AUSTRALIAN CENTRE FOR ROBOTIC VISION

Tutorial::Vision for Robotics

Vision Sensors & Geometry

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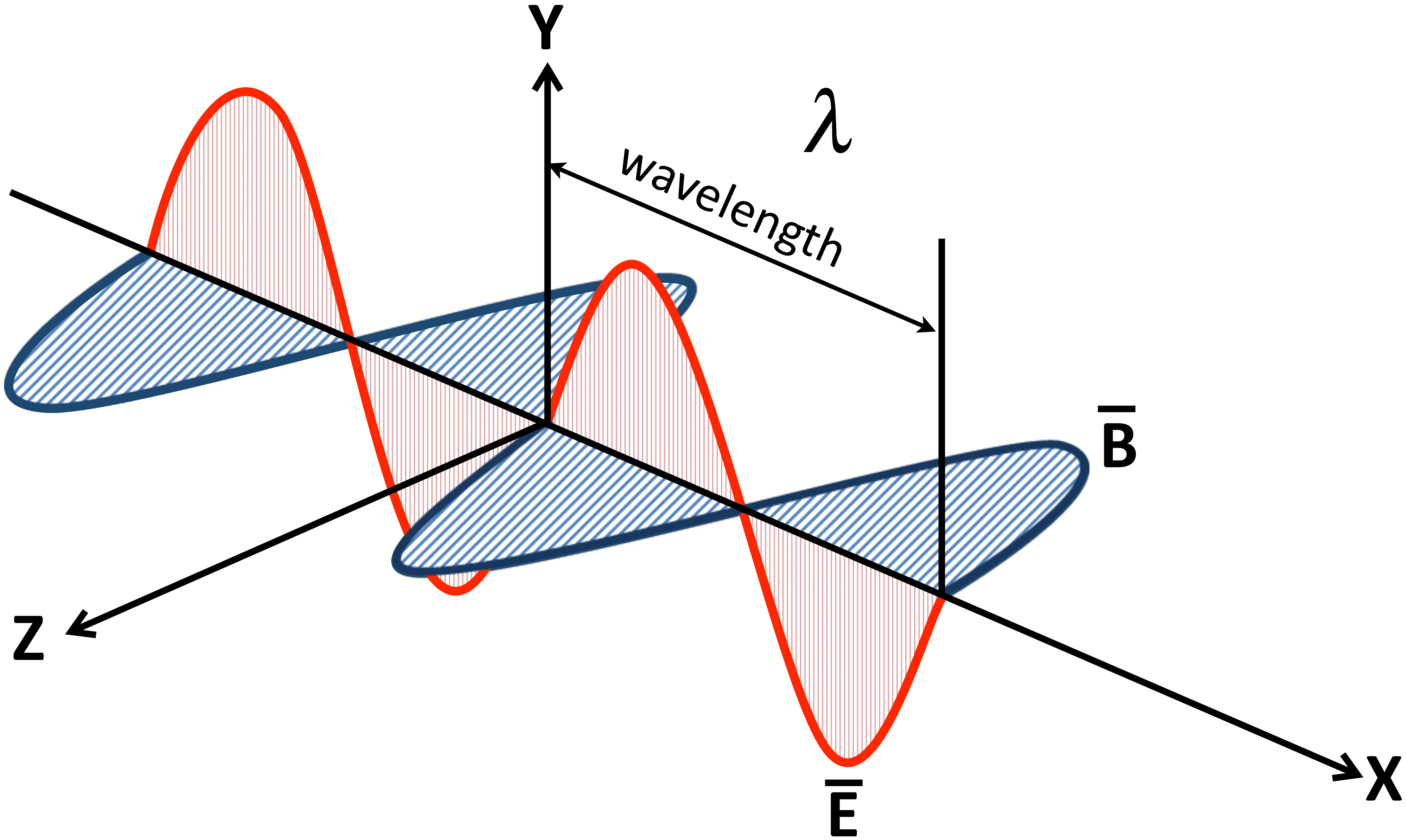
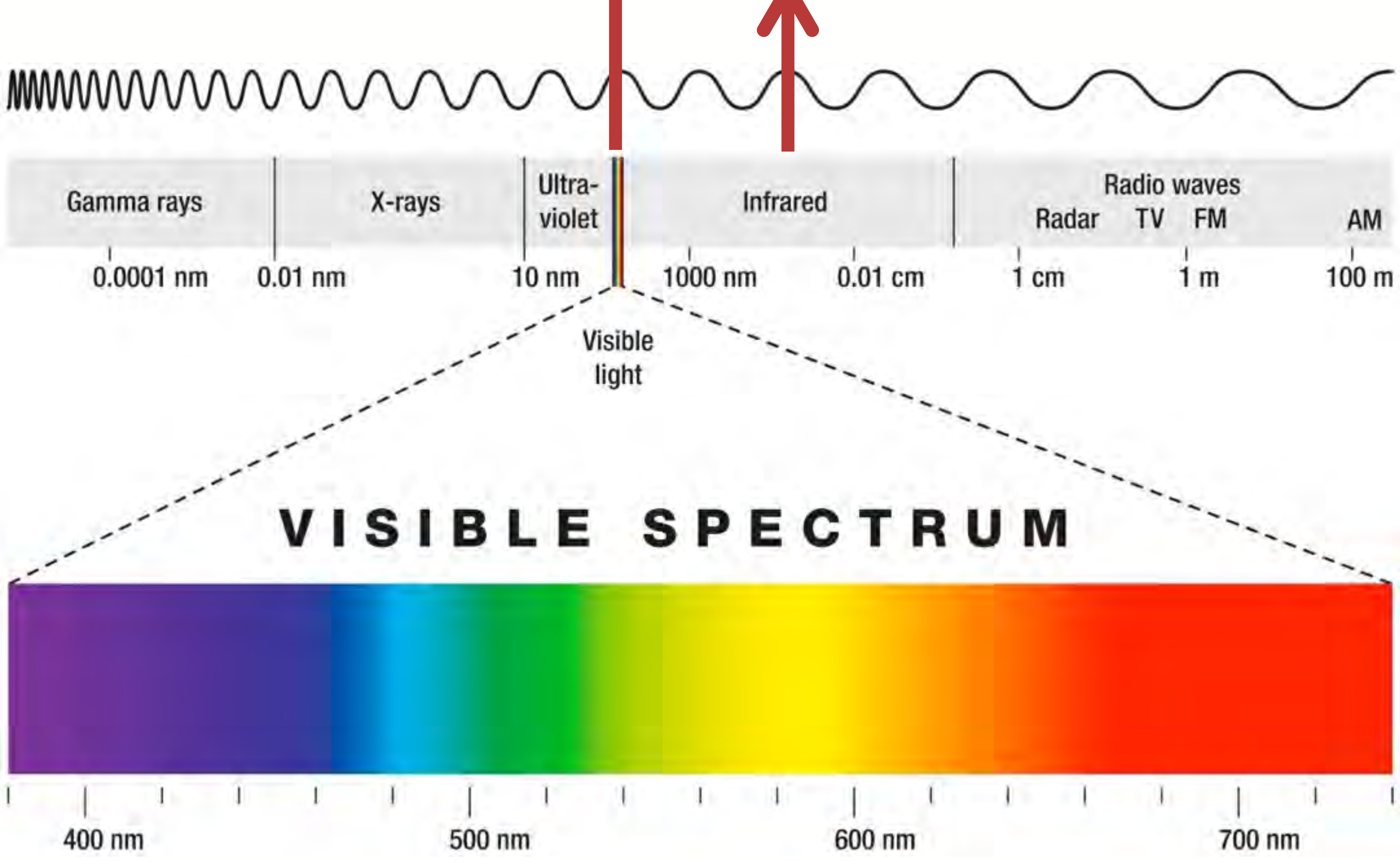
QS STARS AWARDS 2015 REIMAGINE EDUCATION

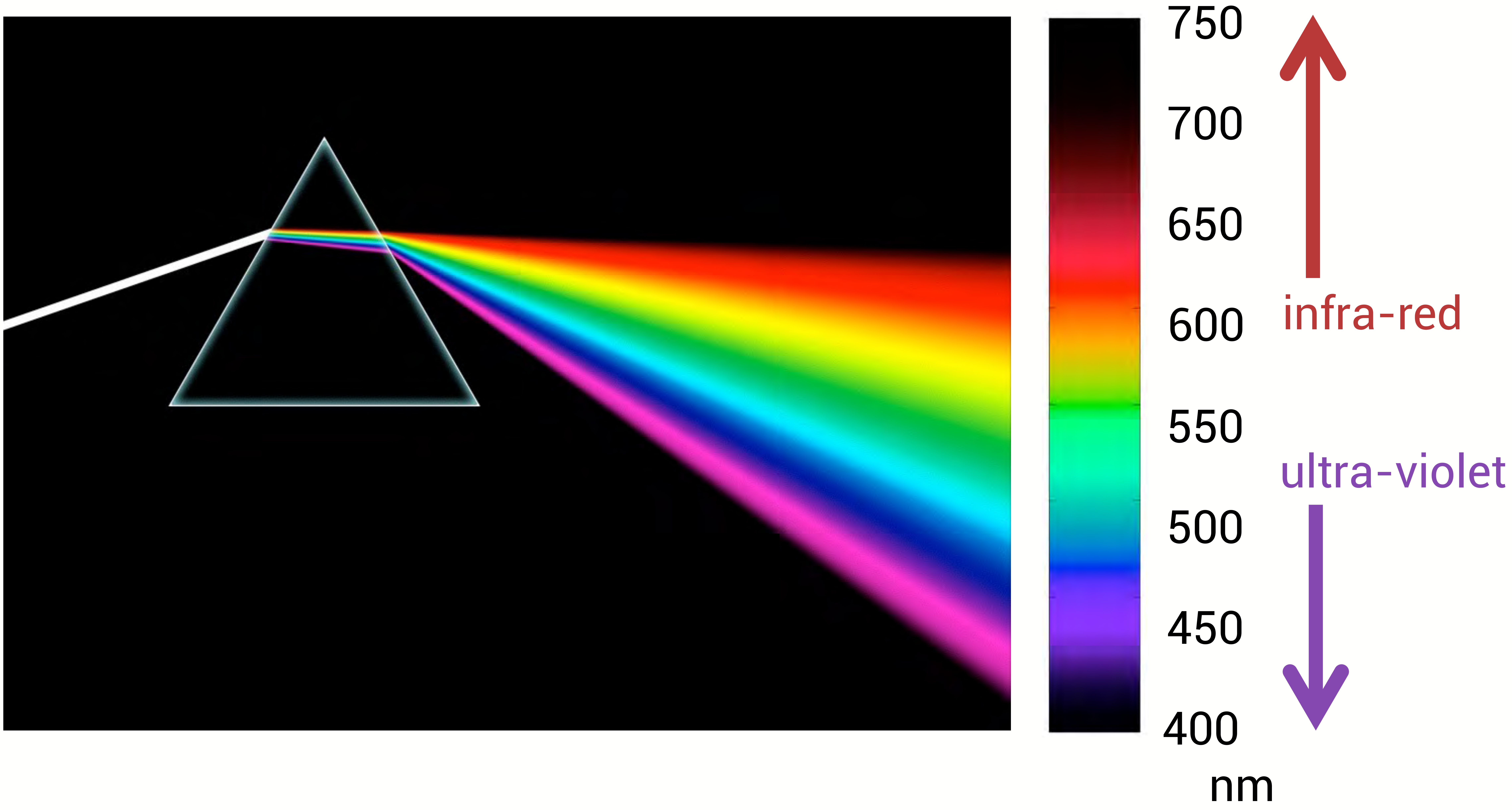
Light is an electromagnetic wave



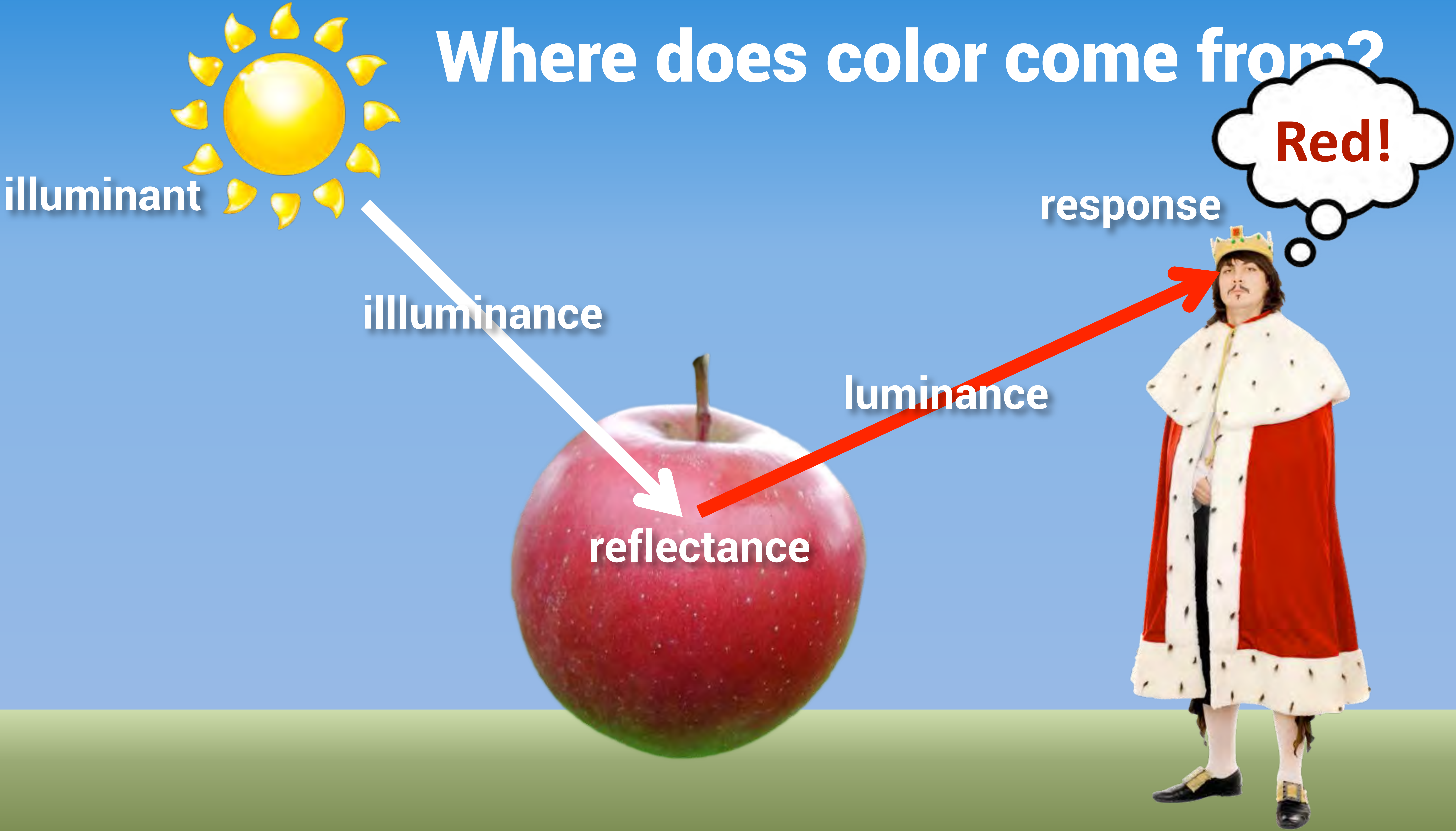
Human **senses**

Max Dupain | Public Domain via Wikimedia Commons




750
700
650
600
550
500
450
400
nm
infra-red
ultra-violet

Where does color come from?



Where does color come from?

Sources of light

incandescent

Gluehbirne 2006
Dickbauch

Sun Halo 2013
Janice Marie Foote | CC A2.0.

Electron stimulated

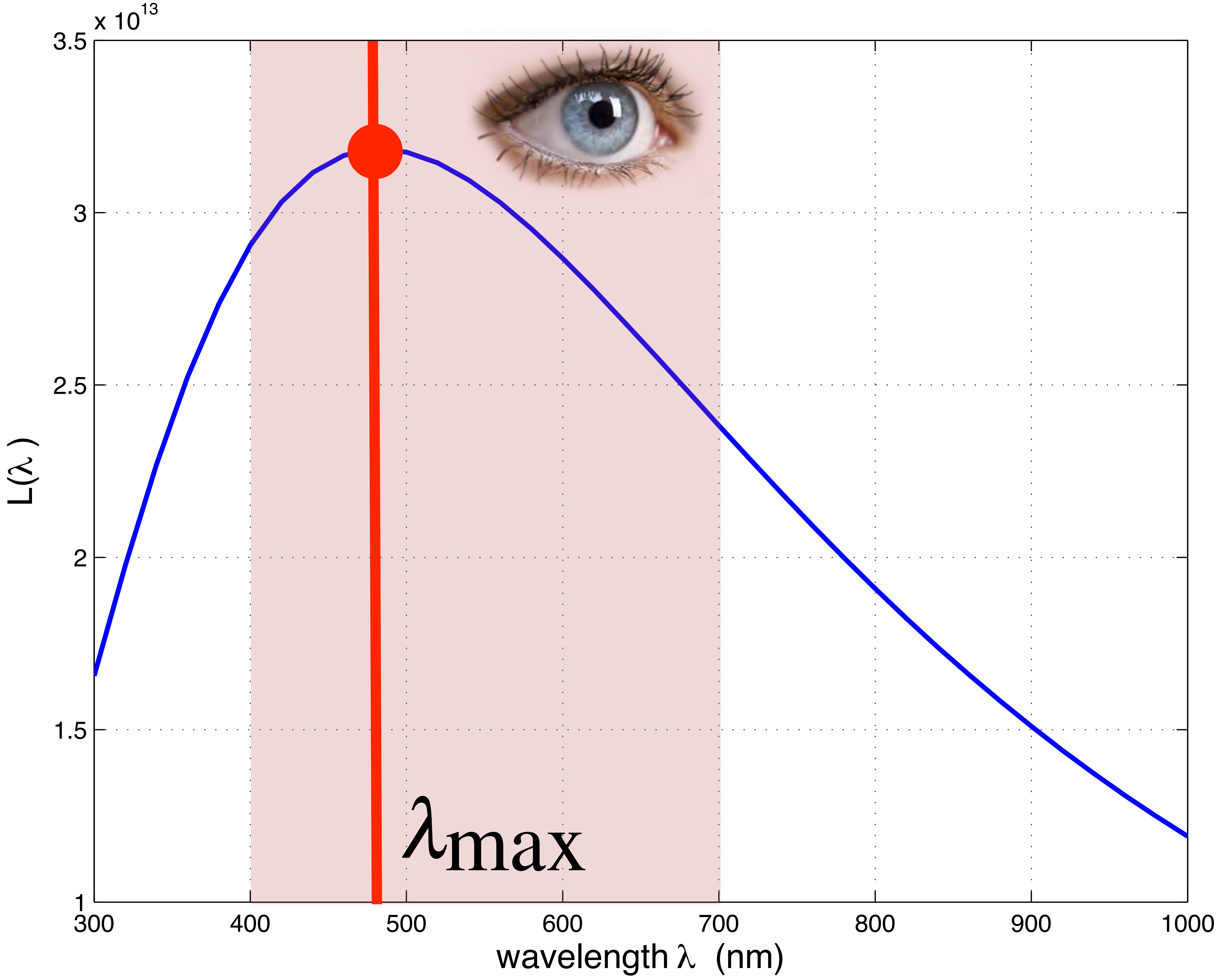
Electro luminescent

RGB LED 2005
PiccoloNamek | CC A3.0.

gas discharge

laser

Blackbody radiators



Gluehbirne 2006

Dickbauch

Sun Halo 2013

Janice Marie Foote | CC A2.0.

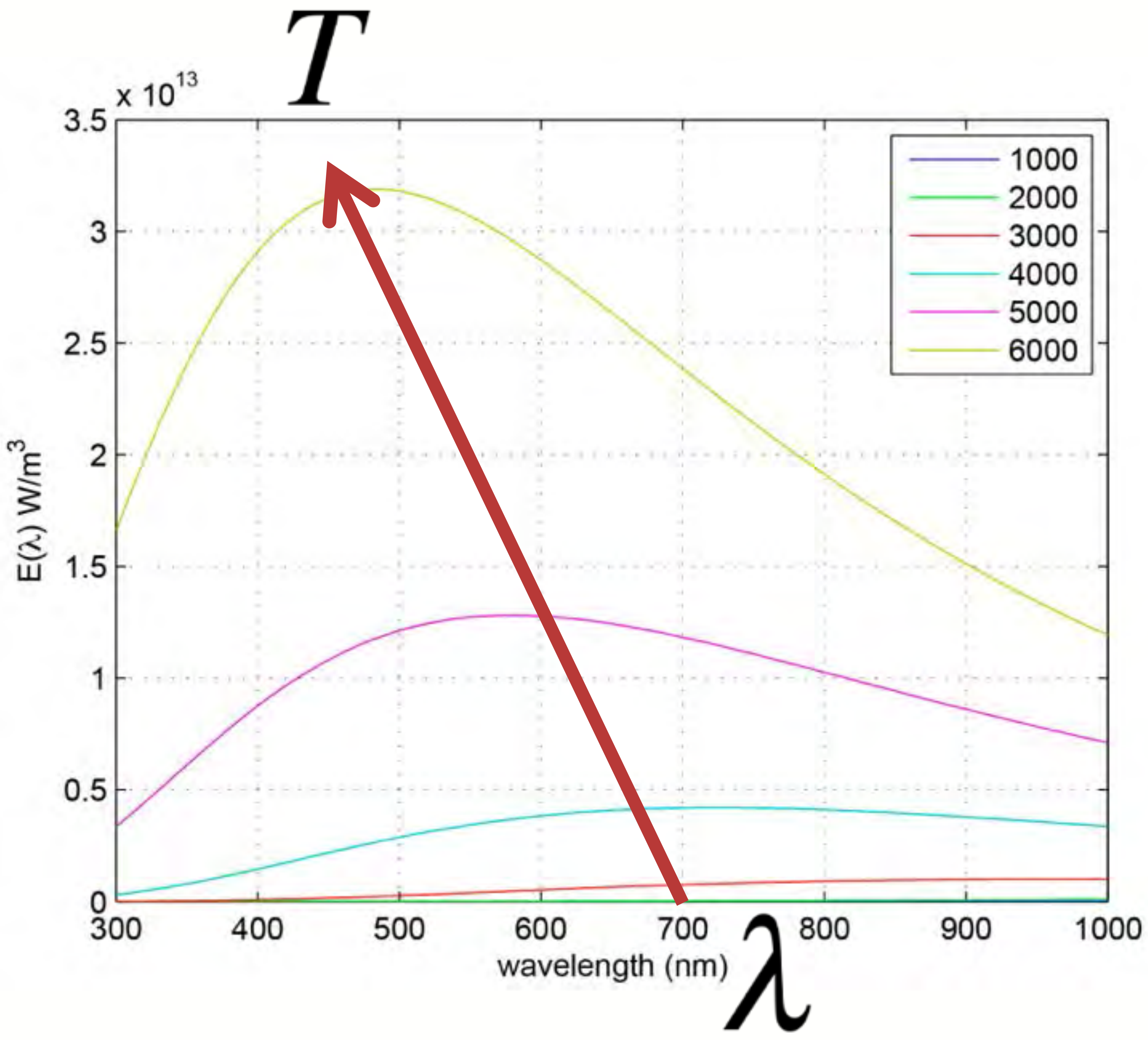
Planck's law

$$E(\lambda) = \frac{2hc^2}{\lambda^5(e^{hc/k\lambda T} - 1)} \mathrm{W/m^2/m}$$

Wien displacement law

$$\lambda_{max} = \frac{2.8978 \times 10^{-3}}{T} \mathrm{m}$$

Color is related to temperature



red hot
~1000K

yellow hot
~1400K

Folklife Festival Welsh Blacksmith 2009
Mr. T. in DC | CC A2.0.

white hot
~1800K

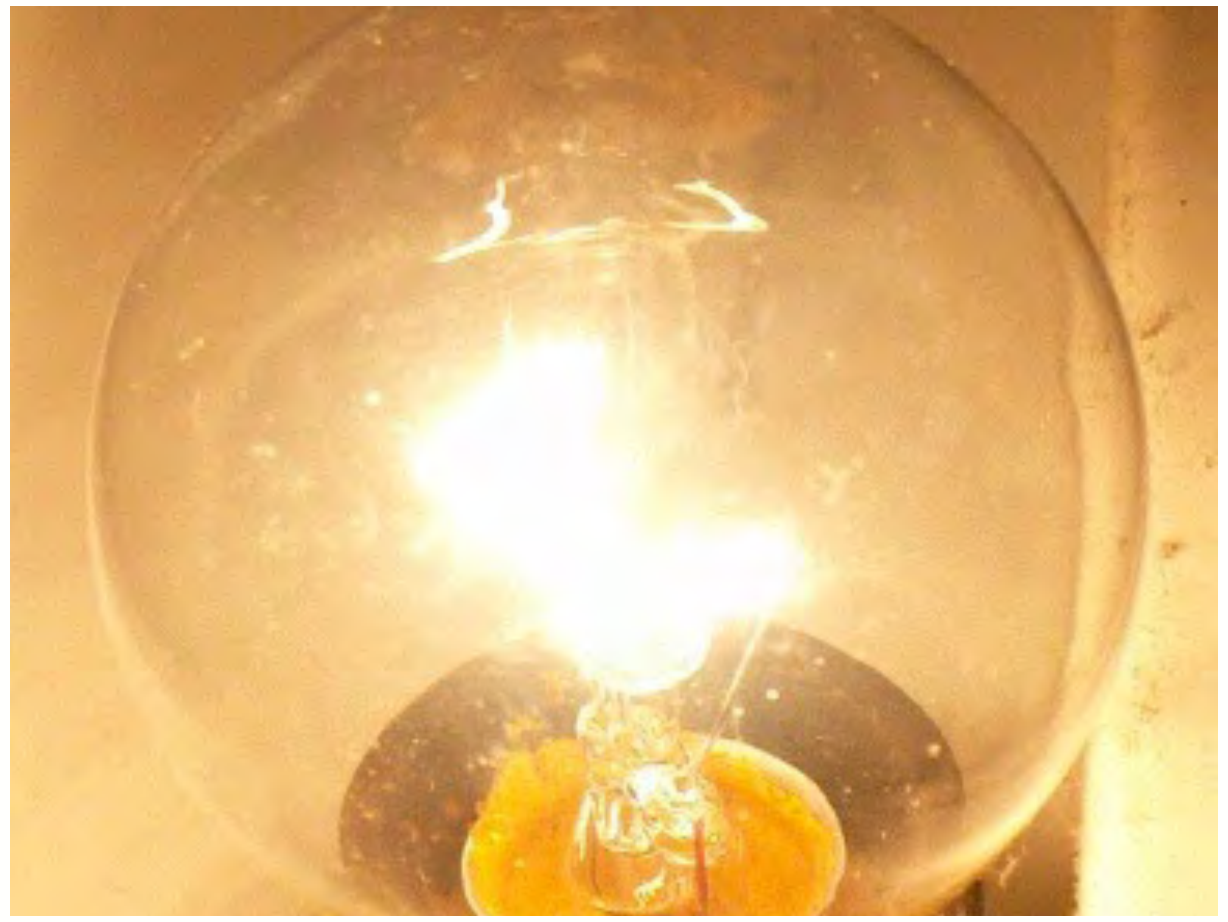
Blacksmith at work 2010
Derek Key | CC A2.0.

