule:

$$\left(\frac{d}{ds}s\right)(f) = \frac{d}{ds}(sf) = f + s\frac{df}{ds} \quad \text{and} \quad \left(s\,\frac{d}{ds}\right)(f) = s\frac{df}{ds}$$

So

$$\left(\frac{d}{ds}s \;-s\;\frac{d}{ds}\right)(f) = f \Longrightarrow \frac{d}{ds}s \;-s\;\frac{d}{ds} = 1$$

Or using the commutator notation:

$$\left[\frac{d}{ds}, s\right] = \frac{d}{ds}s - s\frac{d}{ds} = 1$$

Set $p := \frac{d}{ds}$ and $q := s \times \cdot$, therefore

$$[p, q] = pq - qp = 1$$

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Preliminary remarks

Back to the first example

In the first example we have used

$$\Pi = \frac{1}{s^3}\ \frac{d^2}{ds^2}\ s^2$$

Here we look at p^2q^2. Since $pq = qp + 1$ we have

$$\begin{aligned} p^2q^2 &= p(qp+1)q = pqpq + pq = (qp+1)(qp+1) + (qp+1) \\ &= qpqp + 3qp + 2 = q(qp+1)p + 3qp + 2 = q^2p^2 + 4qp + 2 \end{aligned}$$

Thus we find again, since $p := \frac{d}{ds},\ q := s \times \cdot$:

$$\Pi = \frac{1}{s}\frac{d^2}{ds^2} + \frac{4}{s^2}\frac{d}{ds} + \frac{2}{s^3}$$

Preliminary remarks

Back to the first example

Now let us note that $\frac{d}{ds}\ s^2\ \frac{d}{ds}s$ reads as

$$pq^2pq = q^3p^2 + 4q^2p + 2q$$

which means that these two operators $\frac{1}{s^3}\ \frac{d^2}{ds^2}\ s^2$ and $\frac{1}{s^3}\ \frac{d}{ds}\ s^2\ \frac{d}{ds}s$ are the same, they can be written as

$$\frac{1}{s}\frac{d^2}{ds^2} + \frac{4}{s^2}\frac{d}{ds} + \frac{2}{s^3}$$

☞ Use a "canonical form"

Preliminary remarks

Weyl Algebra

- Let us note that A $:= \mathbb{R}[s]\left[\frac{d}{ds}\right]$ has a Weyl Algebra structure (non commutative since $\left[\frac{d}{ds}, s\right] = 1$)

- Thus a canonical basis of A is $\left\{ s^i \frac{d^j}{ds^j} \middle| (i,j) \in \mathbb{N} \right\}$

- Any $F \in$ A can be rewritten into its *canonical form*

$$F = \sum_{i,j} \lambda_{ij} s^i \frac{d^j}{ds^j}, \ \lambda_{ij} \in \mathbb{R} \tag{5}$$

Preliminary remarks
Weyl Algebra

☞ One can associate to the Weyl Algebra A an algebra B defined as the differential operators on $\frac{d}{ds}$ with coefficients in $\mathbb{R}(s)$

$$\mathsf{B} := \mathbb{R}(s)\left[\frac{d}{ds}\right]$$

☞ Any $F \in \mathsf{B}$ can be rewritten into its *canonical form*

$$F = \sum_{i,j} \lambda_{ij} g_i(s) \frac{d^j}{ds^j}, \quad \text{with} \quad g_i(s) \in \mathbb{R}(s) \tag{6}$$

Parameter estimation for sinusoidal biased signal

For a sinusoidal biased noisy signal, to estimate the triplet (amplitude, phase, frequency) is of practical importance for most engineers:

- signal demodulation in communication,
- voltage control of boost converter in power electronics,
- circadian rhythm in biology,
- ...
- modal identification for a flexible beam which plays a central role in AFM : the amplitude of the observed signal is influenced by the interaction forces between the atoms and the 'pointe' (leading to an image of the observed atoms)

Parameter estimation for sinusoidal biased signal

This generic problem consists in estimating the parameters α, ϕ and $\omega > 0$ for the signal

$$z = \alpha \sin(\omega t + \phi) \tag{7}$$

using a biased noisy measure (from sensor):

$$y = \alpha \sin(\omega t + \phi) + \beta + \varpi, \tag{8}$$

where β is an unknown constant bias and ϖ is the noise.

