# Non-A

## Non-Asymptotic estimation for online systems


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Non-A Inria Lille - Nord Europe


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☞ Papers can be found at http://hal.inria.fr/

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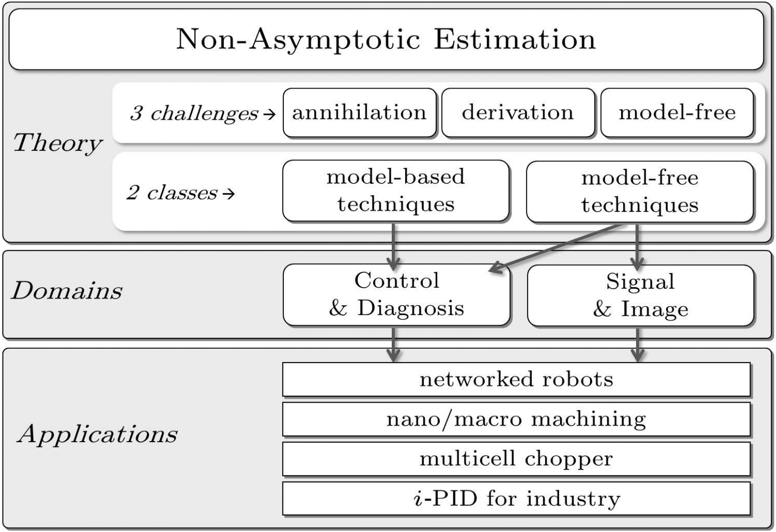
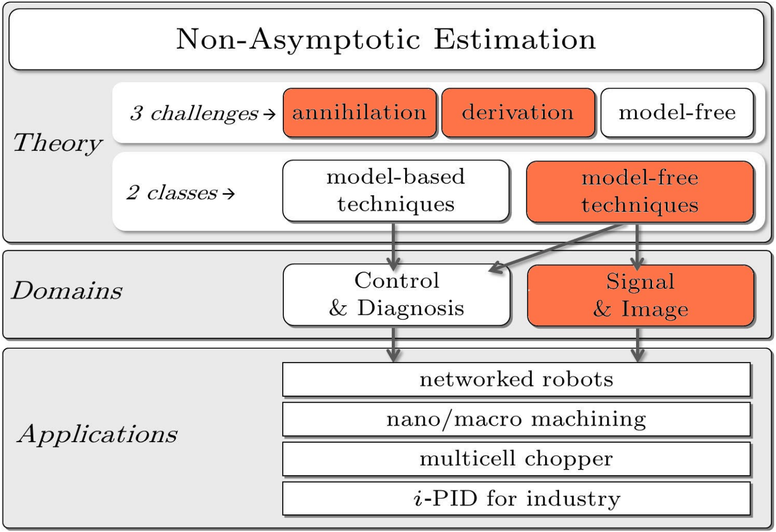
# Non-A challenges and applications



# Non-A challenges and applications



# Main concern: Estimation

☞ Parametric estimation :

$$y = F(x, \Theta) + n, \quad (1)$$

- $y$ is the measured signal,
- $F$ a functional of the "true" signal $x$ depending on parameters $\Theta$,
- $n$ is a noise corrupting the observation.

$\Rightarrow$ Find a "good" $\Theta_{\approx}$ (Existing results: probabilistic knowledge of $n$).

☞ Non-A new standpoint, of algebraic flavour:

- differential algebra which plays w.r.t ODE a similar rôle to commutative algebra w.r.t AE;
- module theory (linear algebra over rings -instead of fields- which are not necessarily commutative);
- operational calculus which was a most classical tool among control and mechanical engineers.

# Estimation and its applications

☞ Automatic control:

- identification of uncertain parameters in the system equations, including delays, switches (lin./nonlin., open/closed loop);
- fault detection and isolation;
- observer-based chaotic synchronization.

☞ Signal, image and video processing:

- noise removal, estimation,
- signal modeling, restoration, . . .
- data compression, decoding for error correcting codes,
- detection of abrupt changes, . . .

☞ Both Control and Signal:

- "model-free control"

$$\boxed{\dot{y}(t) = a y(t) + u(t) + \gamma_0.} \tag{2}$$

Unknown: $a$ parameter to be identified and $\gamma_0$ constant perturbation.

☞ Operational calculus:

$$s\widehat{y}(s) = a\widehat{y}(s) + \widehat{u}(s) + y_0 + \frac{\gamma_0}{s}.$$

☞ Algebra: differential operator $\boxed{\frac{1}{s^\nu}\frac{d}{ds}s}$ $(\nu = 3, y_0 = 0)$ leads to:

$$\boxed{a = \frac{2\int_0^T d\lambda \int_0^\lambda y(t)dt - \int_0^T ty(t)dt + \int_0^T d\lambda \int_0^\lambda tu(t)dt - \int_0^T d\lambda \int_0^\lambda d\sigma \int_0^\sigma u(t)dt}{\int_0^T d\lambda \int_0^\lambda d\sigma \int_0^\sigma y(t)dt - \int_0^T d\lambda \int_0^\lambda ty(t)dt}.} \tag{3}$$

# Still simple: numerical differentiation

2nd-order derivative, estimated on the basis of a 2nd order Taylor polynomial, considering a sliding window with *small size* $T$ :

$$x(t) = x(0) + x^{(1)}(0)t + \frac{1}{2}x^{(2)}(0)t^2, \ t \in [0, T]$$

or

$$X(s) = \frac{1}{s}x(0) + \frac{1}{s^2}x^{(1)}(0) + \frac{1}{s^3}x^{(2)}(0)$$

is given by the integral formula:

$$x^{(2)}(0) = \frac{5!}{T^5}\int_0^T \left(\tau^2 - 4(T-\tau)\tau + (T-\tau)^2\right) x(\tau)d\tau$$

with the corresponding operator:

$$\frac{1}{s^3}\ \frac{d^2}{ds^2}\ s^2$$

# A look on algebra

For the 2nd order derivative estimation, we have applied the operator:

$$\Pi = \frac{1}{s^3}\ \frac{d^2}{ds^2}\ s^2 \in \mathbb{R}(s)\left[\frac{d}{ds}\right]$$

☞ Weyl Algebra structure ⇒ "canonical form":

$$\Pi = \frac{1}{s}\frac{d^2}{ds^2} + \frac{4}{s^2}\frac{d}{ds} + \frac{2}{s^3}$$

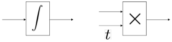
Weyl Algebra: let $p = \frac{d}{ds}$ and $q = s\times$, then $[p, q] = pq - qp = 1$.

# Non-A core

These examples show how Non-A techniques proceed. They are:

- Algebraic: operators (Weyl Algebra),
- Non-Asymptotic ($a$ or $x^{(2)}(0)$ are obtained in finite time $T$),
- deterministic: no knowledge of the stat. prop. of the noise,

with the following properties:

- two basic operations 

- $T > 0$ can be very small $\Rightarrow$ fast estimation.
- iterative integrals (convolution) $\Rightarrow$ filtering - mean processing.