Color **temperature**

T=2000–
3000 K

Gluehbirne 2006
Dickbauch

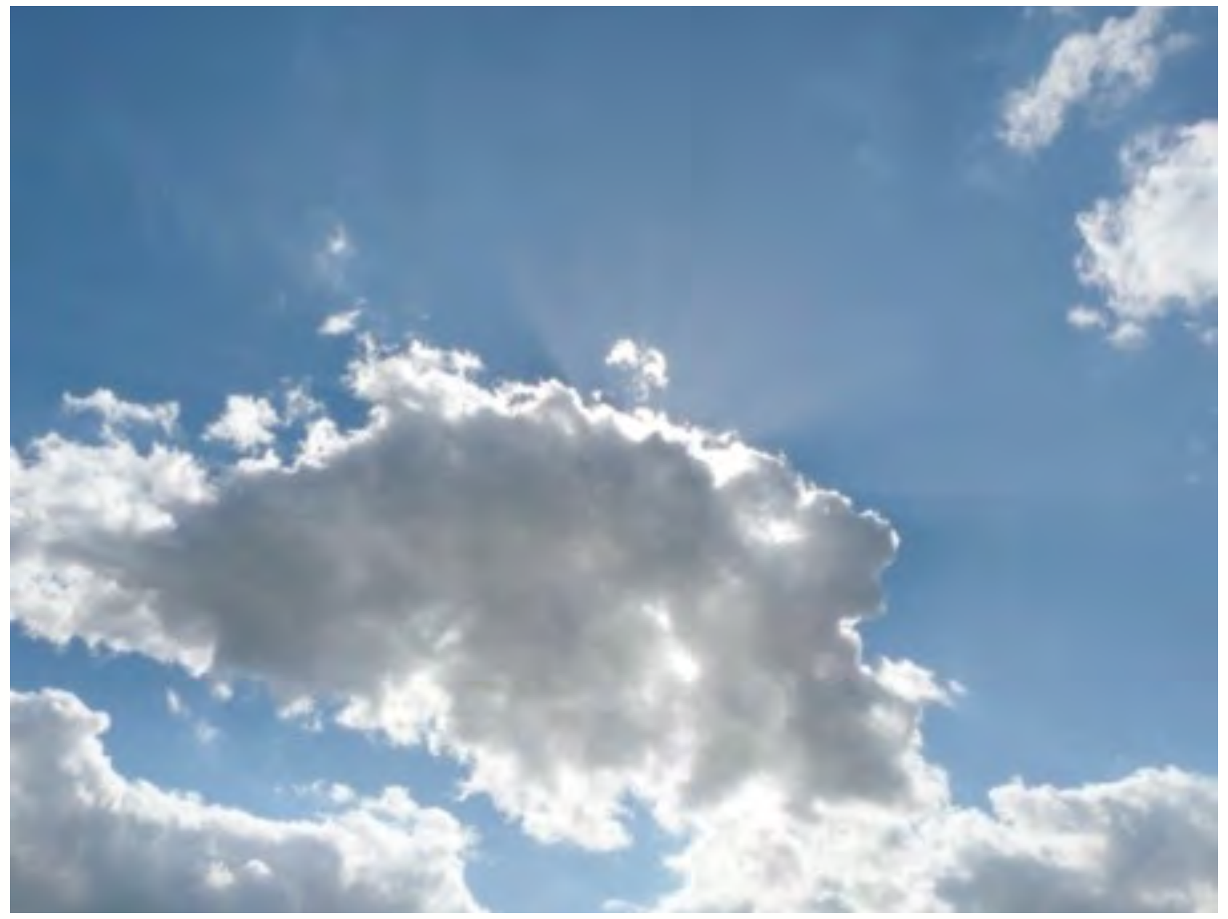
T=3000 K

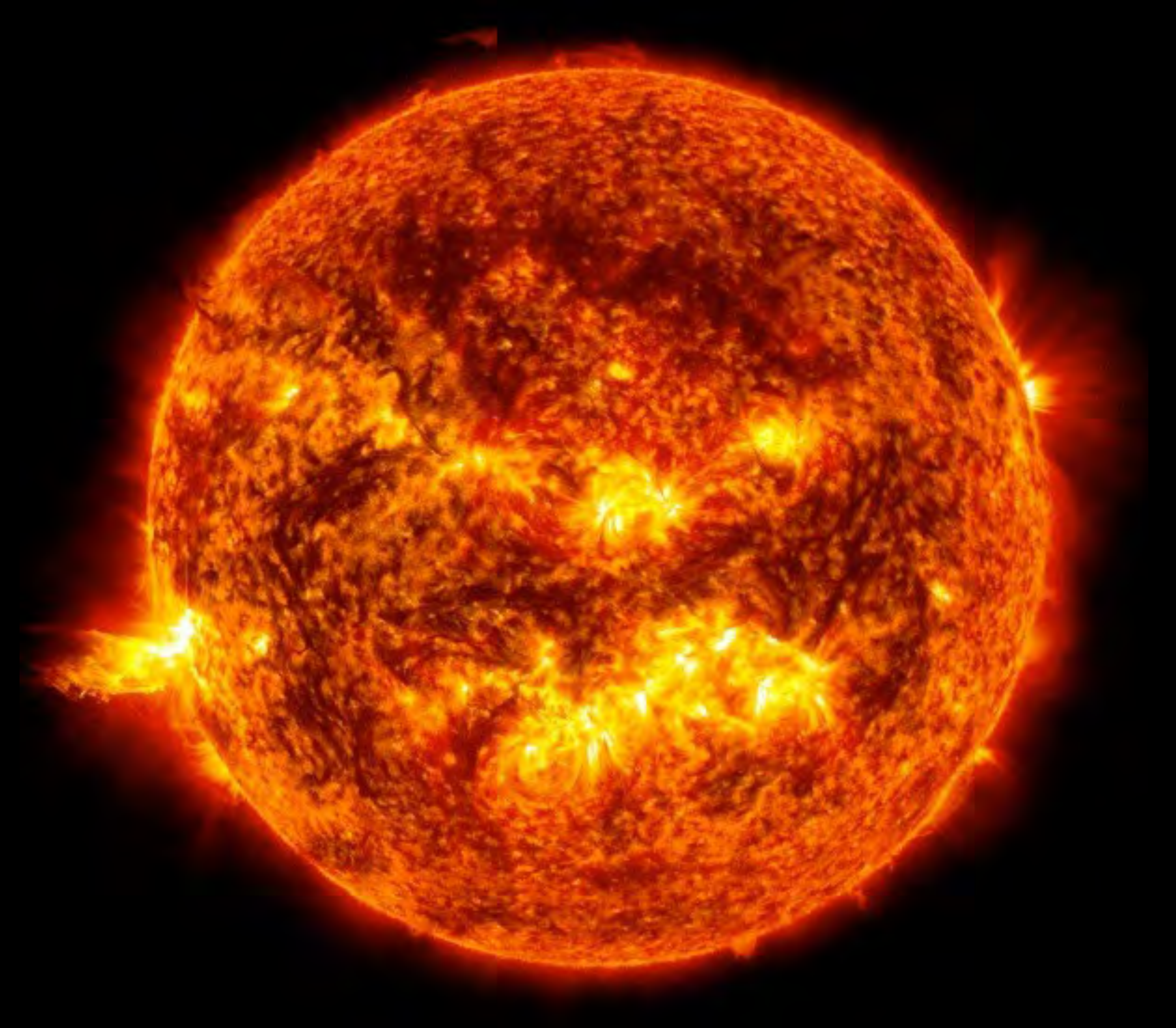


Sun Halo 2013
Janice Marie Foote | CC A2.0.

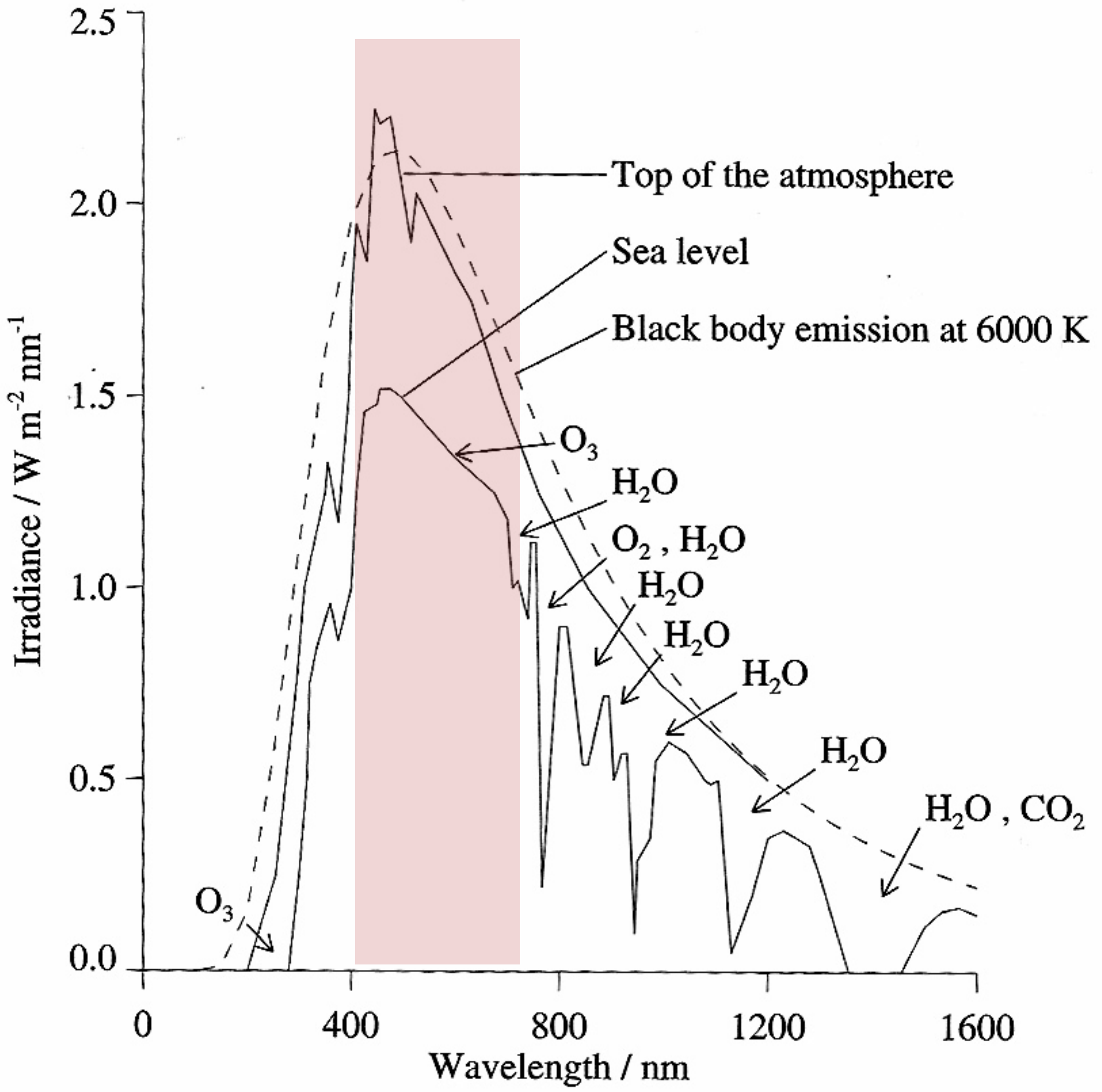
T=5000–
5400 K

T=8000–
10000 K

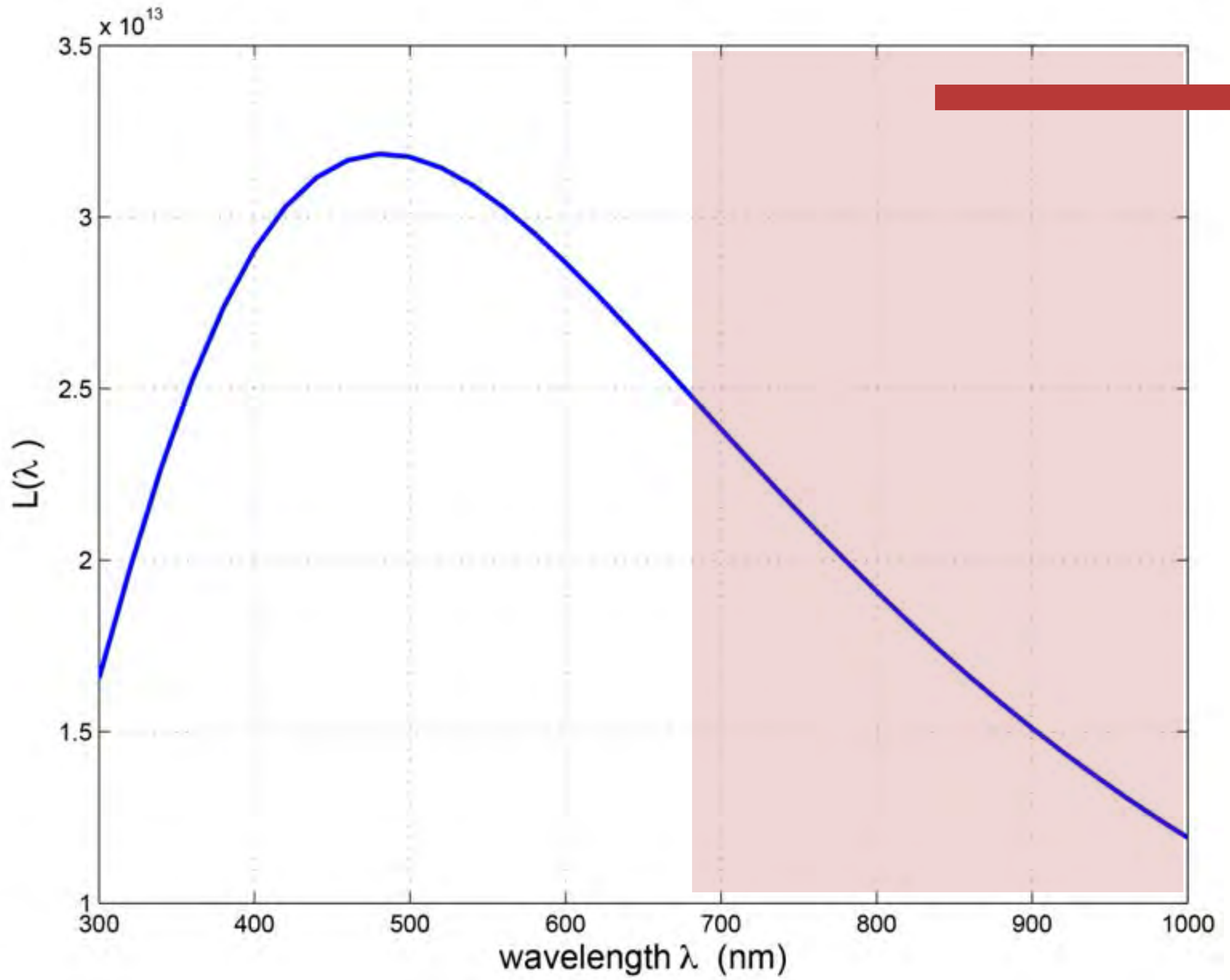




NASA Goddard Photo and Video



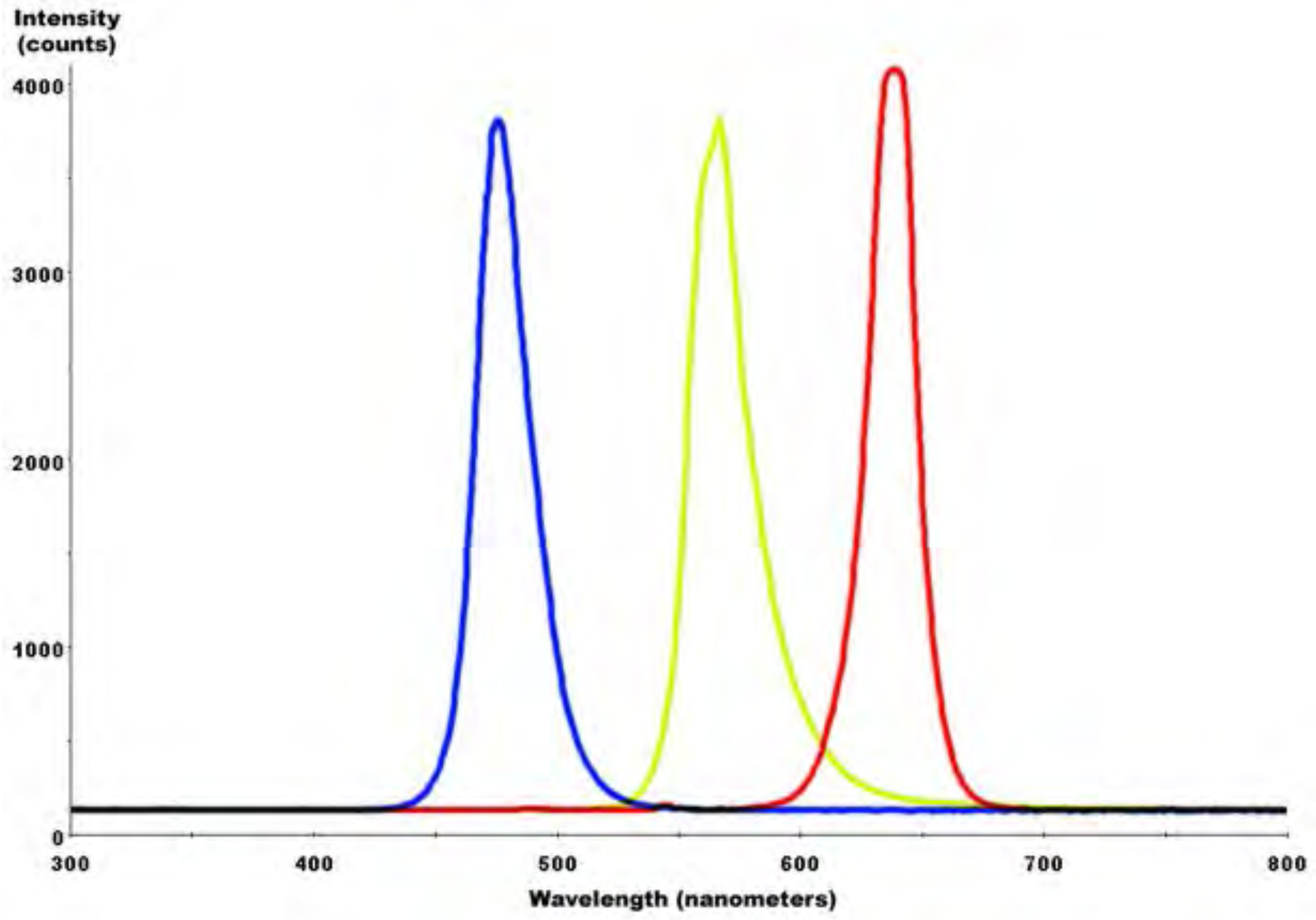
Thermal radiation



lots of energy at longer wavelengths

$$\lambda_{\text{max}} = \frac{2.8978 \times 10^{-3}}{T}\ \text{m}$$

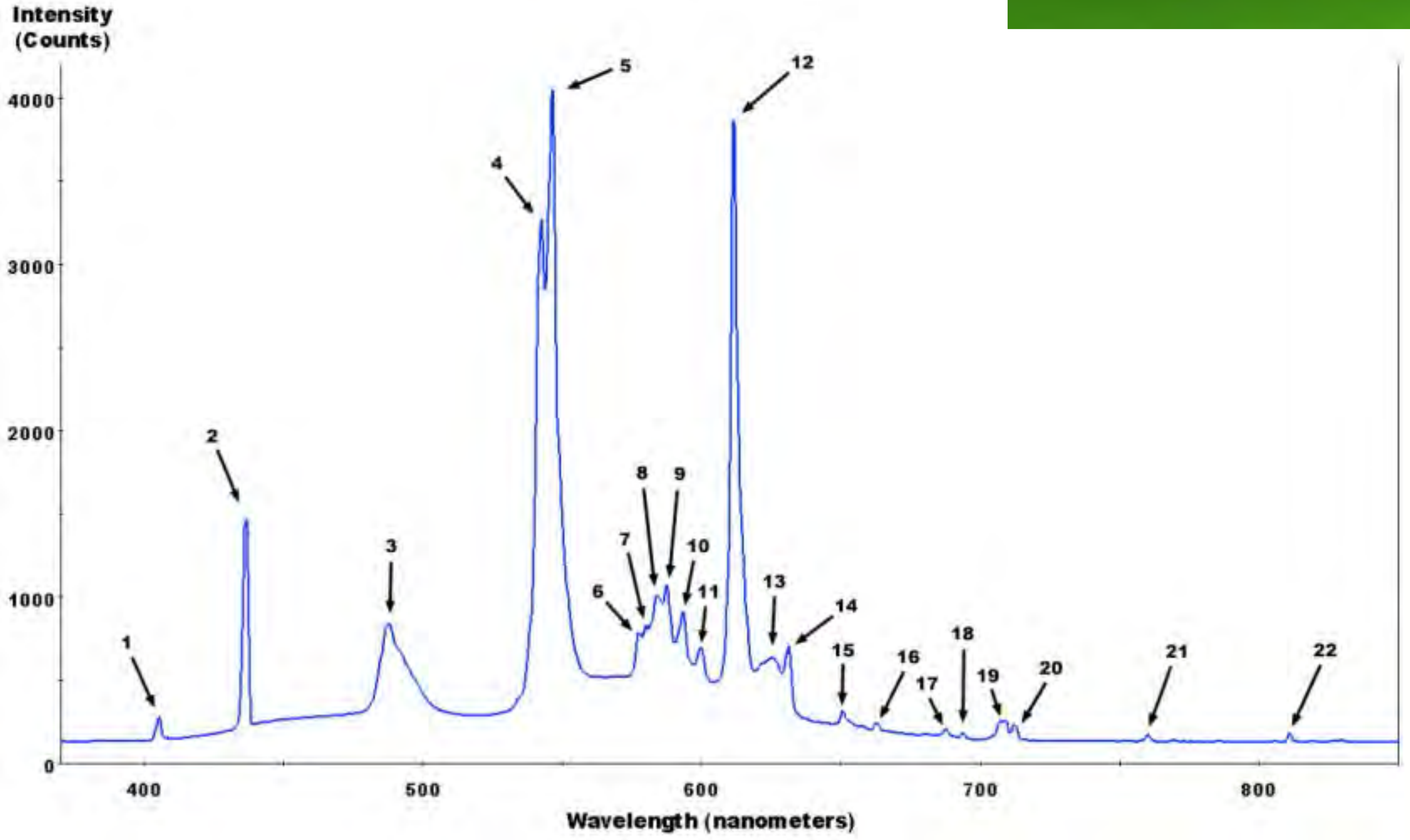
Non-blackbody illuminants



Deglr6328 | CC A3.0.