Parameter estimation for sinusoidal biased signal

Some other technics:

- Least Square method,
- EKF,
- non linear adaptive observers
- ...

None of these technics give the triplet within a sufficiently small time window (fraction of the signal period) and in a robust manner using noisy measures. The only quite satisfactory solutions were provided by Hebert Sira-Ramirez and Dayan Liu et al. (unbiased case).

☞ Here using some knowledge about minimal annihilators, we'll obtain a less noise sensitive solution...

Parameter estimation of a triplet for sinusoidal biased signal

☞ signal $x = z + \beta$ biased signal (7) satisfies the following ODE

$$\ddot{x} + \omega^2(x - \beta) = 0 \tag{9}$$

☞ $\Theta_{\text{est}} = \{\theta_1 := \omega^2, \theta_2 := -\alpha \sin(\phi) = -x(0) + \beta, \theta_3 := -\alpha\omega \cos(\phi) = -\dot{x}(0)\}$

☞ $\Theta_{\overline{\text{est}}} = \{\theta_4 := -\beta\}$.

Parameter estimation of a triplet for sinusoidal biased signal

☞ Equation (9) in the operational domain can be rewritten into

$$\mathcal{R}\left(s, X(s), \Theta_{\text{est}}, \Theta_{\overline{\text{est}}}\right): \quad P(X(s)) + Q + \overline{Q} = 0 \qquad (10)$$

with $P = s(s^2 + \theta_1)$, $Q = s^2\theta_2 + s\theta_3, \overline{Q} = (s^2 + \theta_1)\theta_4$

☞ Thus we are looking for annihilators $\Pi \in \mathbb{R}(s)\left[\frac{d}{ds}\right]$ such that $\Pi\left(\overline{Q}\right) = 0$

☞ Annihilators $\frac{1}{s^4}\frac{d^3}{ds^3}, \frac{1}{s^5}\frac{d^4}{ds^4}s$ and many others work $\Rightarrow$ Use of Weyl Algebra Structure and its corresponding canonical form (Theorem by Stafford)

Parameter estimation of a triplet for sinusoidal biased signal

1. Algebraic elimination of all terms in the $\Theta_{\overline{\text{est}}}$ variables: differential operator can kill $\overline{Q}$ (annihilator), described by a single operator in $\mathbb{R}_{\Theta_{\text{est}}}(s)\left[\frac{d}{ds}\right]$ [1].
2. Obtaining a set of equations in Θ_{est} with good numerical properties: Use canonical forms (for annihilator),
3. Back to the time domain (solve system):

$$\mathcal{L}^{-1}\left(\frac{1}{s^m}\frac{d^pX(s)}{ds^p}\right) = \frac{(-1)^p t^{m+p}}{(m-1)!}\int_0^1 w^{m-1,p}(\tau)x(t\tau)d\tau \qquad (11)$$

with $w^{m,p}(t) = (1-t)^m t^p, \forall\, p, m \in \mathbb{N}, m \geq 1$. Choose m, p as small as possible (noise sensitivity).

[1] The polynomial ring in $\frac{d}{ds}$ with coefficients in $\mathbb{R}_{\Theta_{\overline{\text{est}}}}[s]$

Parameter estimation of a triplet for sinusoidal biased signal

☞ Minimal annihilator $\Pi_{\min} = s^2 \frac{d^2}{ds^2} - s \frac{d}{ds}$ applied to (10) gives $\Pi_{\min}(Q) = -s\theta_3$ and

$$\Pi_{\min}(P \cdot X) = (s^5 + s^3\theta_1)\frac{d^2X}{ds^2} + (5s^4 + s^2\theta_1)\frac{dX}{ds} + (3s^3 - s\theta_1)X$$

☞ One family of algebraically dependent relations.