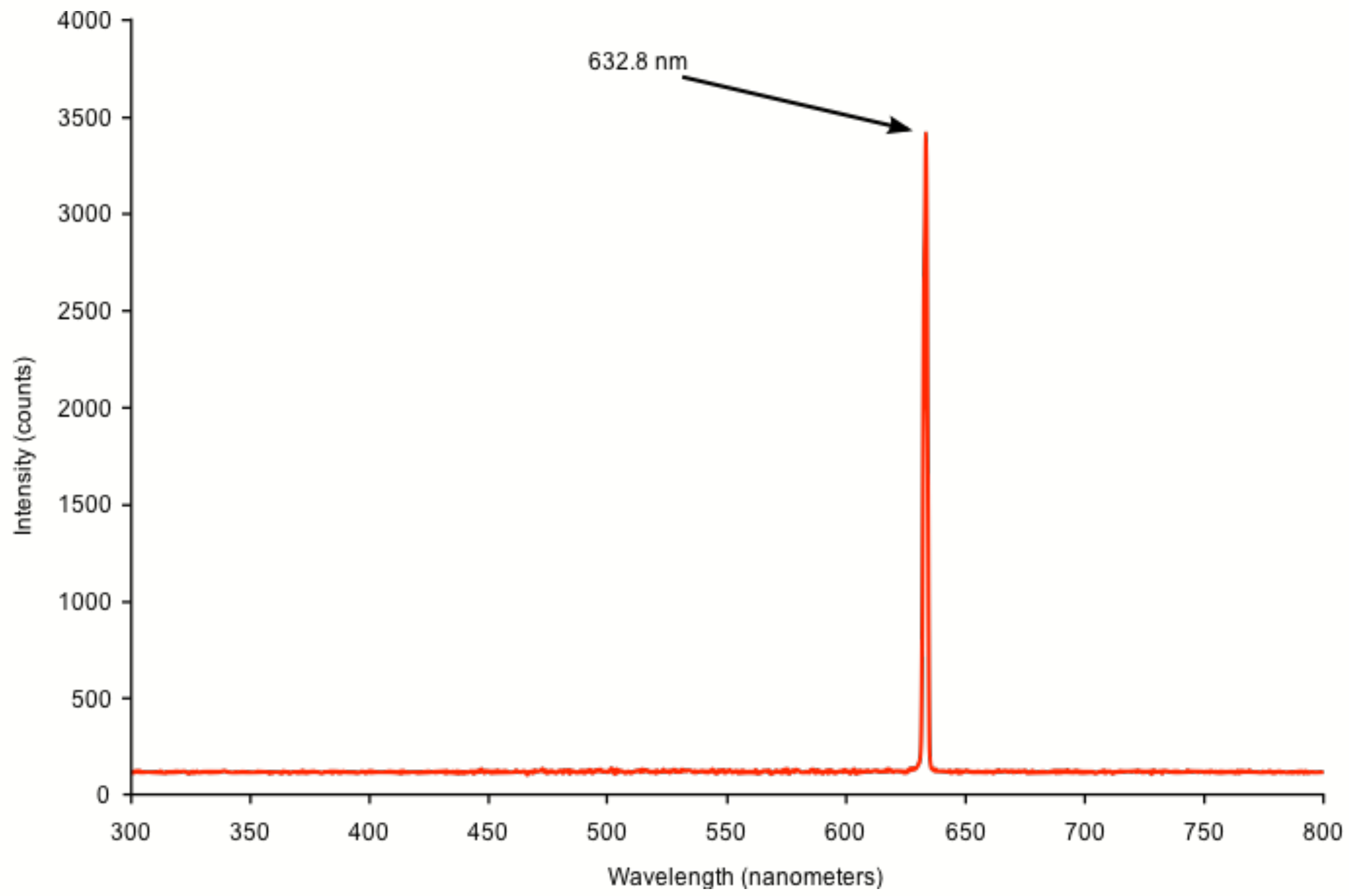
RGB LED 2005

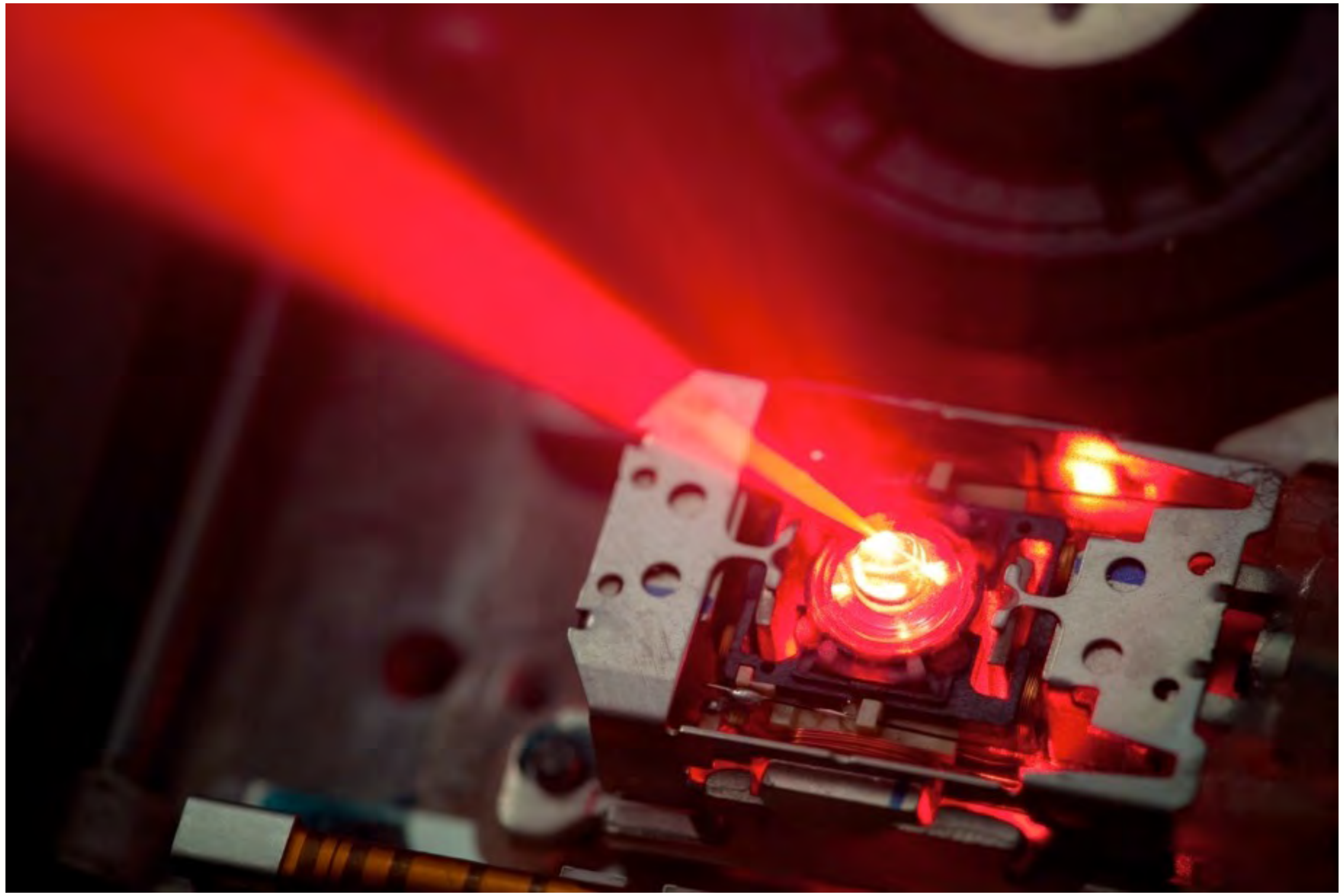
PiccoloNamek | CC A3.0.



Deglr6328 | CC A3.0.

Non-blackbody illuminants

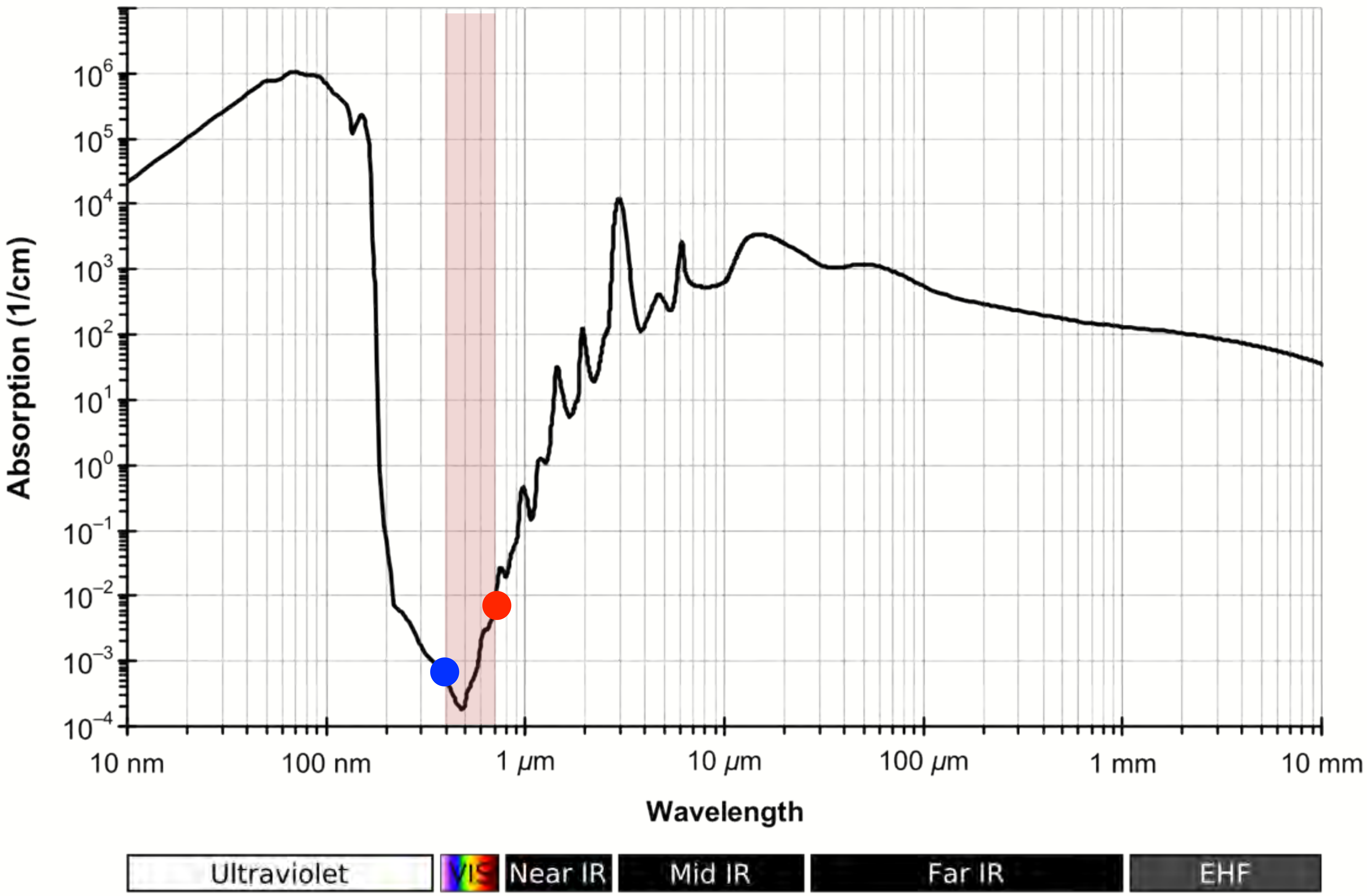




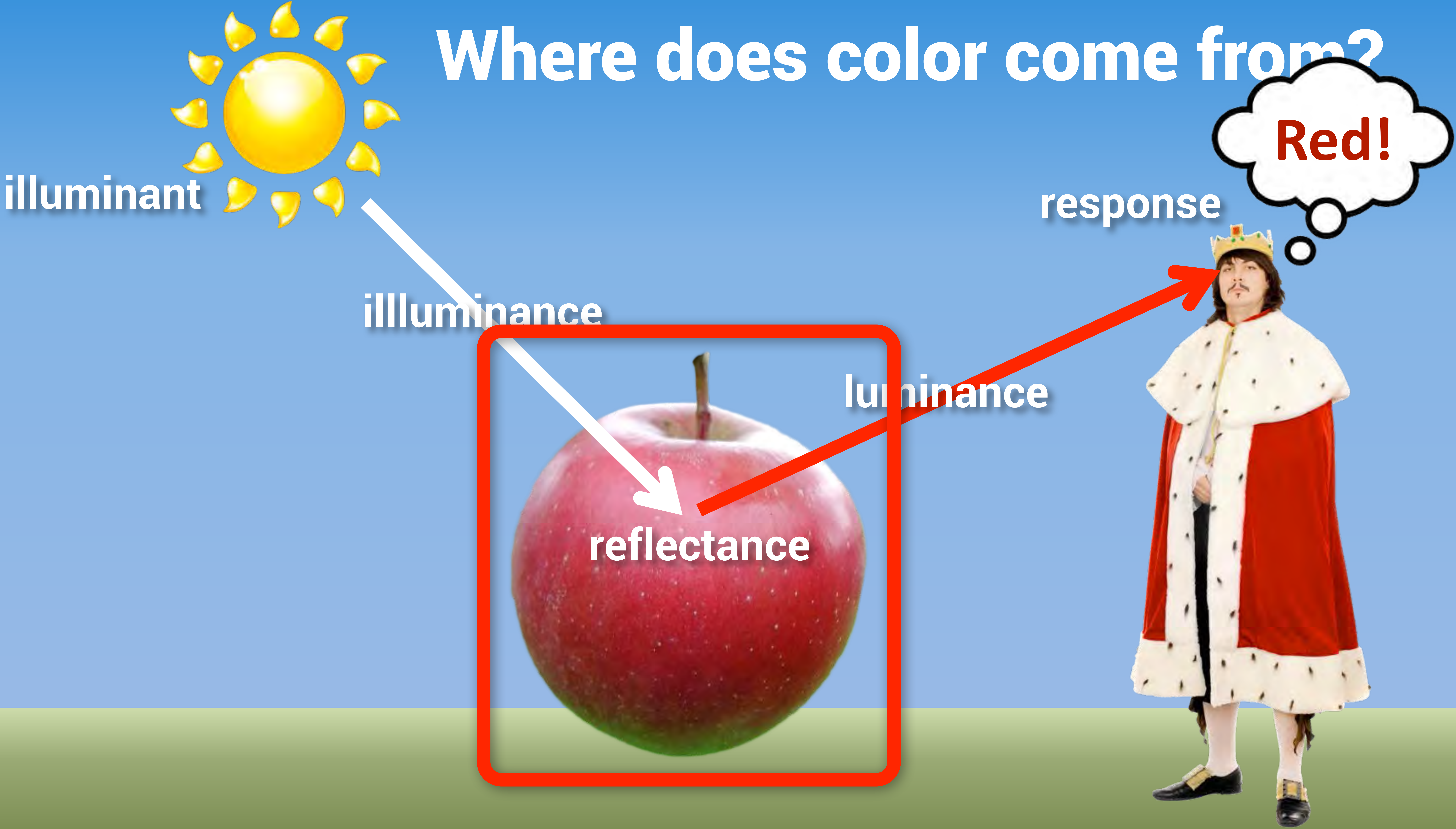
By derivative work: Papa November (talk) 2008

Deglr6328 | CC A3.0.

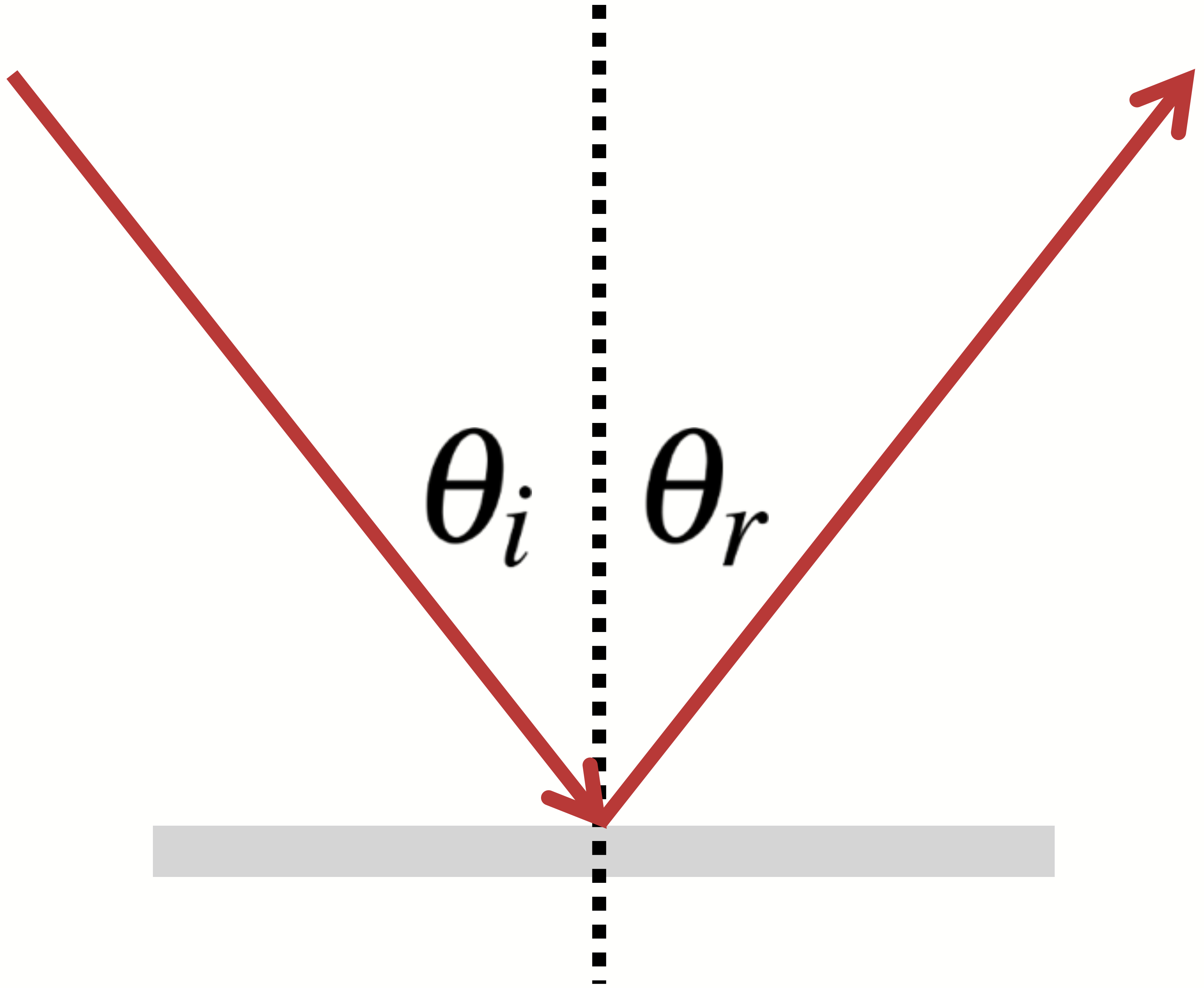
Color change **underwater**



Where does color come from?



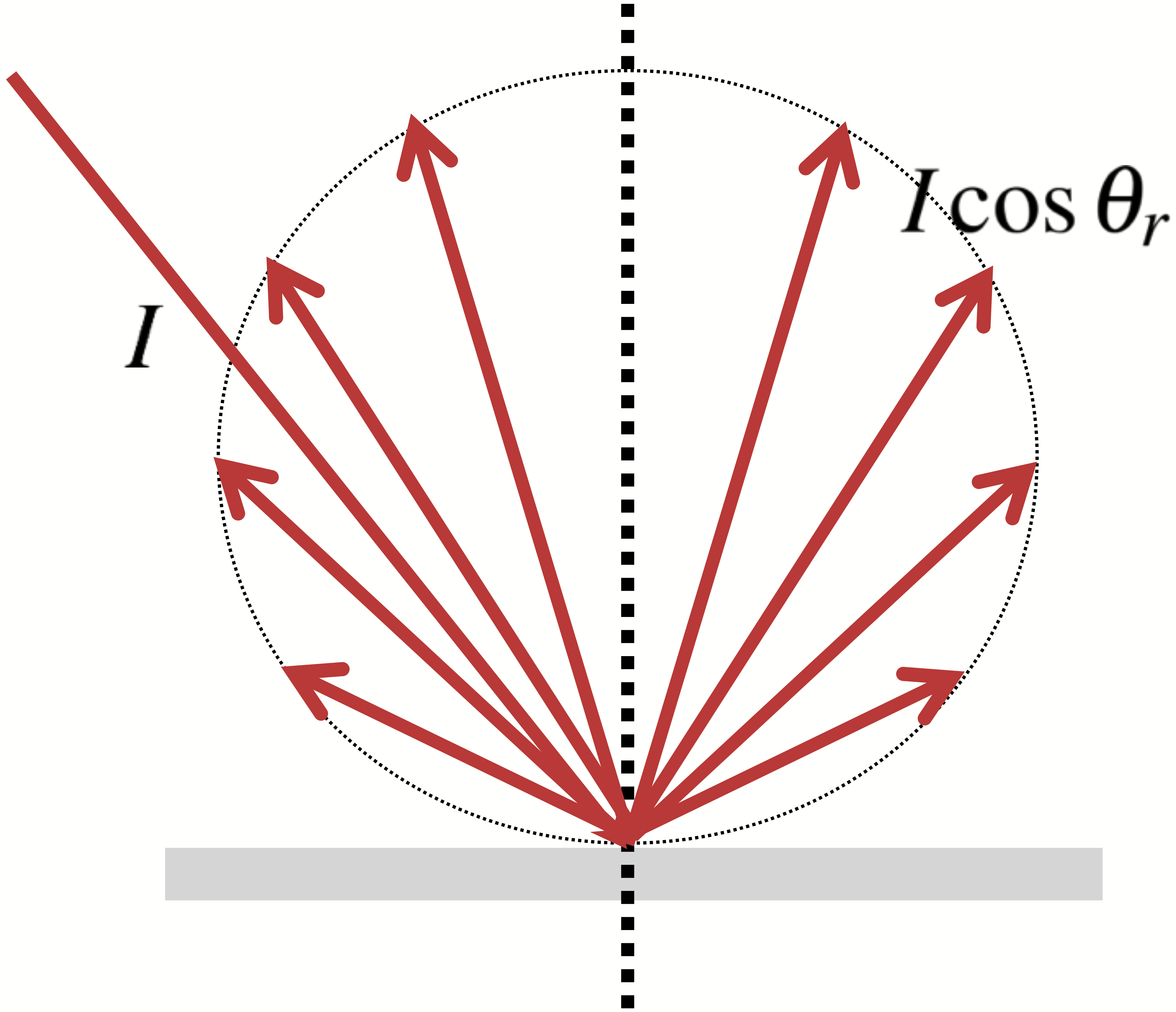
Reflection of light

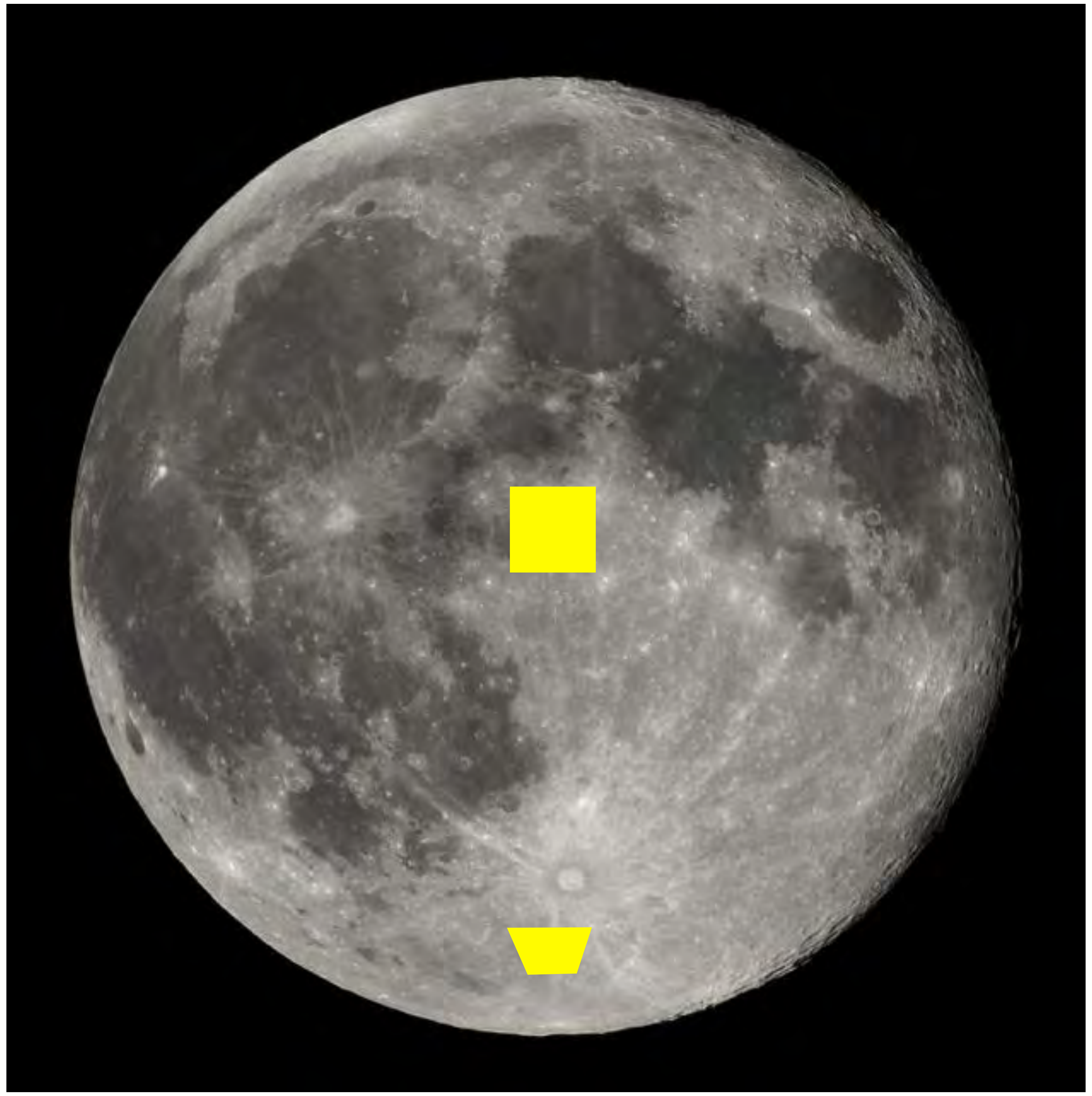


- Specular reflection
  ⤷ angle of incidence equals angle of reflection

STScI & NASA

Reflection of light

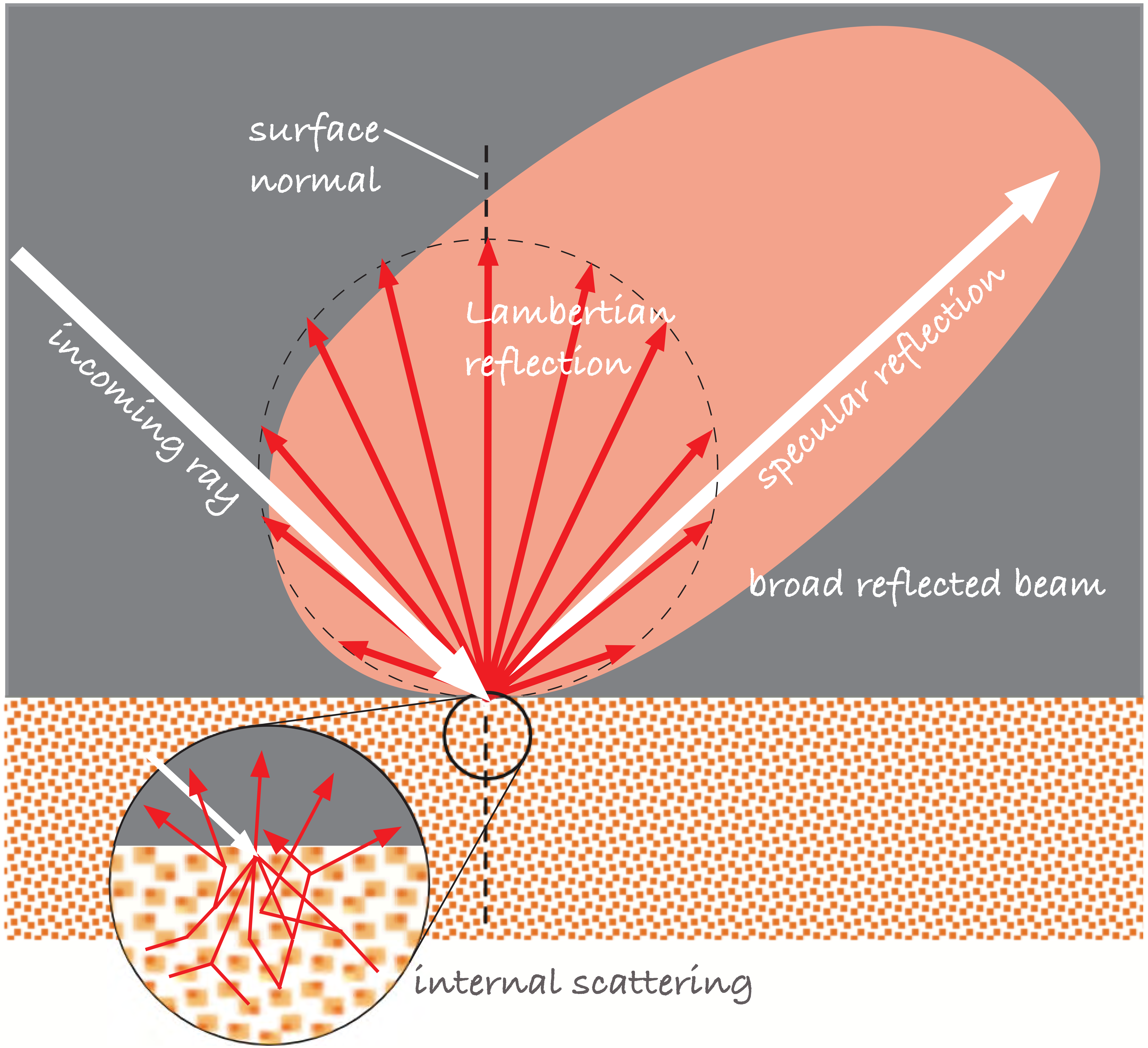




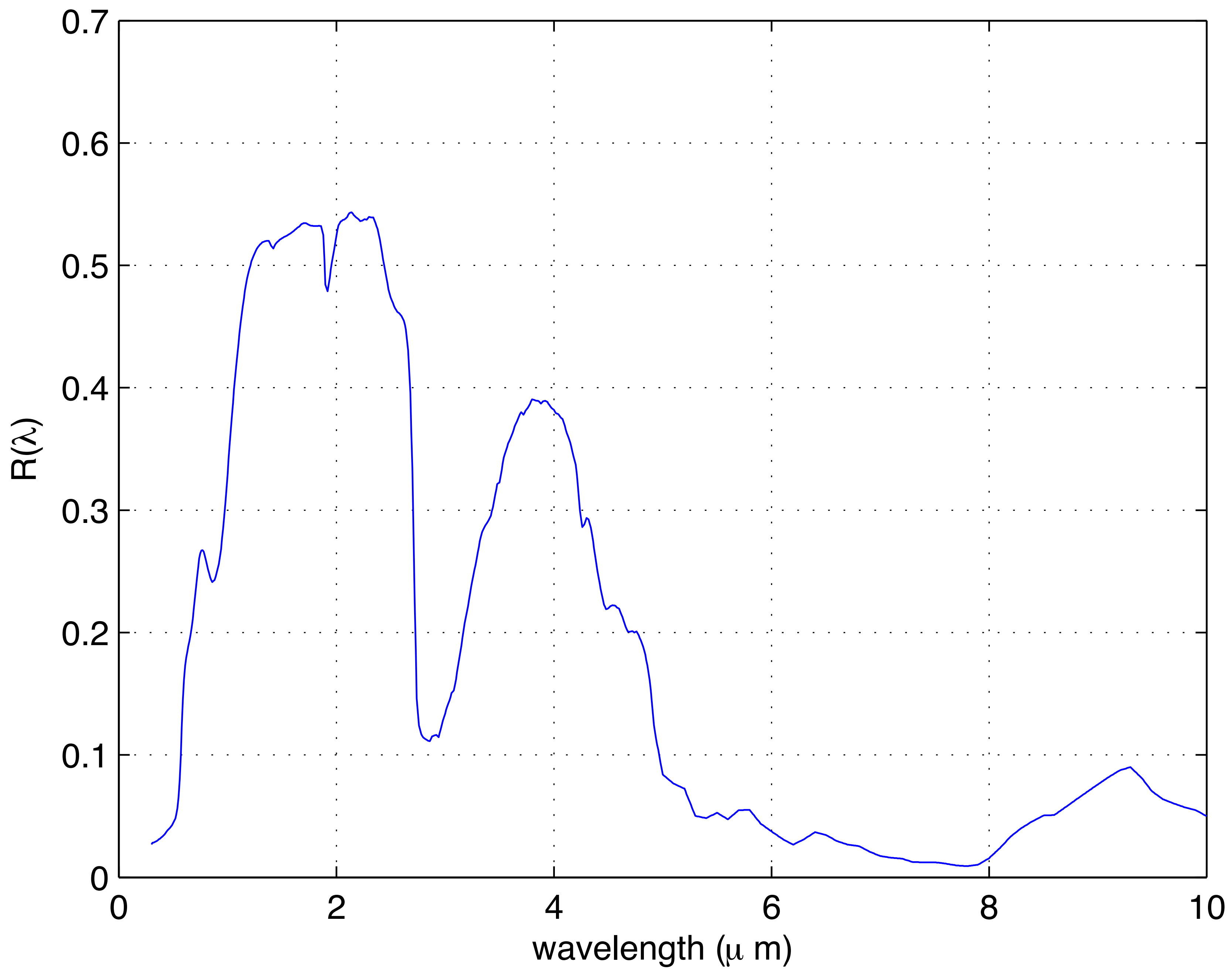
Johann Heinrich Lambert
1728–1777

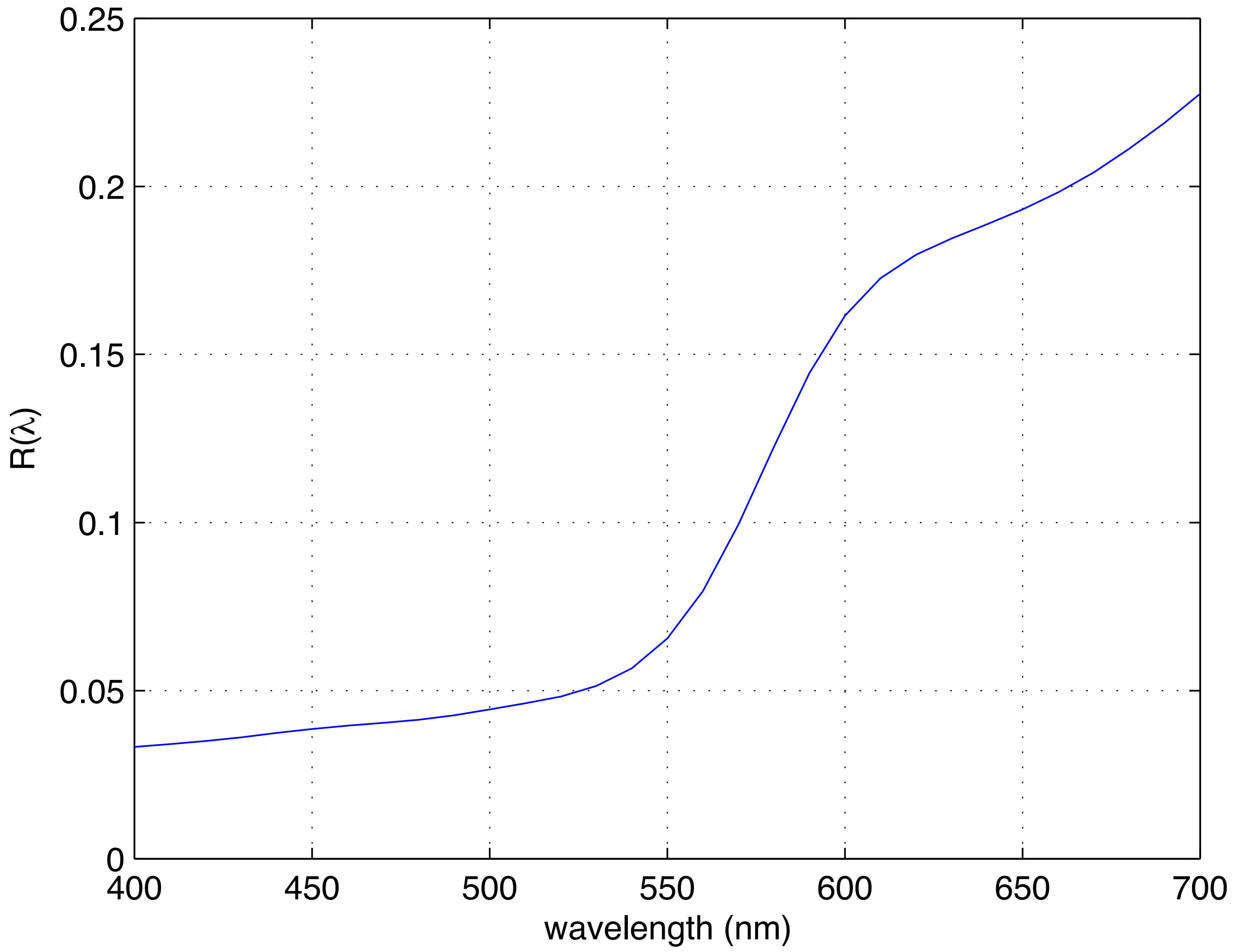
- Lambertian reflection
  - ↳ diffuse/matte surface
  - ↳ brightness **invariant** to observer's angle of view

Dichromatic reflectance model



Reflectance depends on wavelength





With kind permission of **Springer Science+Business Media**.
Data from ASTER, Baldridge et al. 2009.

Reflectance depends on wavelength

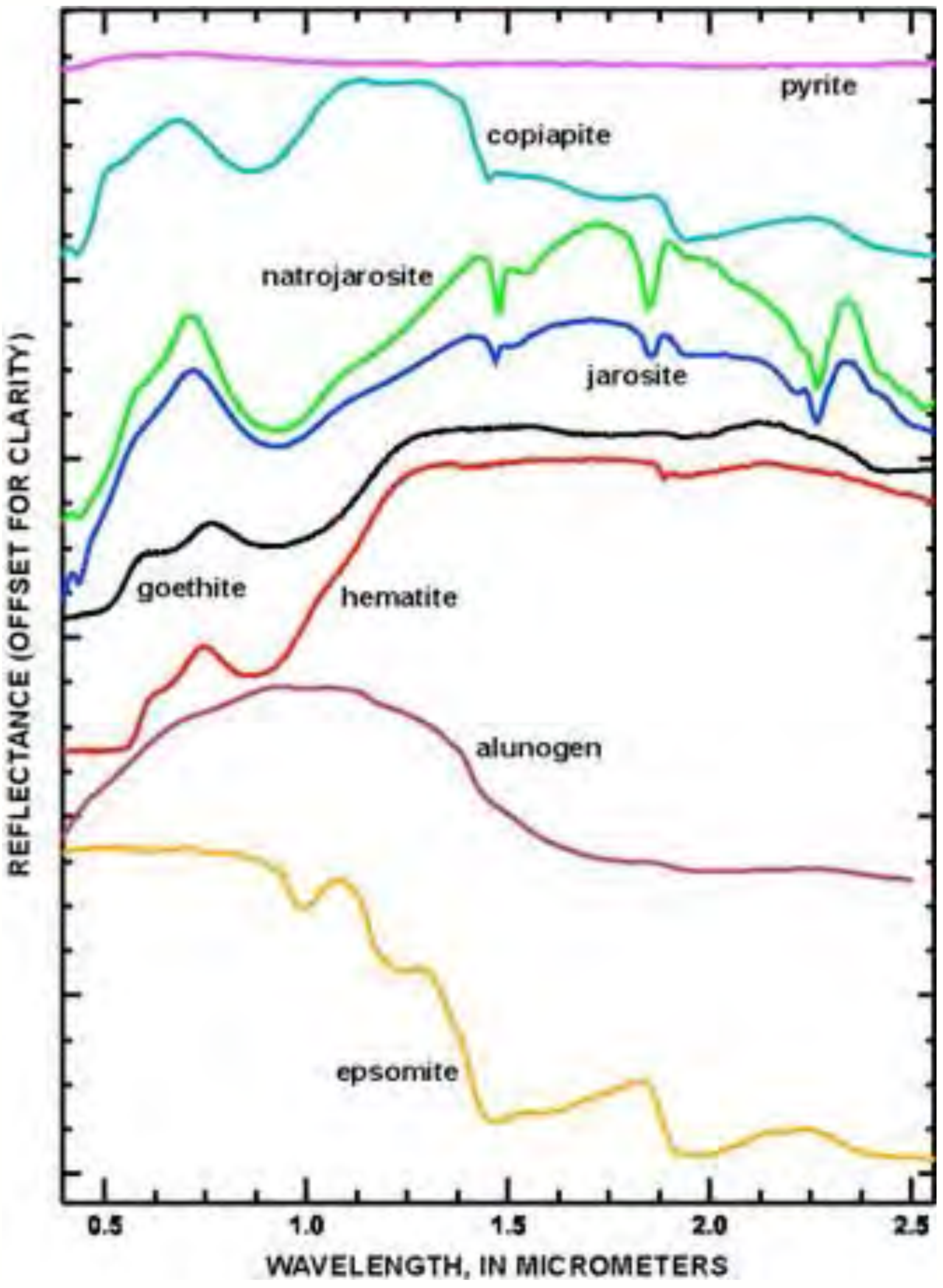
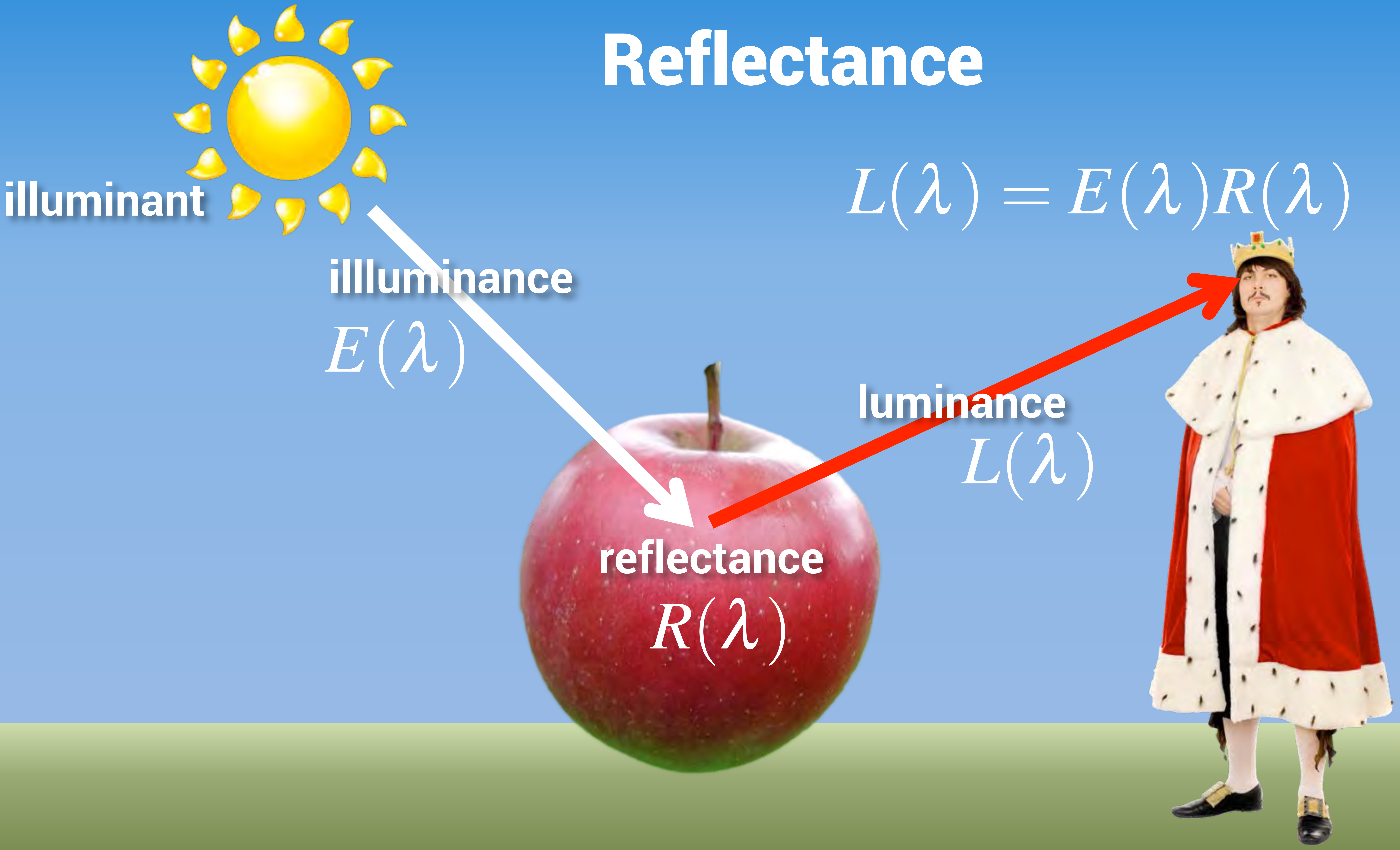


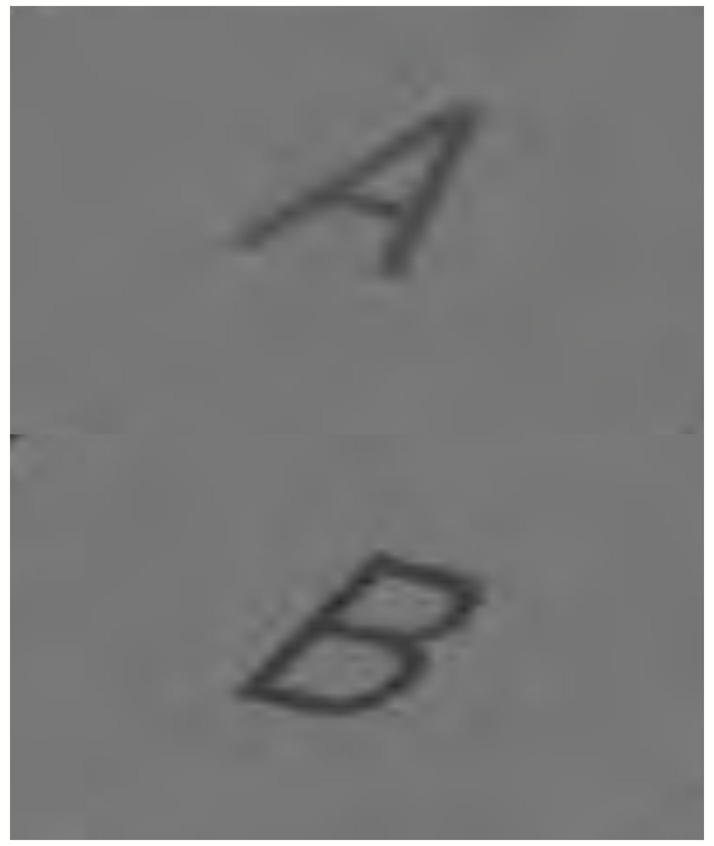
Figure: Rockwell, B. W, McDougal, R. R., Gent, C. A. & the United States Environmental Protection Agency