Parameter estimation of a triplet for sinusoidal biased signal

In order to linearly identify the two parameters θ_1, θ_3 we need $\Pi = (a_1(s)\frac{d}{ds} + a_0(s))\Pi_{\min}$

$$\Pi = g_3(s)\frac{d^3}{ds^3} + g_1(s)\left(s\frac{d^2}{ds^2} - \frac{d}{ds}\right),$$

with $g_1(s) = \sum_{i=1}^{m} \frac{a_i}{s^i}$ et $g_3(s) = \sum_{i=0}^{m} \frac{b_i}{s^i}$, $a_i, b_i \in \mathbb{R}$.

$$\Pi_1 = \frac{1}{s^4}\Pi_{\min} \quad \text{et} \quad \Pi_2 = \frac{1}{s^4}\frac{d^3}{ds^3}.$$

Parameter estimation of a triplet for sinusoidal biased signal

$$\begin{pmatrix} \frac{1}{s^5}\mathcal{O}_1 & -\frac{1}{s^5} \\ \frac{1}{s^4}\mathcal{O}_3 & 0 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{s^5}\mathcal{O}_2 \\ \frac{1}{s^4}\mathcal{O}_4 \end{pmatrix},$$

with

$$\mathcal{O}_1 = s^2 \frac{d^2X(s)}{ds^2} + s\frac{dX(s)}{ds} - X(s),$$

$$\mathcal{O}_2 = s^4 \frac{d^2X(s)}{ds^2} + 5s^3\frac{dX(s)}{ds} + 3s^2X(s),$$

$$\mathcal{O}_3 = s\frac{d^3X(s)}{ds^3} + 3\frac{d^2X(s)}{ds^2},$$

$$\mathcal{O}_4 = s^3\frac{d^3X(s)}{ds^3} + 9s^2\frac{d^2X(s)}{ds^2} + 18s\frac{dX(s)}{ds} + 6X(s).$$

Parameter estimation of a triplet for sinusoidal biased signal

We finally get

$$\theta_1 = \frac{1}{t^2} \frac{\int_0^1 (-w^{0,3}(\tau) + 9w^{1,2}(\tau) - \frac{1}{2}w^{2,1}(\tau) + w^{3,0}(\tau))x(t\tau)d\tau}{\int_0^1 (\frac{-1}{2}w^{2,3}(\tau) + \frac{1}{2}w^{3,2}(\tau))x(t\tau)d\tau},$$

$$\theta_3 = \frac{5!}{t^3} \Bigg(\int_0^1 (w^{0,2}(\tau) - 5w^{1,1}(\tau) + \frac{3}{2}w^{2,0}(\tau))x(t\tau)d\tau - \theta_1 \int_0^1 (\frac{1}{2}w^{2,2}(\tau) - w^{3,1}(\tau) - \frac{1}{4!}w^{4,0}(\tau))x(t\tau)d\tau \Bigg)$$

Parameter estimation of a triplet for sinusoidal biased signal

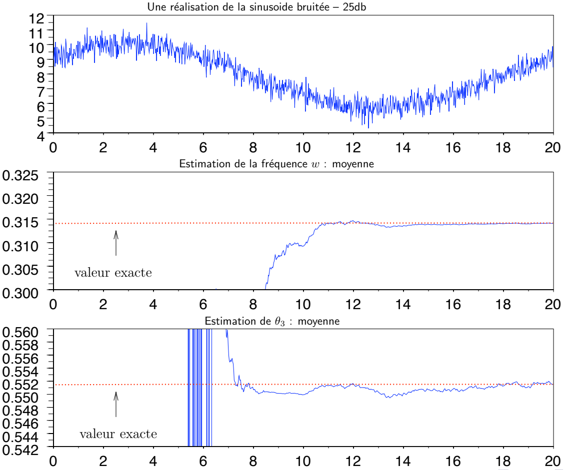
Looking for the minimal annihilator in $\mathbb{R}_{\Theta_{\text{est}}}(s)\left[\frac{d}{ds}\right]$ (be carefull with previous case !) we find

$$\Pi_3 = -\frac{1}{2}\left(\frac{1}{s^4}+\frac{\theta_1}{s^6}\right)\frac{d}{ds}+\frac{1}{s^5}$$

Thus it is possible to get the last parameter

$$\theta_2 = \frac{1}{10t^2\theta_1}\left(\theta_3 t(20-t^2\theta_1)+\int_0^1\left(-t^4\theta_1^2(w^{5,0}(\tau)+5w^{4,1}(\tau))\right.\right.$$
$$\left.\left.+40t^2\theta_1(3w^{2,1}(\tau)-w^{3,0}(\tau))-120(w^{1,0}(\tau)-w^{0,1}(\tau))\right)x(t\tau)d\tau\right).$$

Parameter estimation of a triplet for sinusoidal biased signal



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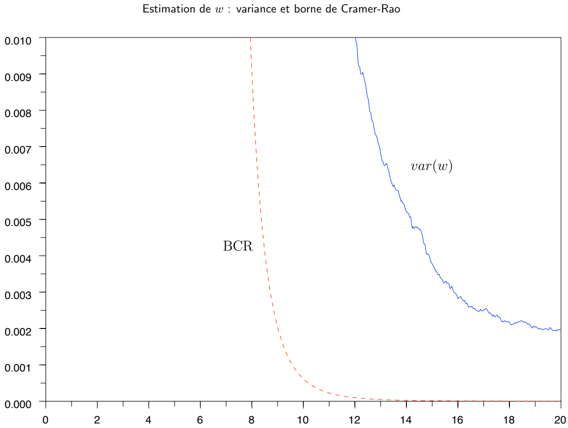

Observation via numerical differentiation

Applications of causal Jacobi estimators to non linear observation

- The Ball and Beam system:

$$\left\{ \begin{array}{rcl} (mr^2+J)\ddot{\theta}+2mr\dot{r}\dot{\theta}+mgr\cos(\theta) & = & u \\ m\ddot{r}+mg\sin(\theta)-mr\dot{\theta}^2 & = & 0. \end{array} \right. \tag{12}$$

- The state vector $(r,\dot{r},\theta,\dot{\theta})^T$ and the vector of outputs $(y_1,y_2)^T=(r,\theta)^T$:

$$\left\{ \begin{array}{rcl} r & = & y_1 \\ \dot{r} & = & -\frac{1}{2my_1\dot{y}_2}\left((my_1^2+J)\ddot{y}_2+mgy_1\cos(y_2)-u\right) \\ \theta & = & y_2 \\ \dot{\theta} & = & \left(\frac{\ddot{y}_1+g\sin(y_2)}{y_1}\right)^{\frac{1}{2}}. \end{array} \right. \tag{13}$$

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Applications of causal Jacobi estimators to non linear observation

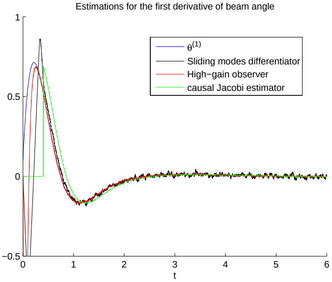
- The Ball and Beam system:

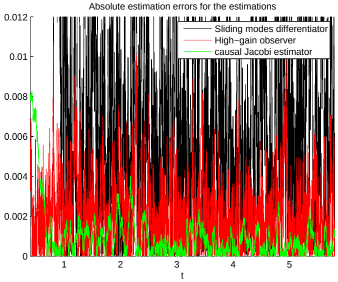
$$\left\{ \begin{array}{rcl} (mr^2 + J)\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\cos(\theta) & = & u \\ m\ddot{r} + mg\sin(\theta) - mr\dot{\theta}^2 & = & 0. \end{array} \right. \quad (12)$$

- The state vector $(r, \dot{r}, \theta, \dot{\theta})^T$ and the vector of outputs $(y_1, y_2)^T = (r, \theta)^T$:

$$\left\{ \begin{array}{rcl} r & = & y_1 \\ \dot{r} & = & -\frac{1}{2my_1\dot{y}_2}\left((my_1^2 + J)\ddot{y}_2 + mgy_1\cos(y_2) - u\right) \\ \theta & = & y_2 \\ \dot{\theta} & = & \left(\frac{\ddot{y}_1 + g\sin(y_2)}{y_1}\right)^{\frac{1}{2}}. \end{array} \right. \quad (13)$$

Observation via numerical differentiation





- The output of θ with $SNR = 24.5\text{dB}$ and $T_s = 10^{-4}$;
- The used causal Jacobi estimator $D^{(n)}_{\kappa,\mu,-T,q} x(t_0 - T\xi_{q+1,\kappa,\mu})$ with $\kappa = \mu = 0$, $q = 1$, $\xi_{q+1,\kappa,\mu} = 0.2764$ and $T = 4000Ts$.