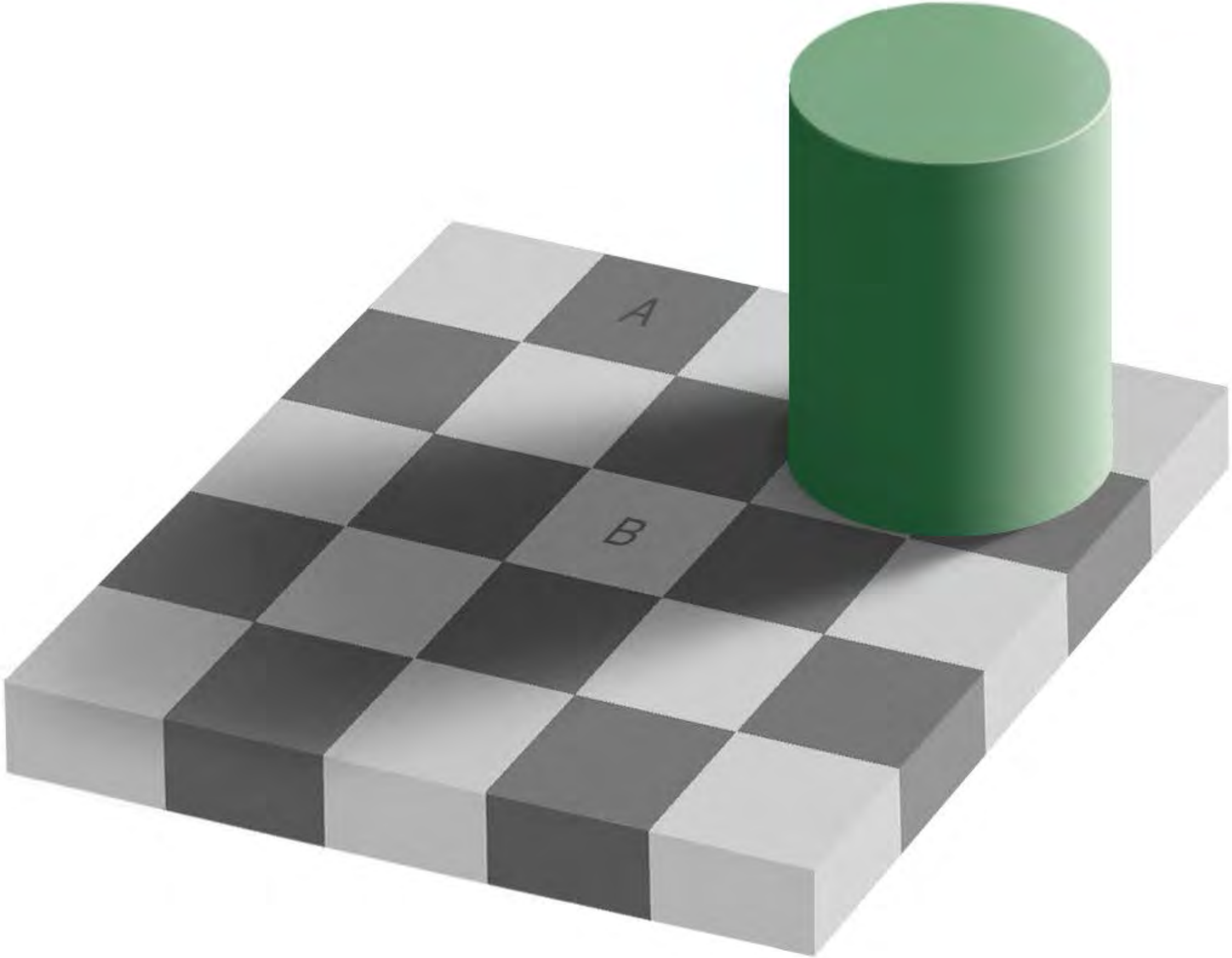
Reflectance

$$L(\lambda) = E(\lambda)R(\lambda)$$



Unpacking reflectance and illumination

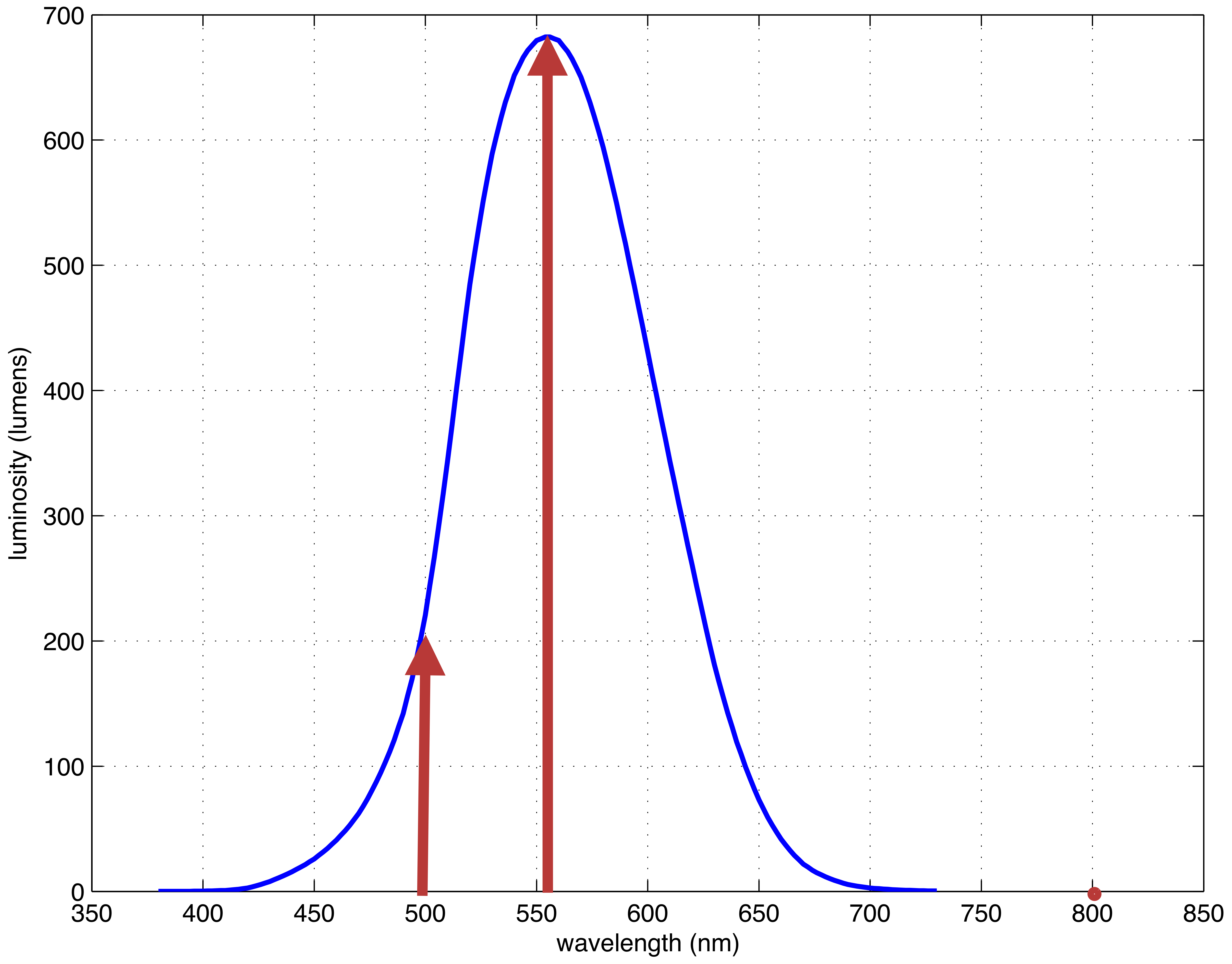


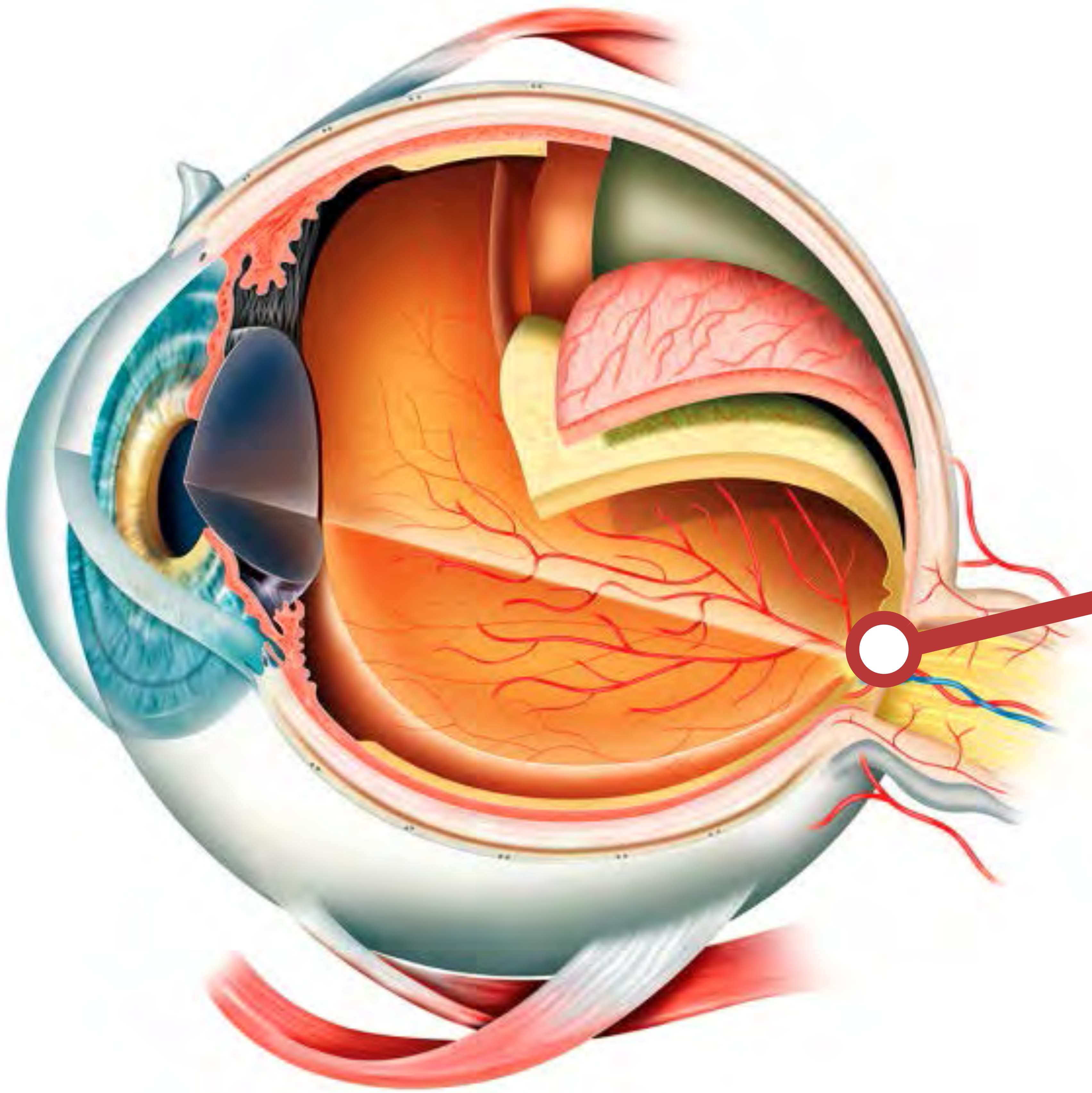


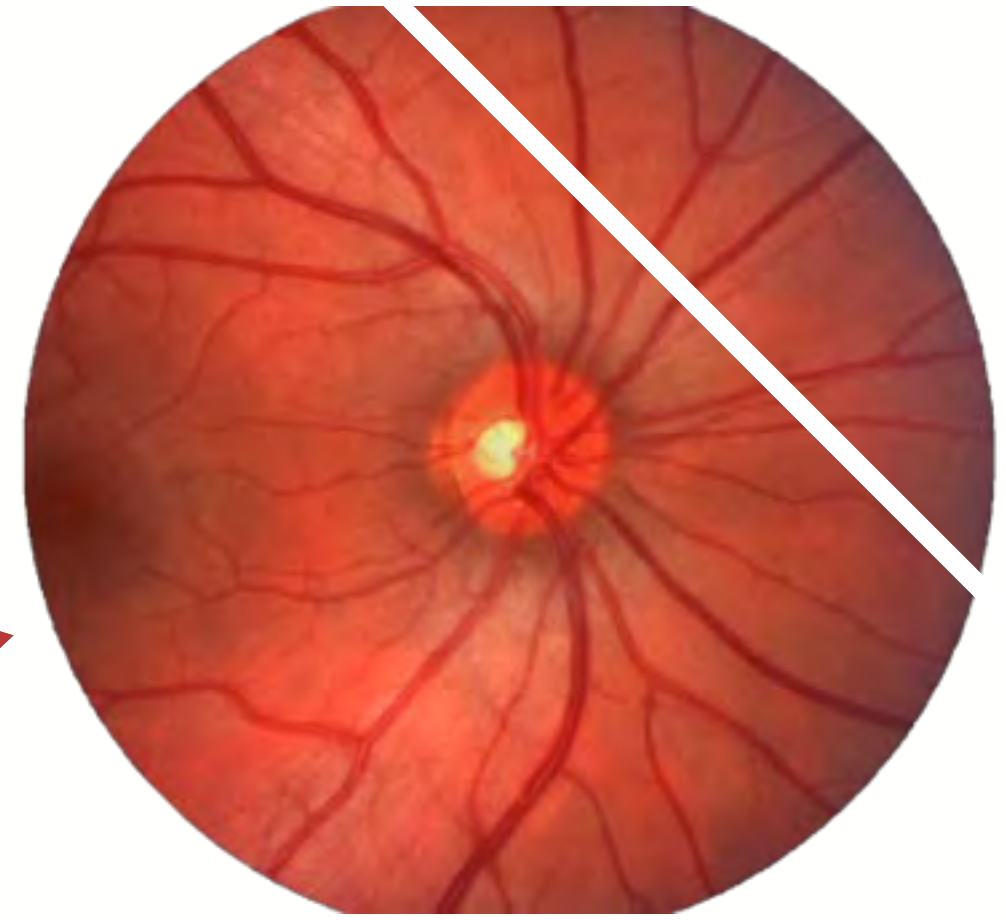
Where does color come from?

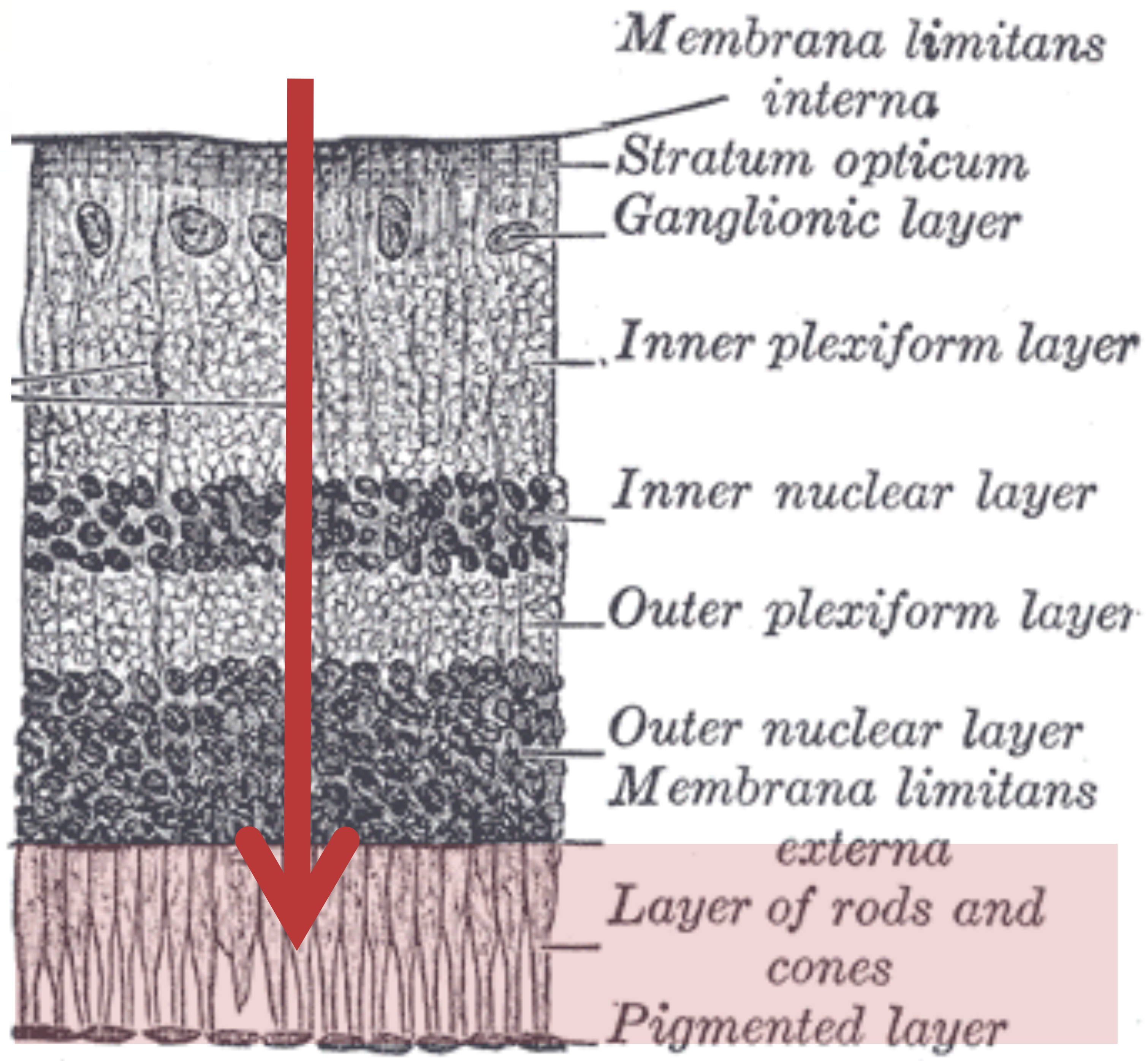
Luminosity function

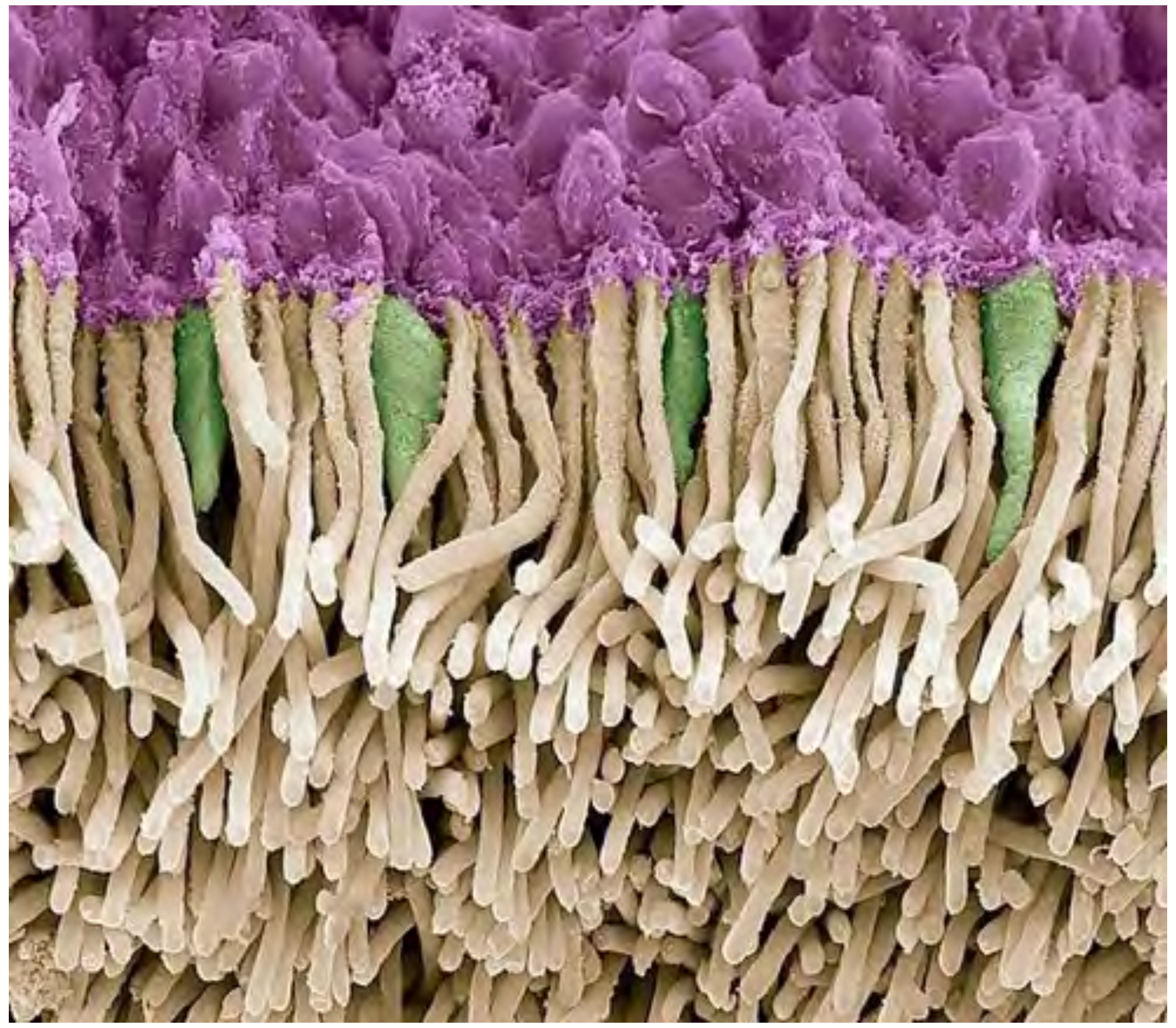
- At **555nm** (green)
  - ⤷ 1W → 683 lumens
- At **500nm** (blue)
  - ⤷ 1W → 220 lumens
- At **800nm** (infrared)
  - ⤷ 1W → 0 lumens



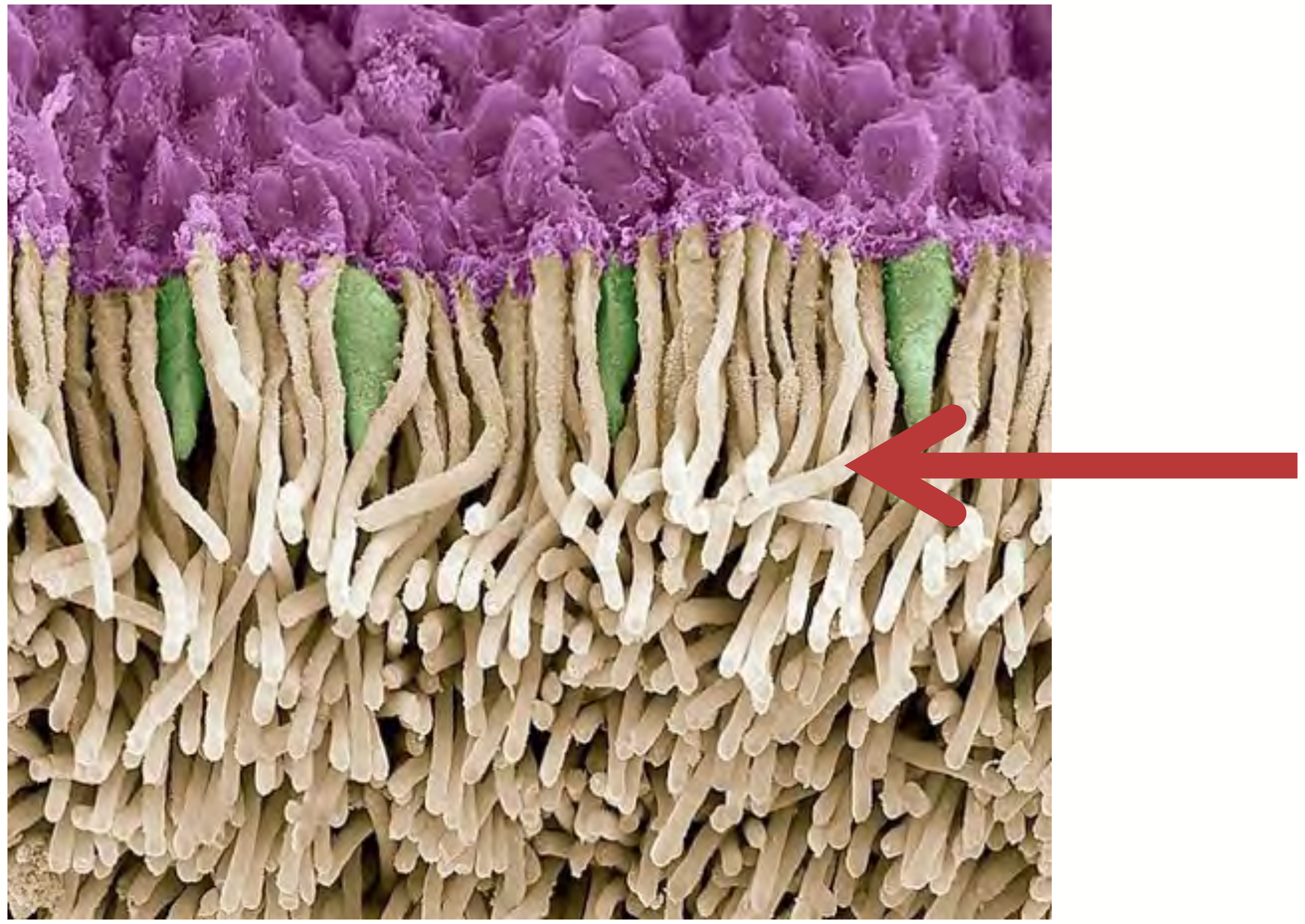




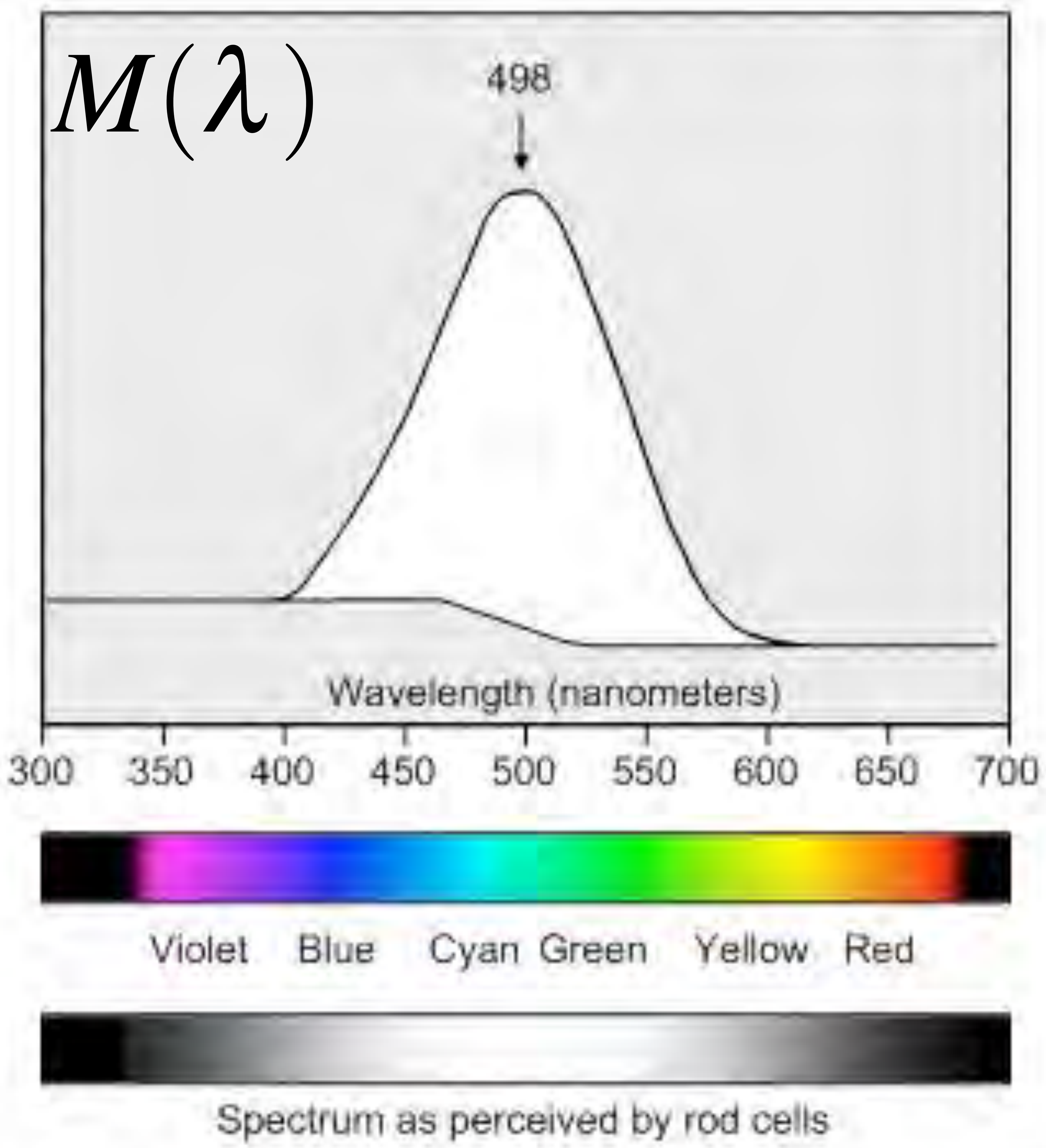

Membrana limitans interna
Stratum opticum
Ganglionic layer
Inner plexiform layer
Inner nuclear layer
Outer plexiform layer
Outer nuclear layer
Membrana limitans externa
Layer of rods and cones
Pigmented layer



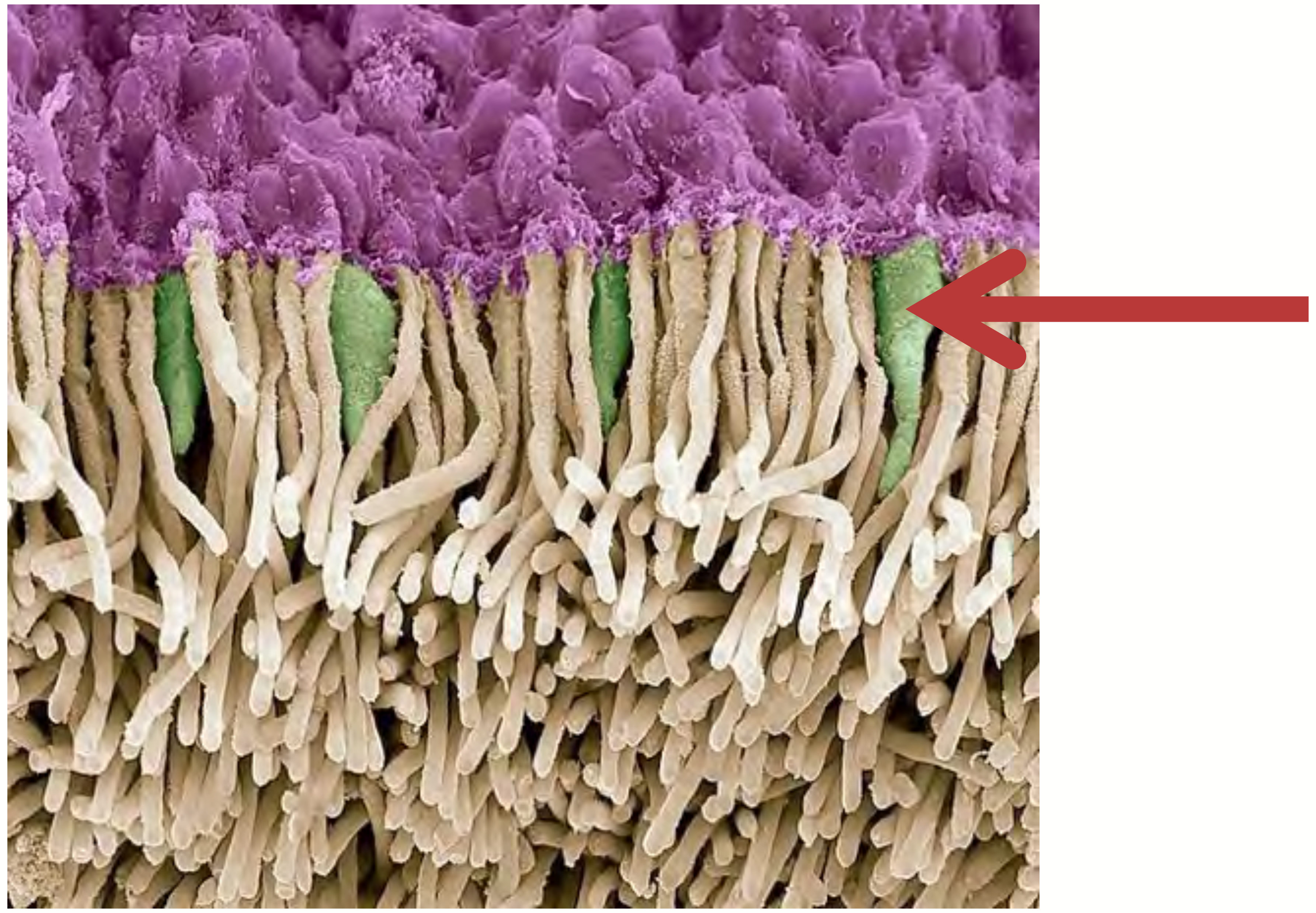
Rod cell response



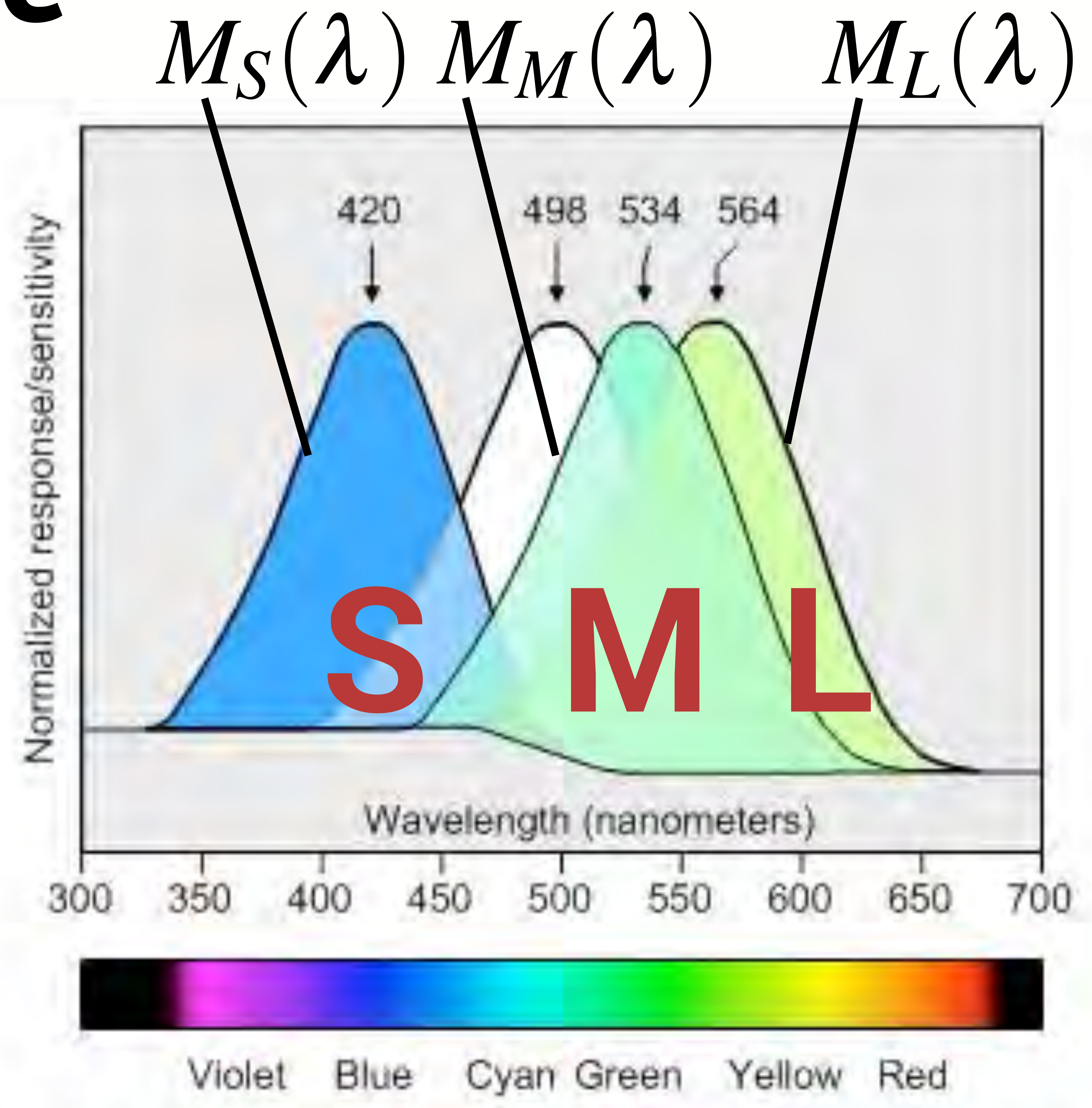
- contain rhodopsin
- low-light vision
- motion sensitive