Observation via numerical differentiation

Comparisons with different criterions

Observer	Convergence time	Number of parameters
Jacobi estimator	T	4 (one for each estimation)
High-gain	known (gain)	2
Sliding modes	known (gain)	2

Observer	Time-delay	Robustness (noise and T_s)
Jacobi estimator	known	good
High-gain	unknown (small)	good
Sliding modes	unknown	bad

Mobile robot localization using a single landmark

State Space Model of the unicycle robot

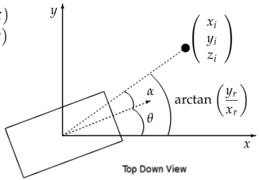
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} u\cos(\theta) \\ u\sin(\theta) \\ \omega \end{bmatrix} \quad (14)$$

$$V = [u\ \omega]^T \quad (15)$$

Measurement model

$$Z = H(X) = \begin{bmatrix} \alpha + \eta_\alpha \\ \beta + \eta_\beta \end{bmatrix} \quad (16)$$

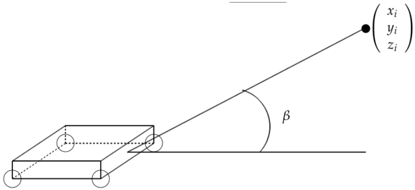
$$x_r = (x_i - x)$$
$$y_r = (y_i - y)$$



Top Down View

Goal

Find K as
$P = [x, y, \theta]^T = K(Z, \dot{Z}, \ddot{Z}, ..., Z^{(n)})$



Three dimensional view

Mobile robot localization using a single landmark

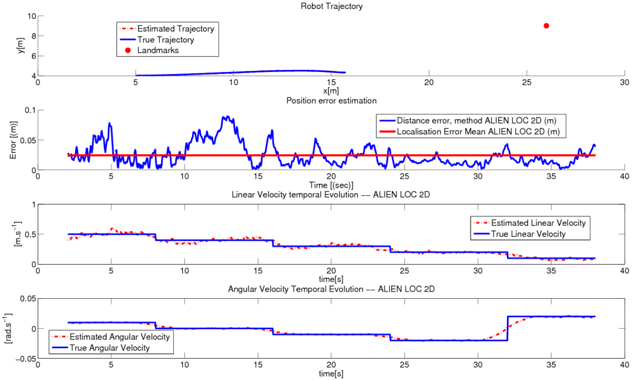
Results in the 3D case

$$\widehat{u} = \frac{z_{Ai}\dot{\beta}_f}{\sin^2 \beta_f \cos \alpha_f} \quad (17)$$

$$\widehat{\omega} = \frac{2\tan(\alpha_f)}{\sin(2\beta_f)}\dot{\beta}_f - \dot{\alpha}_f \quad (18)$$

$$\widehat{X}_r = \begin{bmatrix} \widehat{x}_r \\ \widehat{y}_r \end{bmatrix} = \begin{bmatrix} \dfrac{\widehat{u}\sin(\alpha_f)\cos(\alpha_f+\theta_f)}{(\dot{\alpha}_f+\widehat{\omega})} \\ \dfrac{\widehat{u}\sin(\alpha_f)\sin(\alpha_f+\theta_f)}{(\dot{\alpha}_f+\widehat{\omega})} \end{bmatrix} \quad (19)$$

Mobile robot localization using a single landmark



Mobile robot localization using a single landmark

The algorithm	ALIEN Algorithm	EKF
Needs to know the control input	NO	YES
Needs to know the noise characteristics	NO	YES
Needs to know the orientation of the robot	YES	NO
Needs to be initialized	NO	YES
Needs only one landmarks	YES	NO
Give an confidence interval	NO	YES

Table: Comparison of two localization algorithms hypothesis

Applicative fields

...in control

- *Magnetic levitation*: one magnetic bearing benchmark,
- *Friction*: two benchmarks linear drive actuating a cart-pendulum, and a stepper motor,
- *Multi-cell chopper* (switching component, hybrid system): observer-based control algorithm,
- *Machine tools* (cooperation with ENSAM Lille): high-speed CNC machines,
- *Process engineering*
- *Networked control*: 2 benchmarks (phantom coupled with two 6 Dof Robot manipulator and two inverted pendulum)
- *Collaborative robotics*: 6 benchmarks (2 miabot fleets (one is at eruratechnology center), 3 pekee, and 20 wifibot (with EPI FUN (ex-POPS))

Applicative fields
...in control

Video

Applicative fields

... signal, image and video processing

- *Compression of audio signals,*
- *Demodulation and its theoretical background,*
- *Compression, edge and motion detection of image and video signals,*
- *Multi-user detection,*
- *Direction-of-arrival estimation,*
- *Turbo-codes,*
- *Watermarking,*
- *Cryptography.*