Cone human response



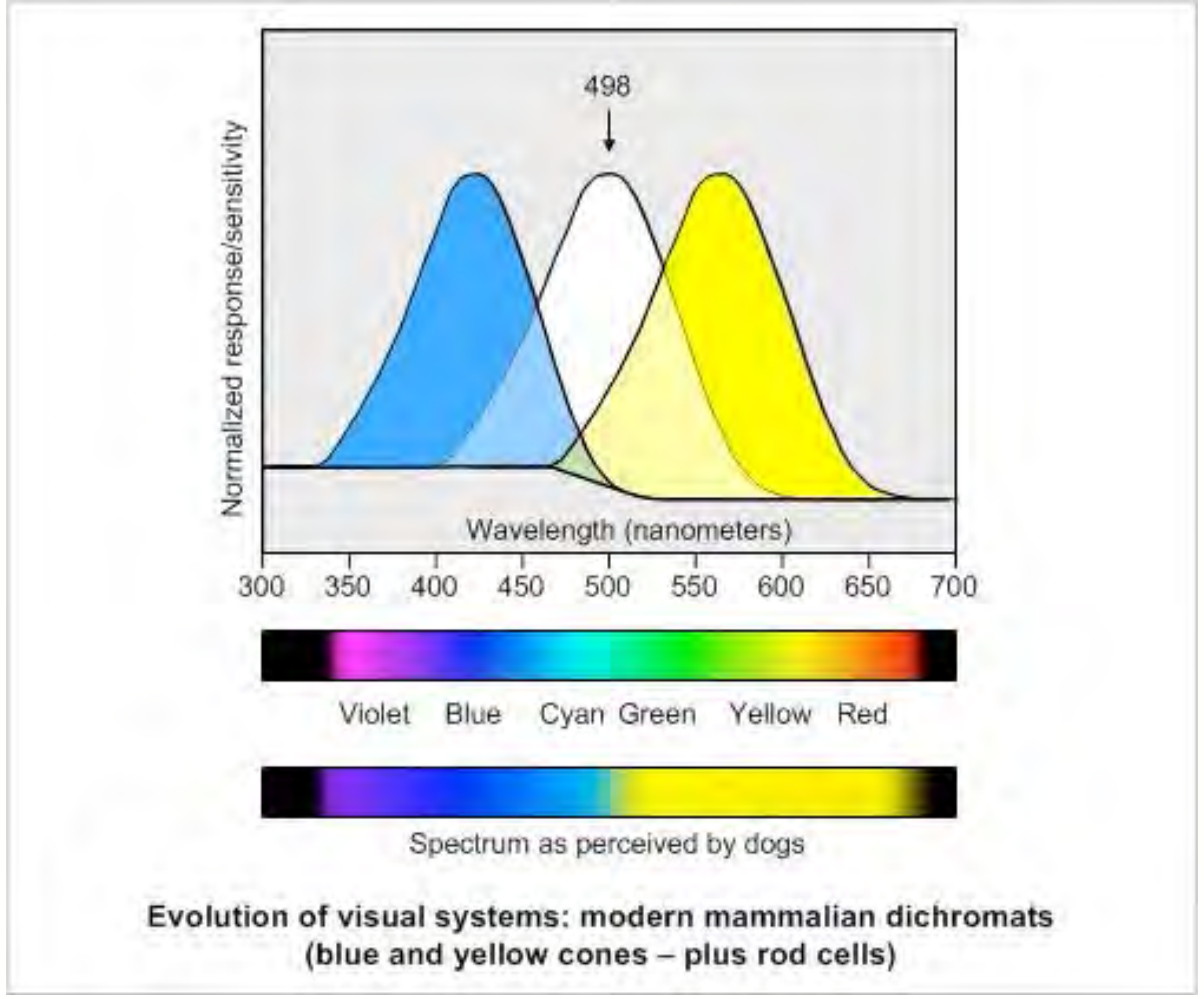
- contain photopsins
- bright-light vision
- three types of cones (**we are trichromats**)
- sensitive to different bands of spectrum

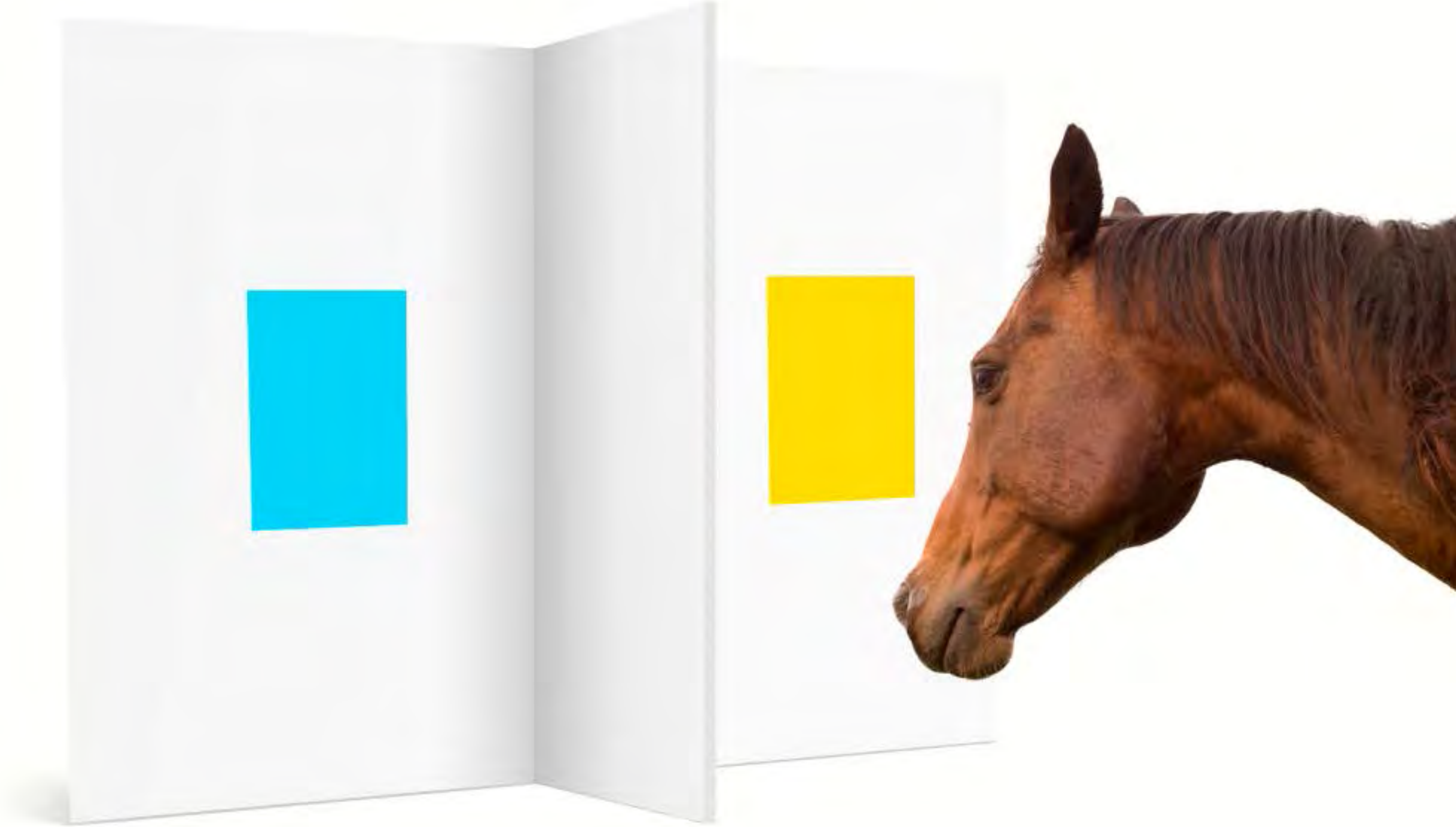


Typical humans (three color cone/pigment types plus rod cells). 2013

Clive "Max" Maxfield | Used with permission.

Dichromats

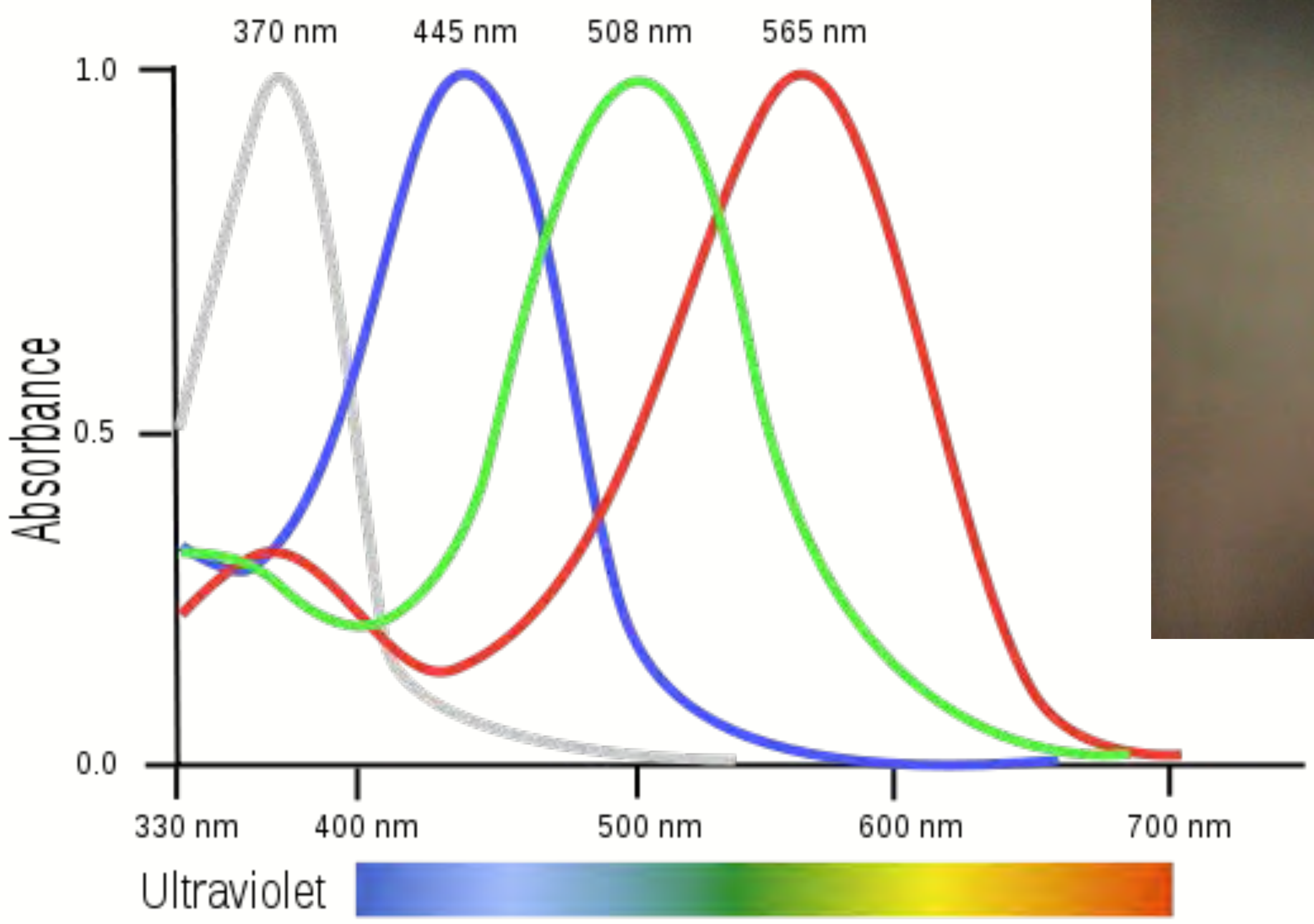




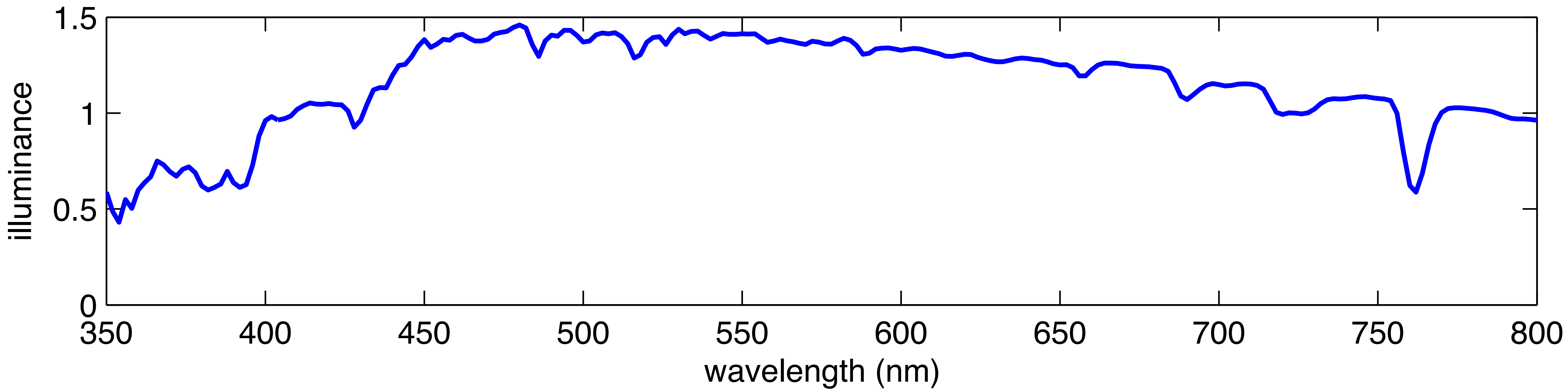
Evolution of visual systems: modern mammalian dichromats (blue and yellow cones- plus rod cells). 2013

Clive "Max" Maxfield | Used with permission.

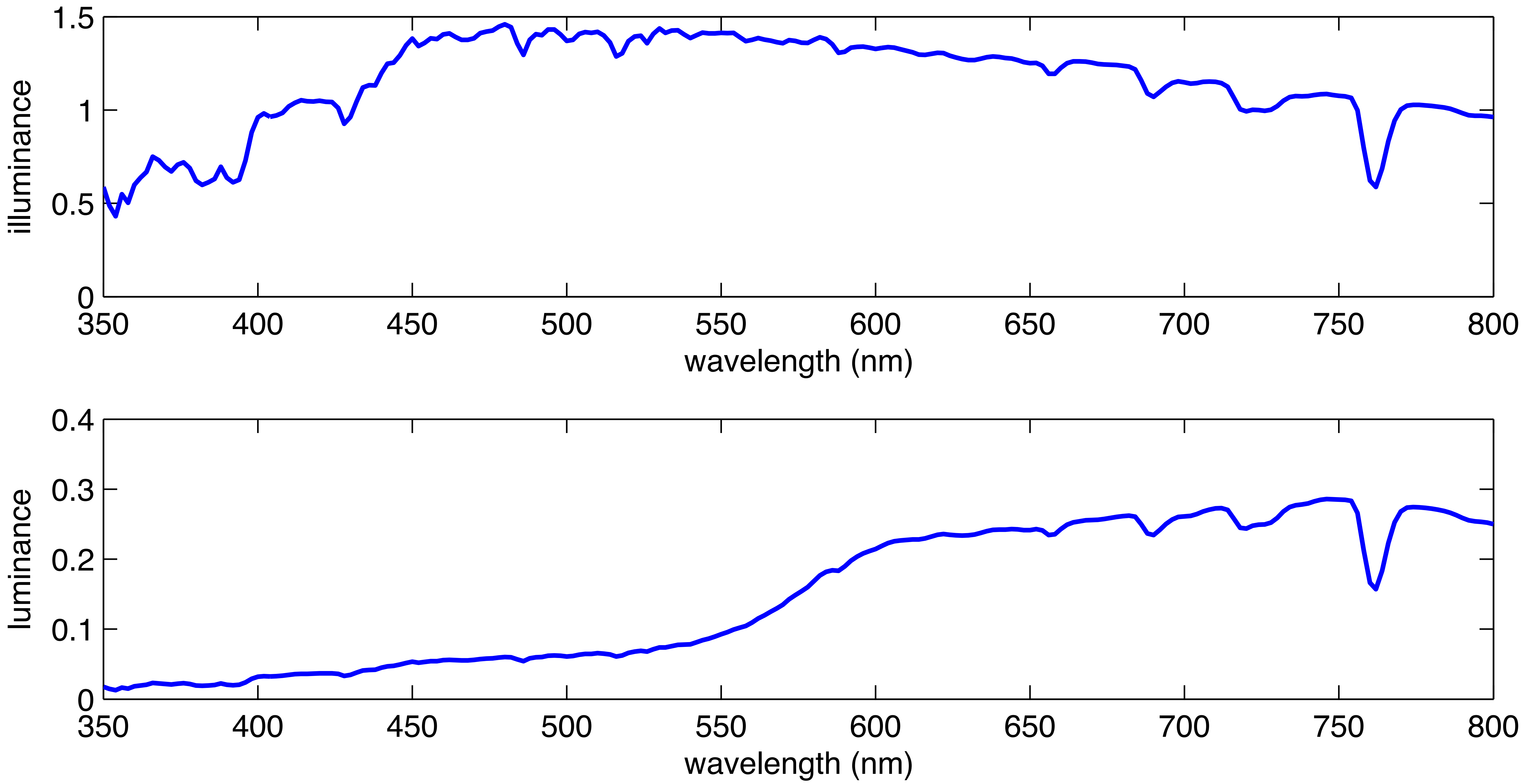
Tetrachromats



Color imaging
spectra

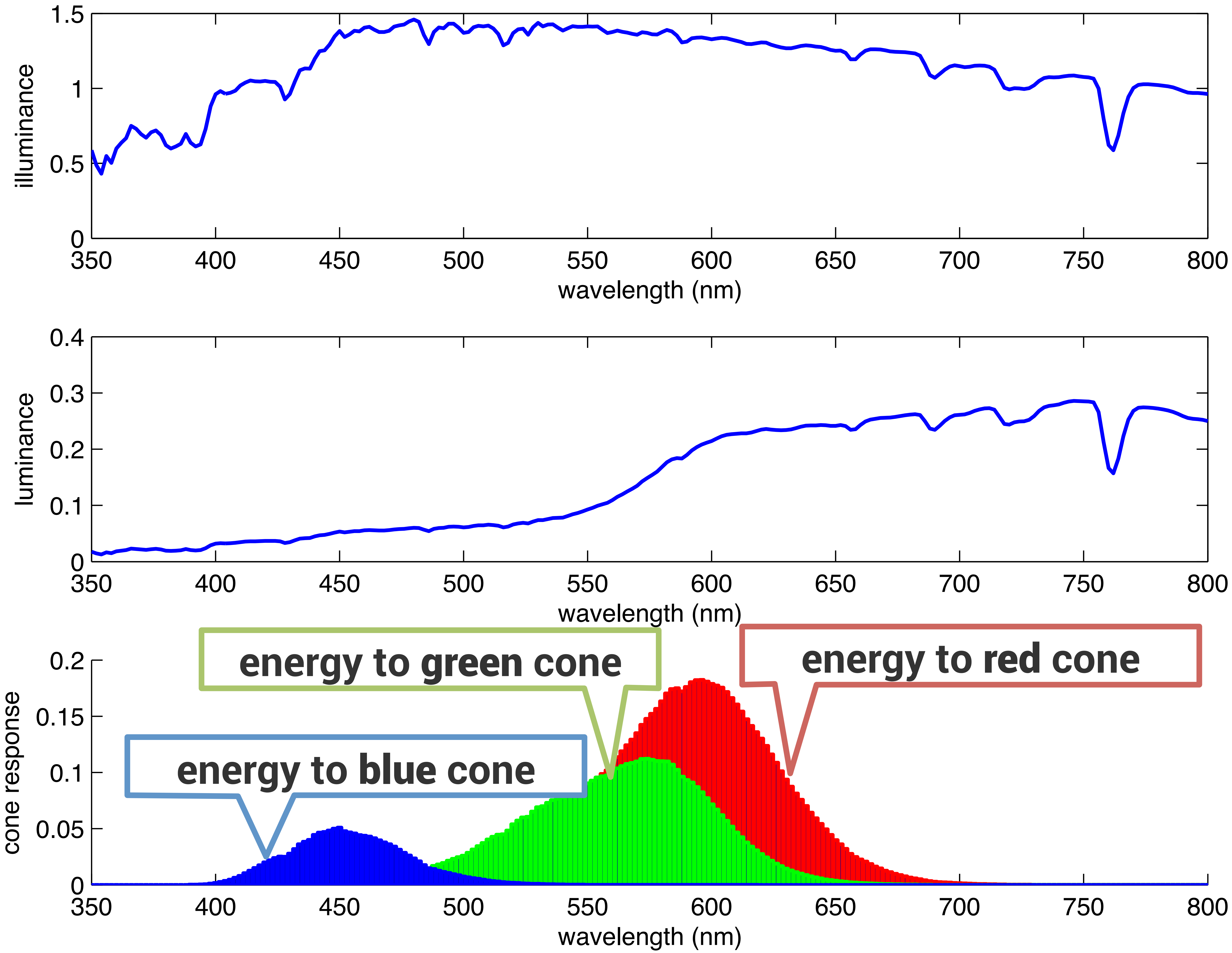


Color imaging
spectra

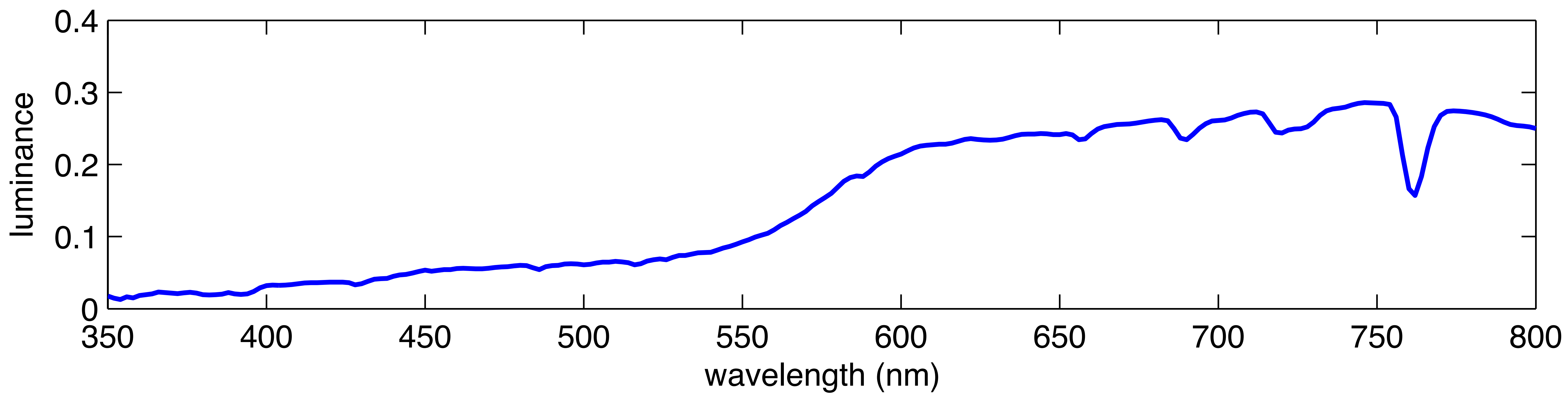


Color imaging
spectra

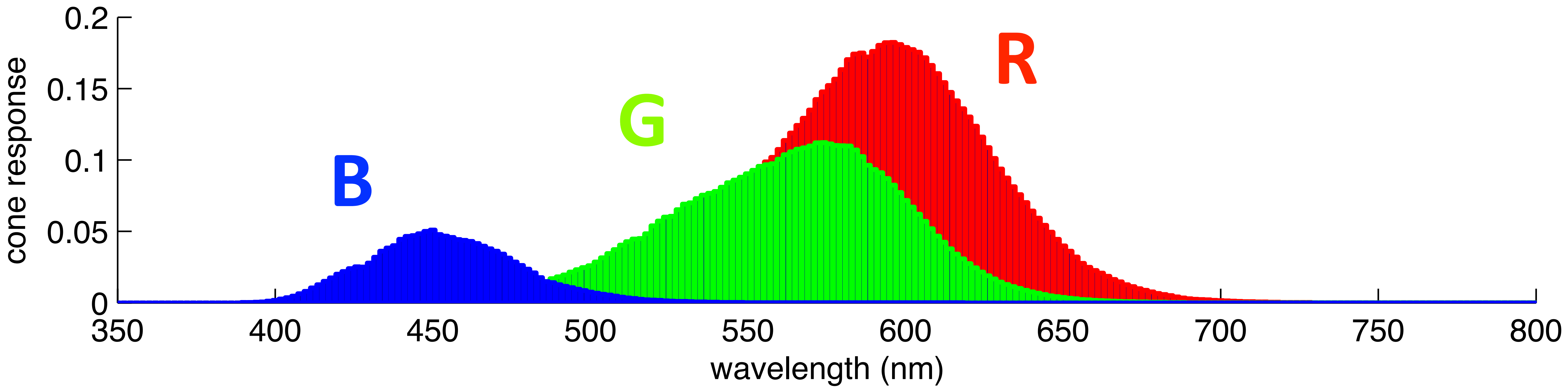
$$\int_\lambda E(\lambda)R(\lambda)M(\lambda)d\lambda$$



Dimension reduction

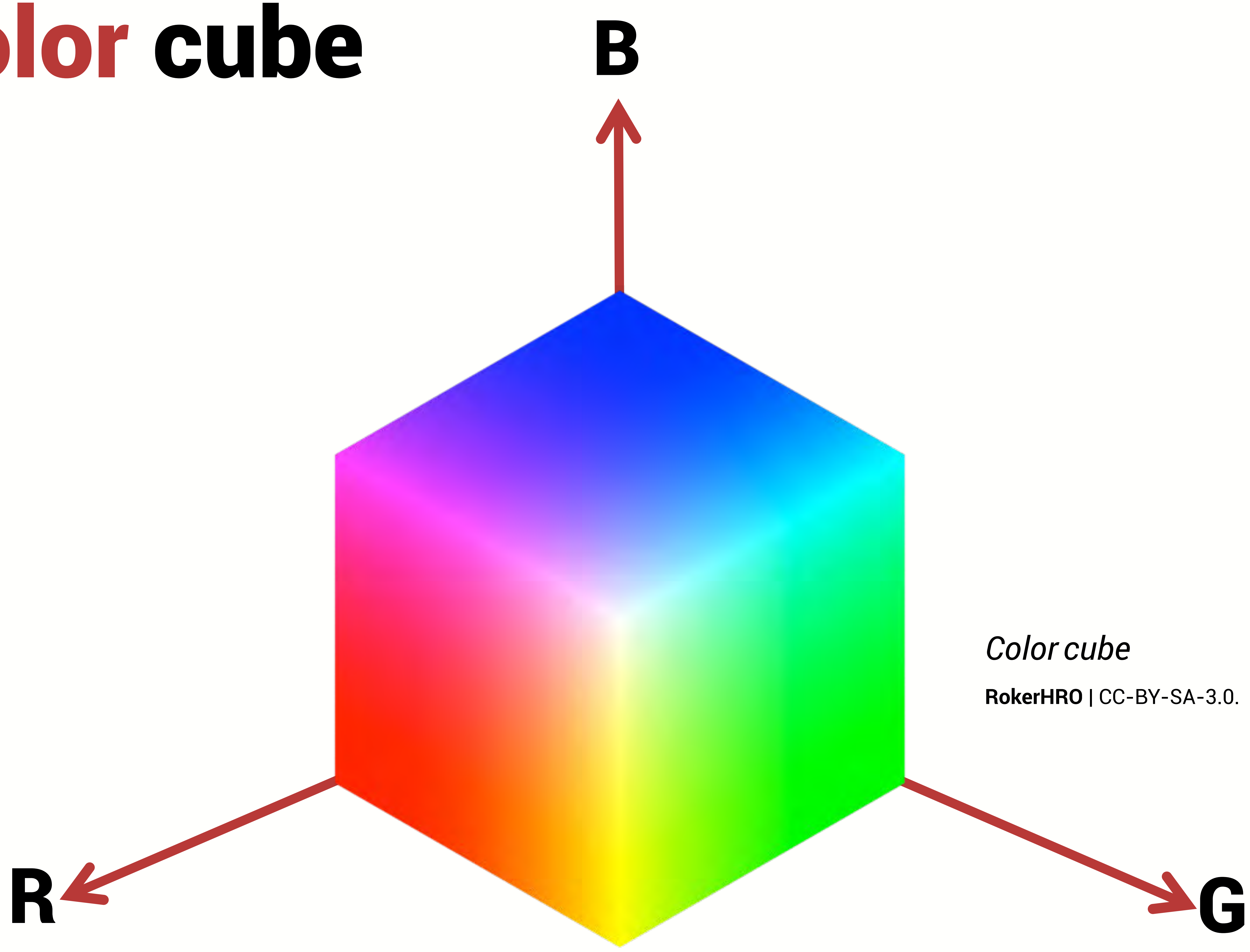


∞ dimensions

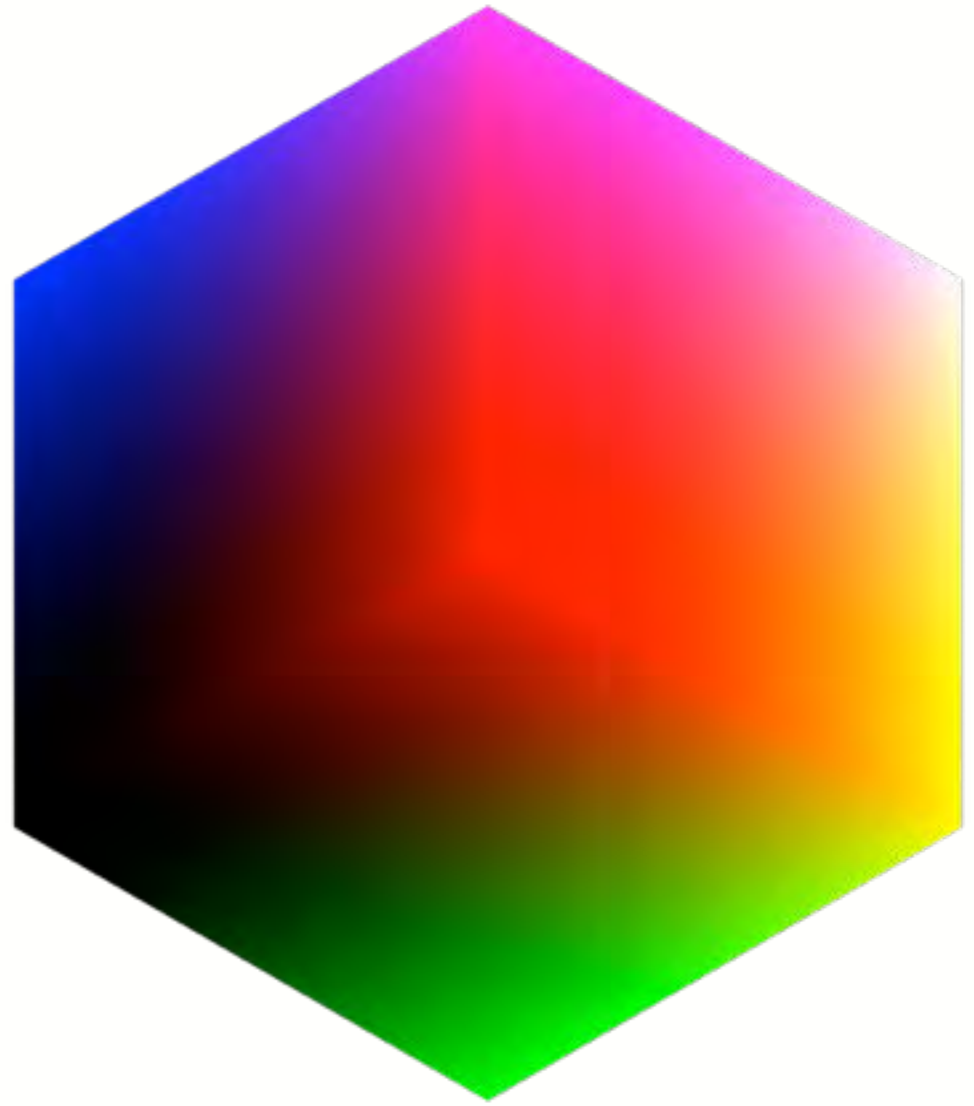


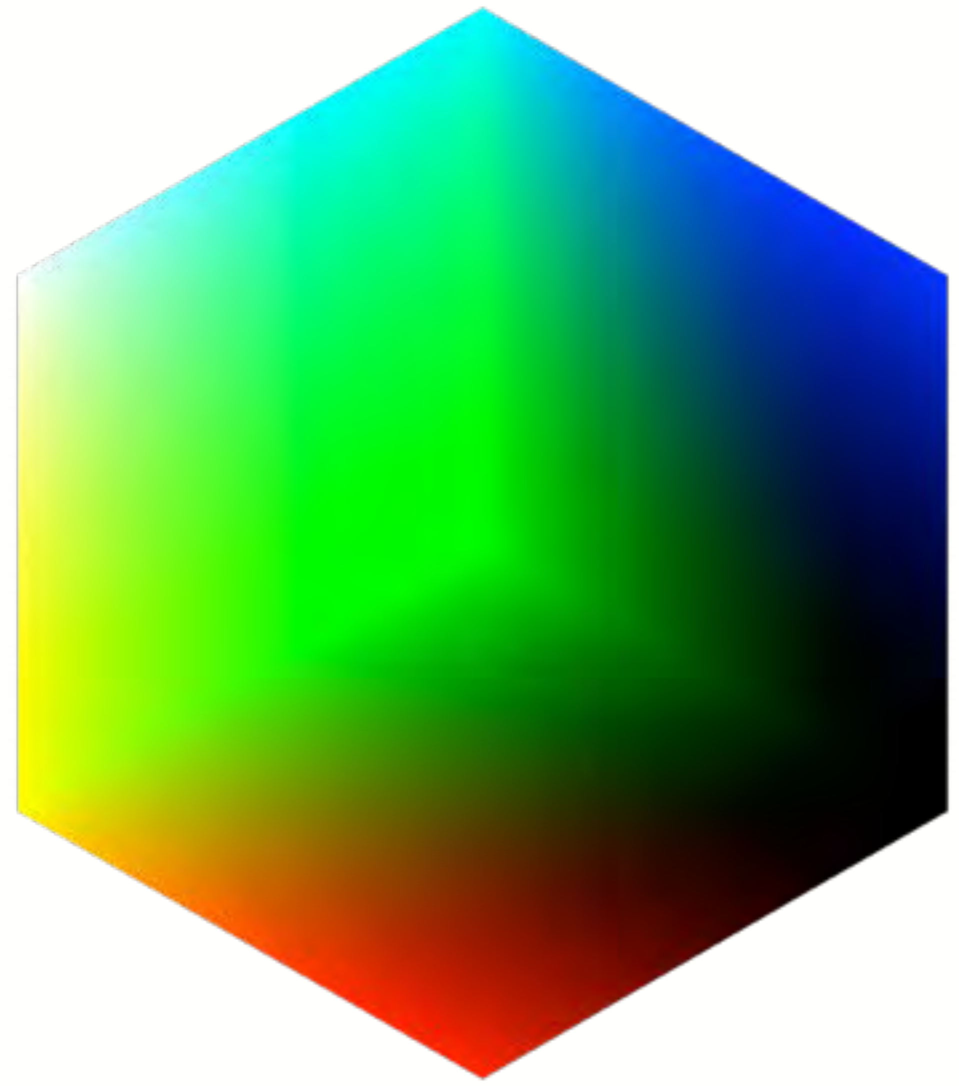
3 dimensions

Color cube



Color cube



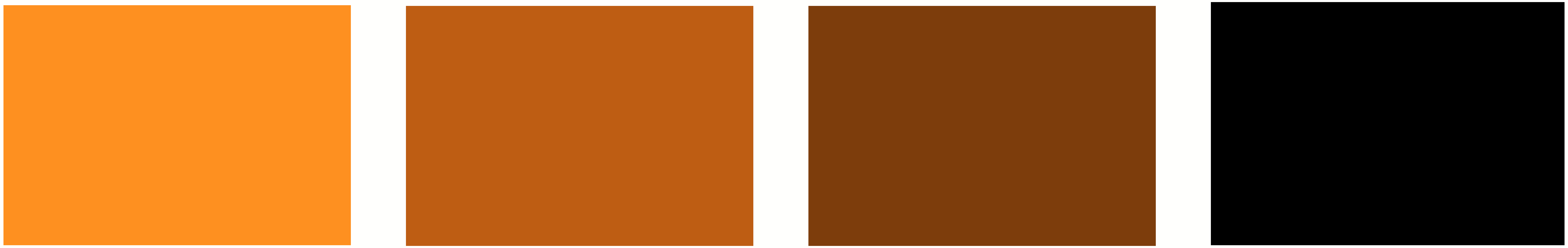


Color names and values

	R	G	B
Slate Grey	112	128	144
Slate Grey Dark	47	79	79
Slate Grey Light	119	136	153
Warm Grey	128	128	105
Ivory Black	41	36	33
Alizarin Crimson	227	38	54
Brick	156	102	31
Cadmium Red Deep	227	23	13
Coral	255	127	80
Deep Pink	255	20	147
English Red	212	61	26
Firebrick	178	34	34
Geranium Lake	227	18	48
Hot Pink	255	105	180
Indian Red	176	23	31
Light Salmon	255	160	122
Madder Lake Deep	227	46	48


(0.6, 0.4, 0.2)
(0.3, 0.2, 0.1)

Color and brightness



100% brightness (253, 124, 27)

75% brightness (175, 74, 18)

50% brightness (105, 46, 13)

0% brightness (0, 0, 0)

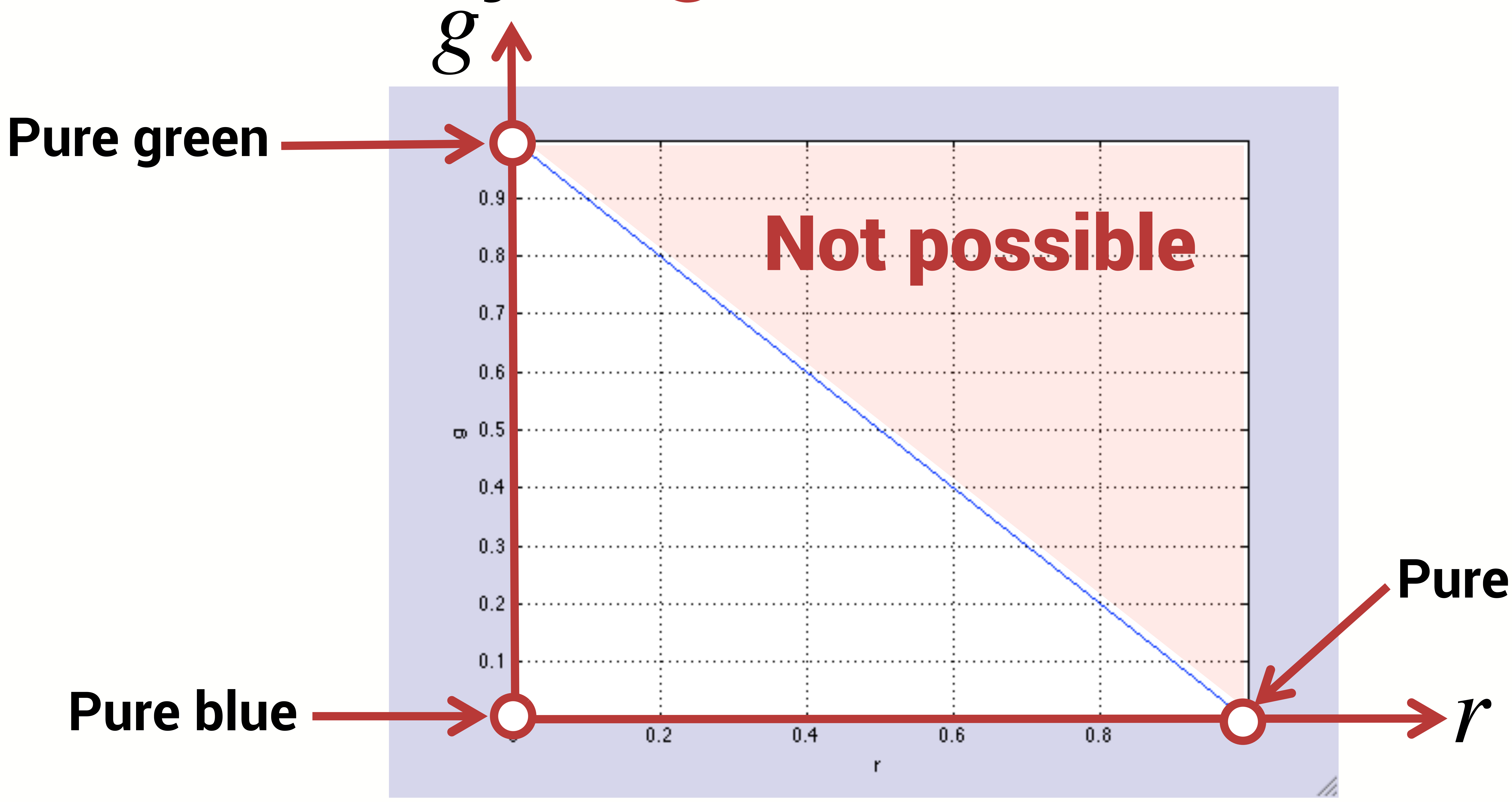
- The eye or camera provides a tristimulus value (R,G,B)
- If lighting level changes the tristimulus is scaled
- Useful to separate color from brightness
- 3 tristimulus value → 1 brightness value, 2 color values

Chromaticity coordinates

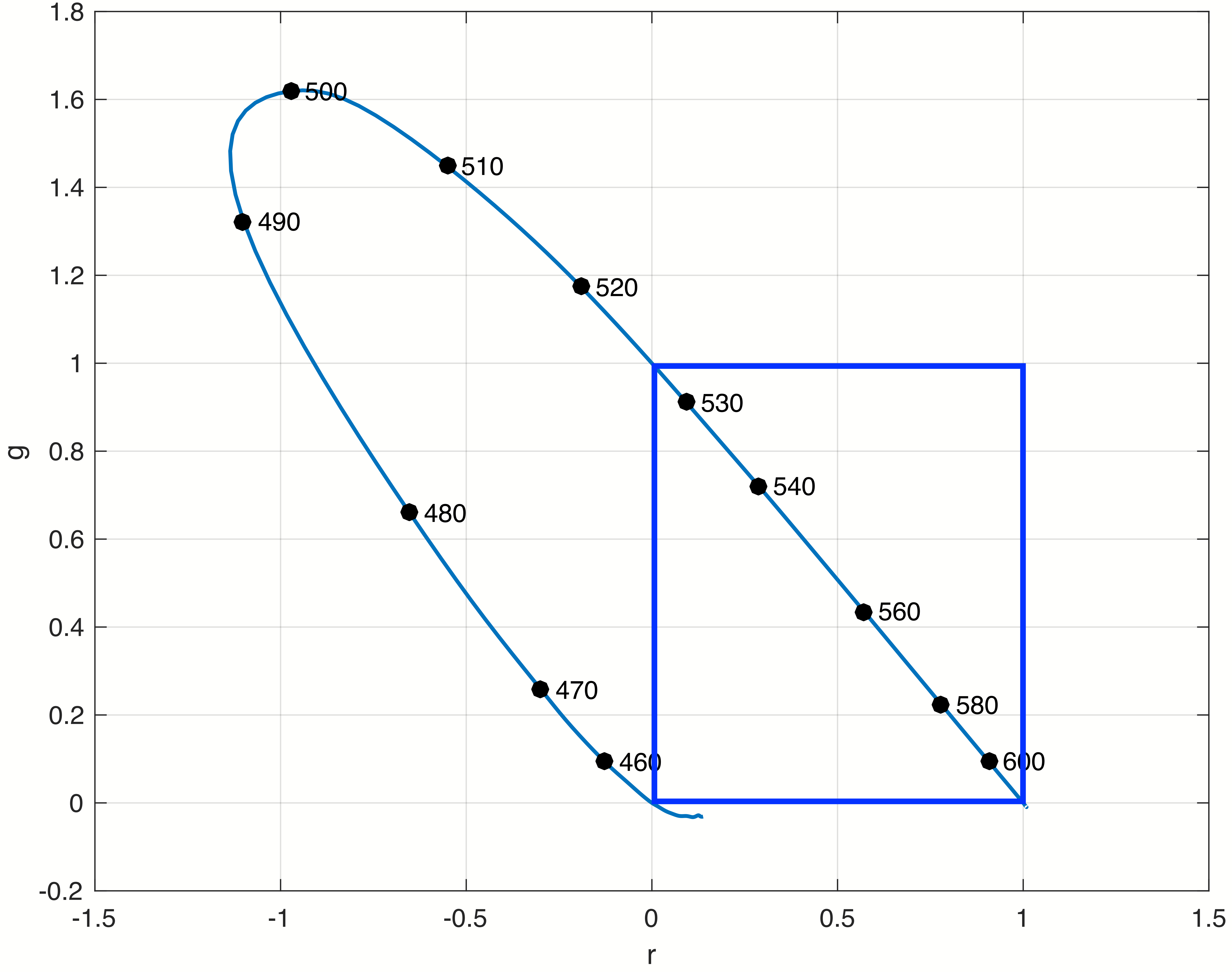
$$r = \frac{R}{R+G+B},\ g = \frac{G}{R+G+B},\ b = \frac{B}{R+G+B}$$

- All values in the range 0 to 1
- Since r+g+b=1 we only need to consider two values, eg. (r,g)

Chromaticity diagram

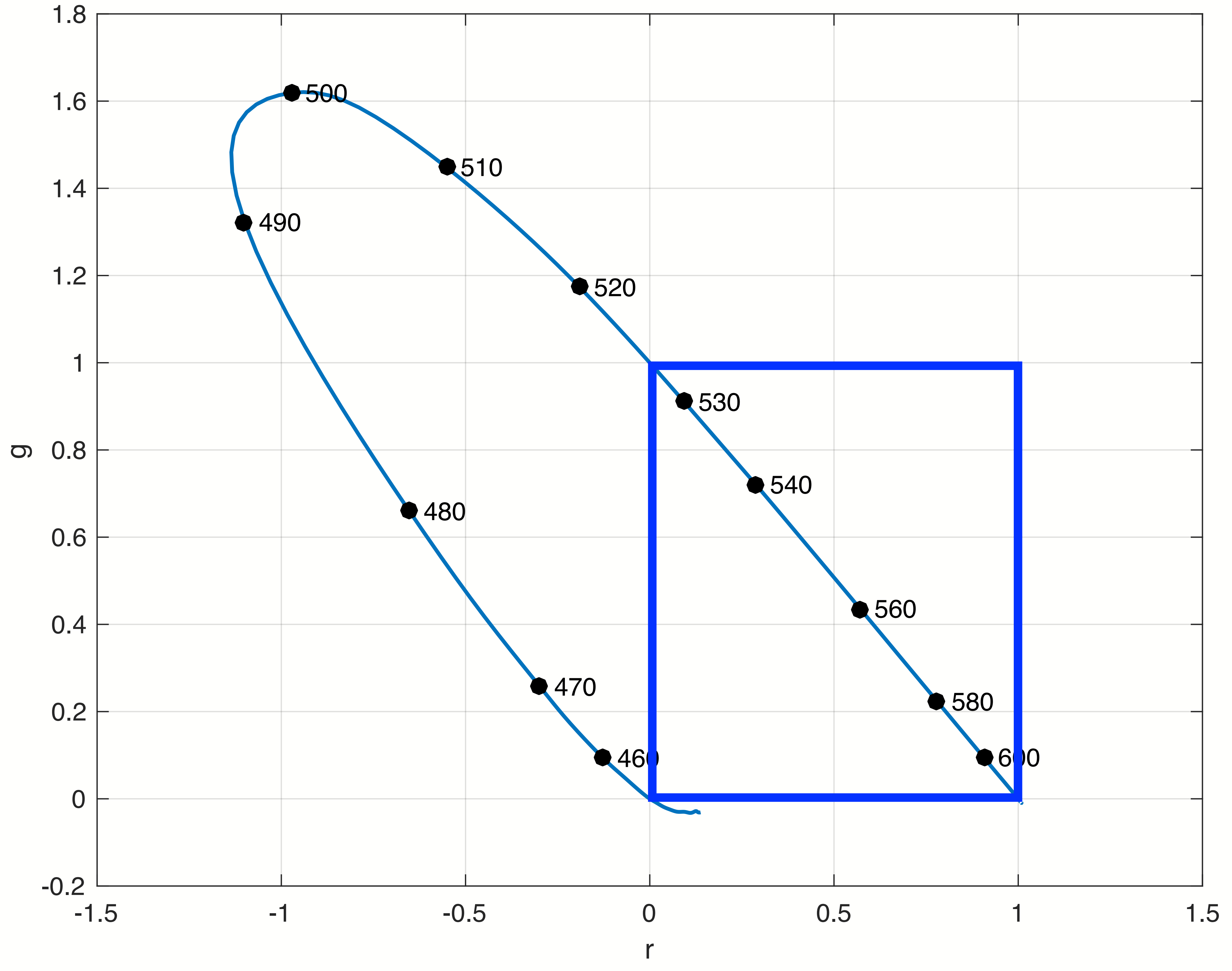


Chromaticity diagram for spectral locus

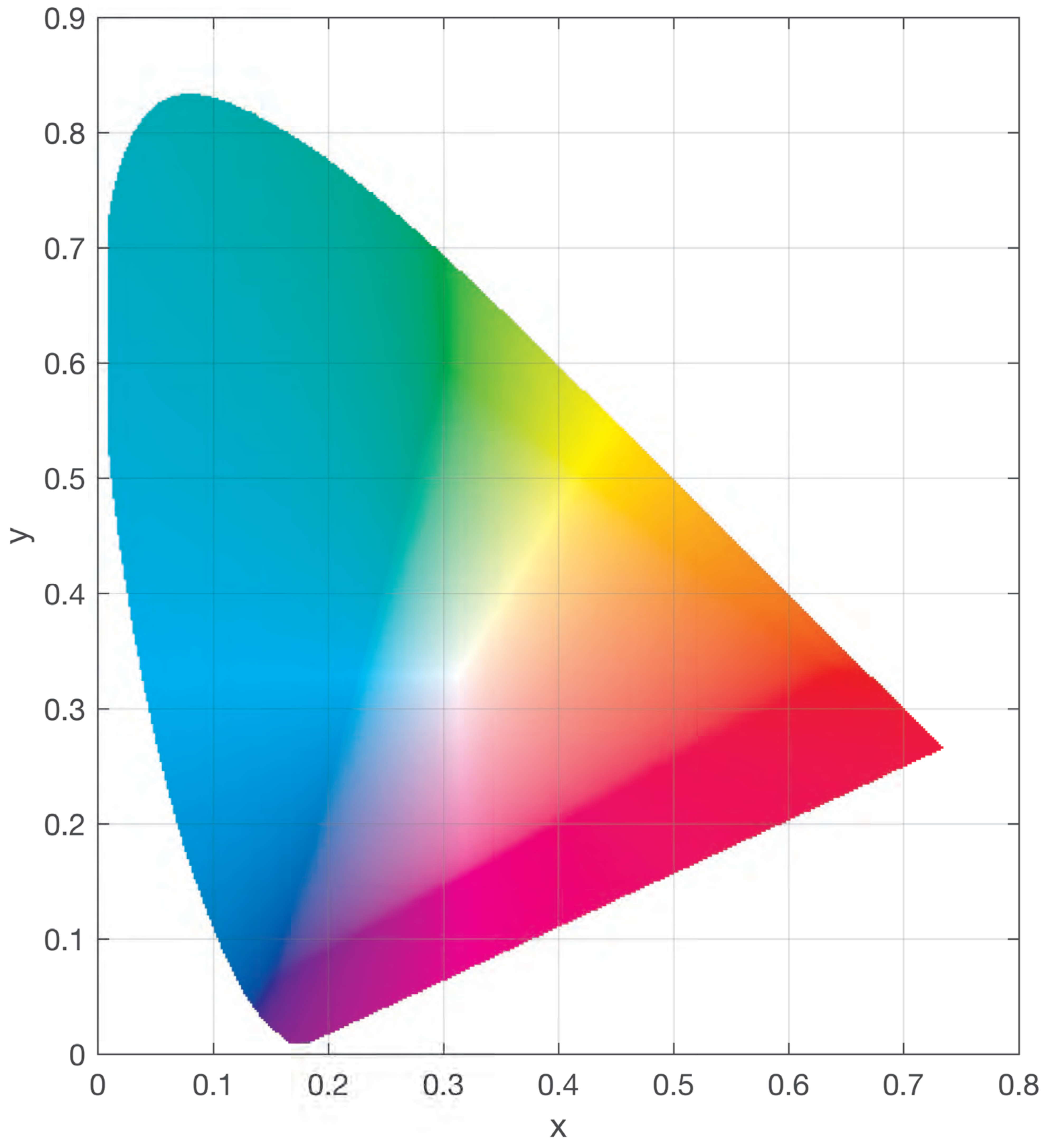


- Many spectral colours require negative amounts of red light!
- RGB cannot represent all possible colors

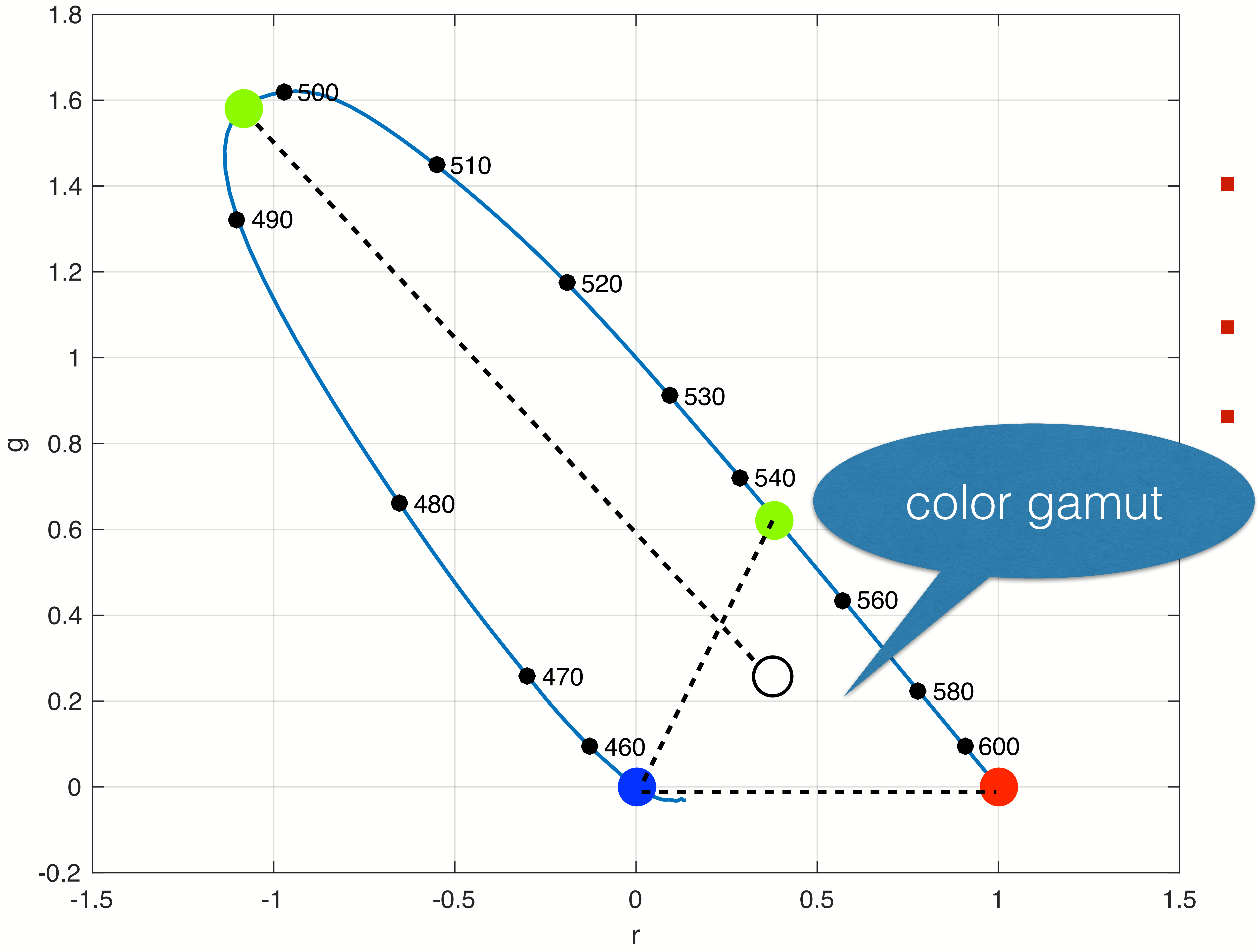
Chromaticity diagram for spectral locus



- Transform into xy colour space
- Based on imaginary XYZ primaries

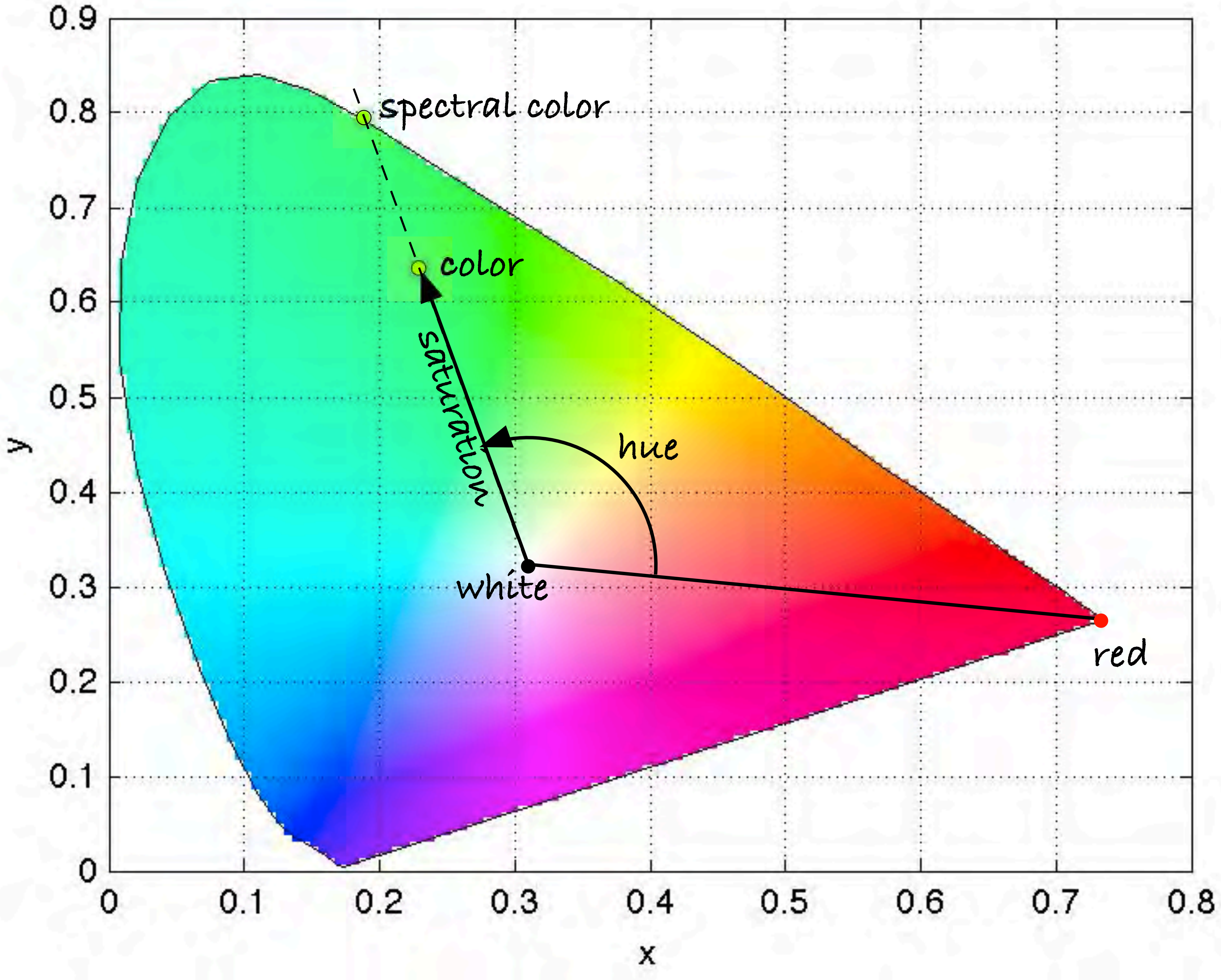


Color gamut



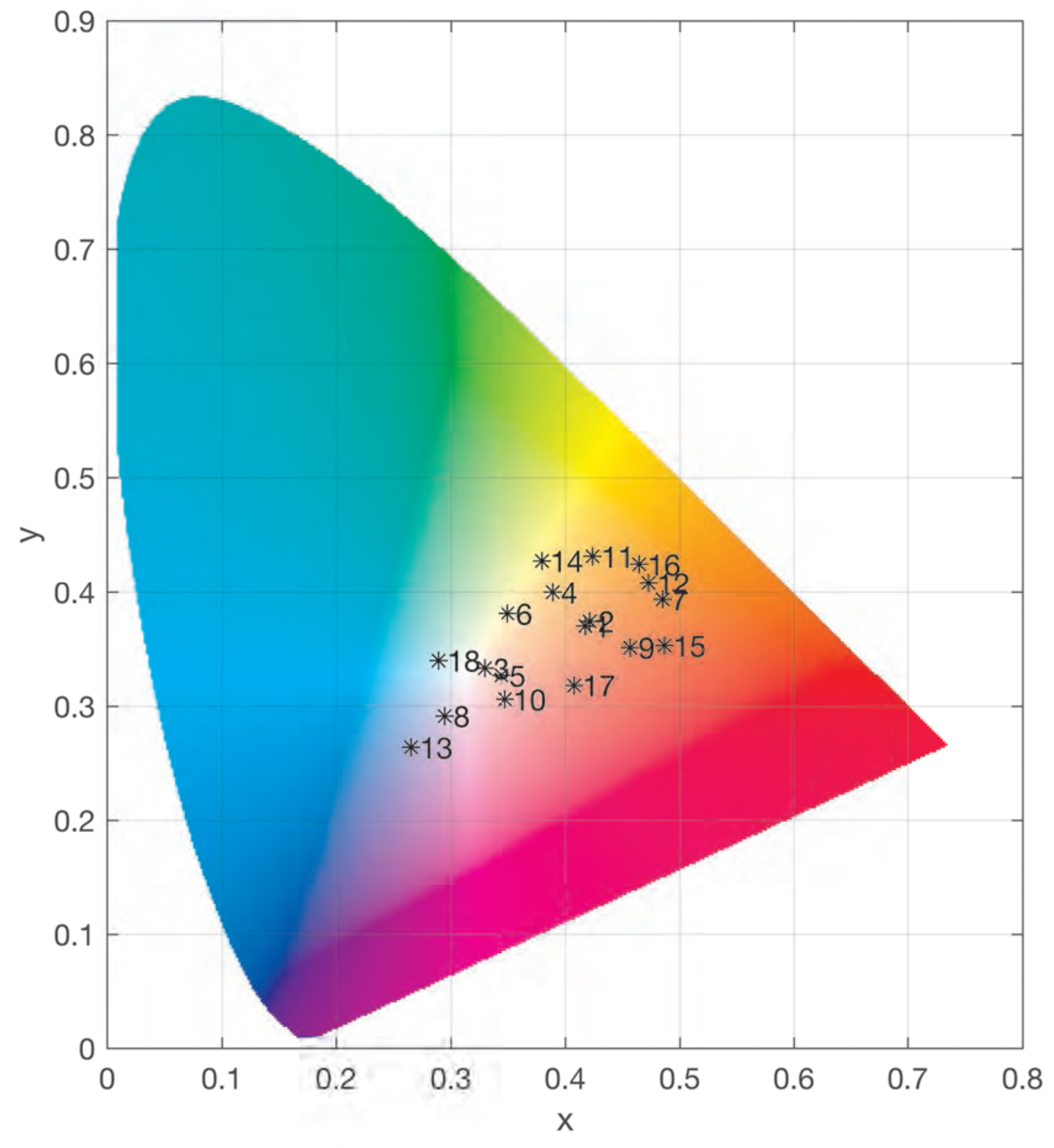
- The set of colours that can be mixed from the 3 primary colours
- Does not include all possible colors!
- There are no 3 physically achievable primaries that can be mixed to form all colors

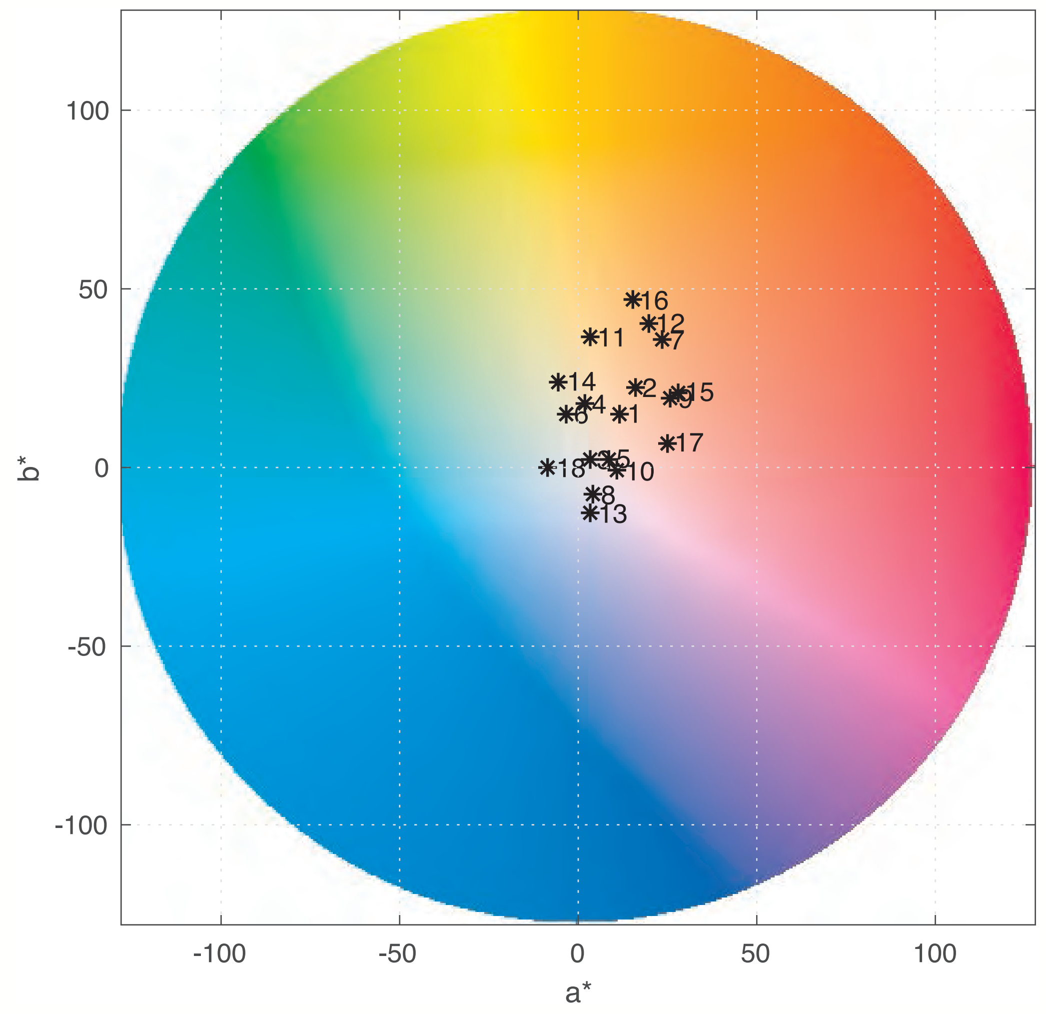
Other color spaces



- **Hue-Saturation**-Value (HSV)
- Two colour dimensions represented in polar form
- Angle is hue
  - 0-360deg
- Normalised length is saturation
  - 1 is pure colour (spectral color)
  - < 1, mixed with some white (pastel color)
  - 0 is white

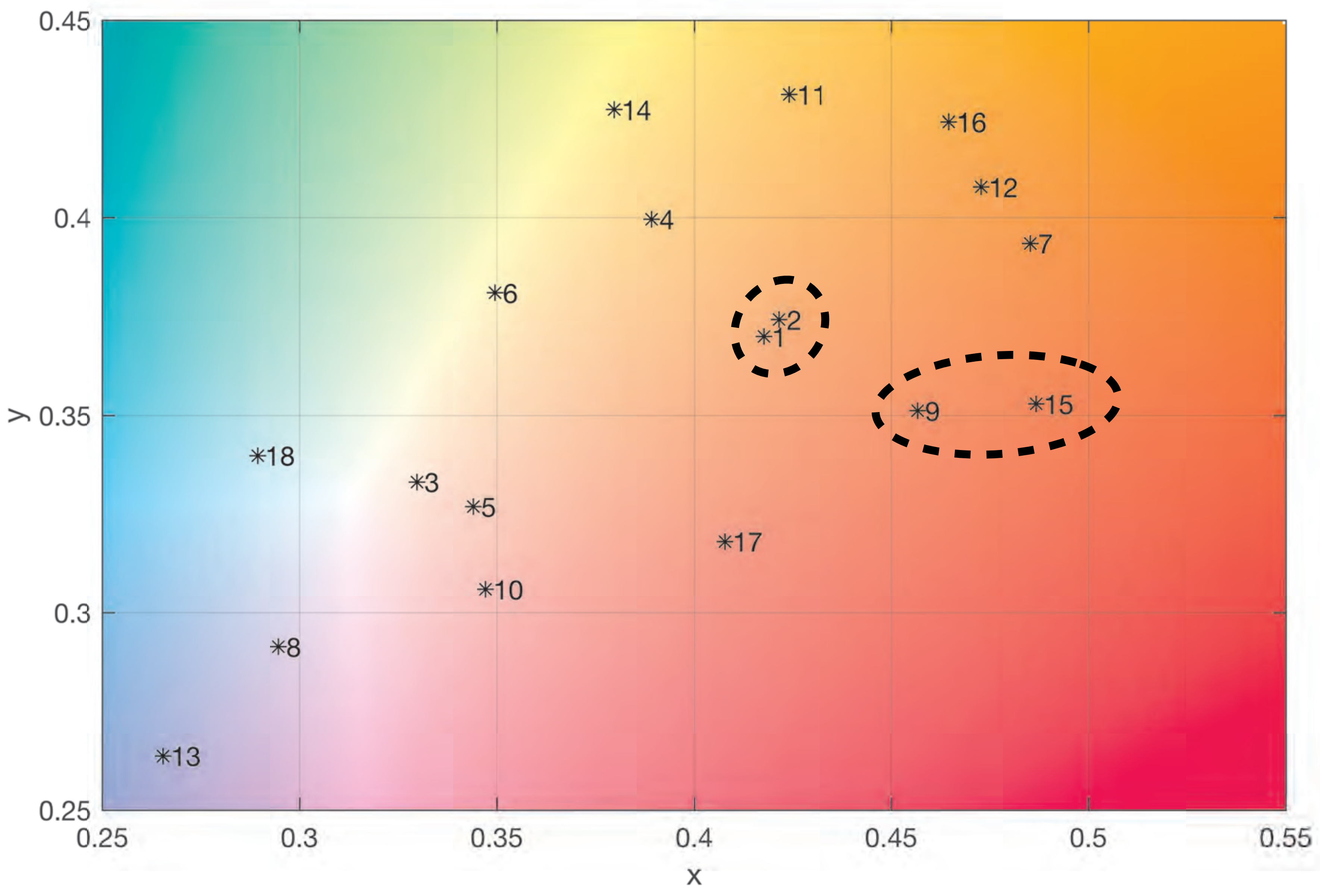
Other color spaces

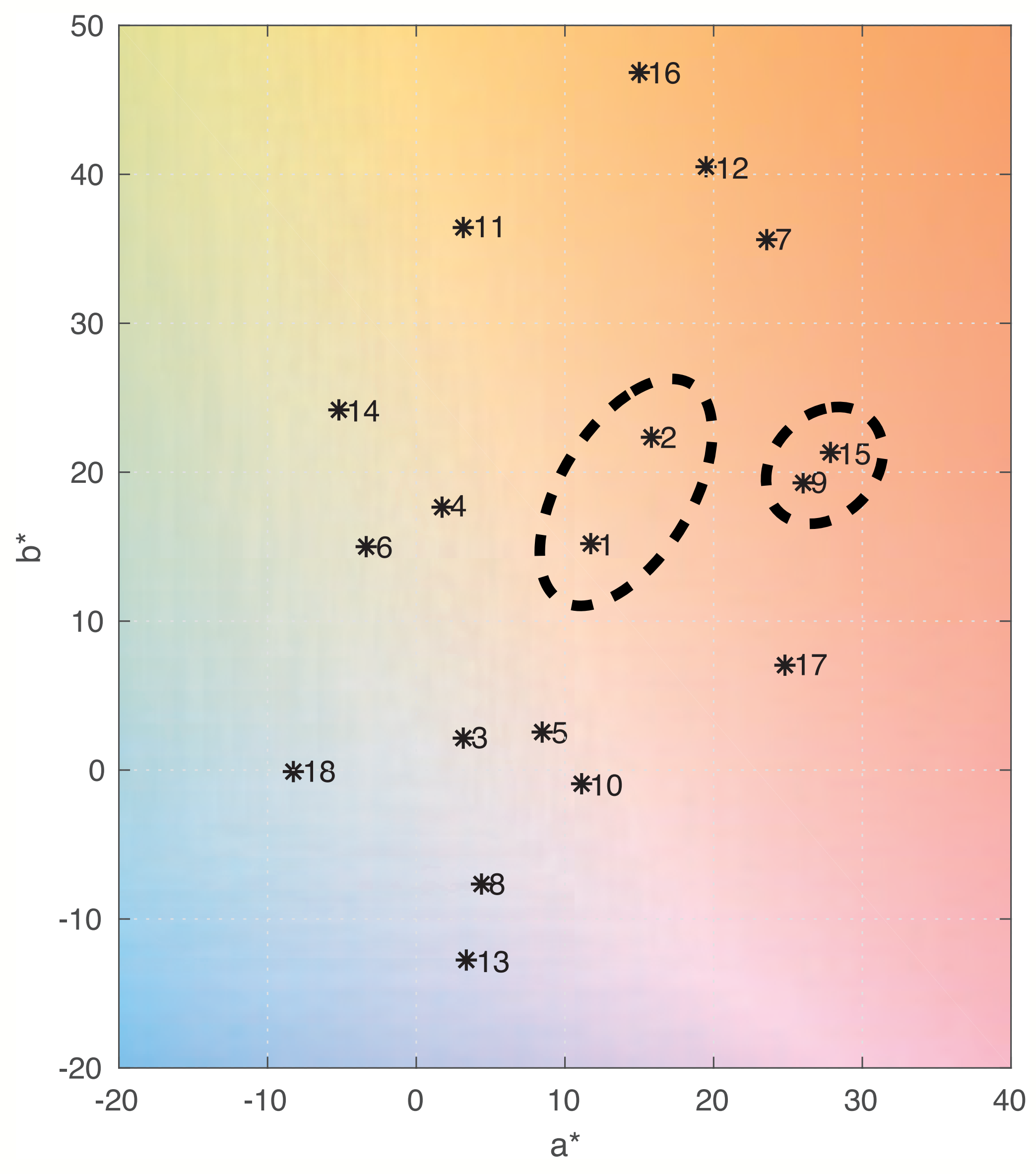




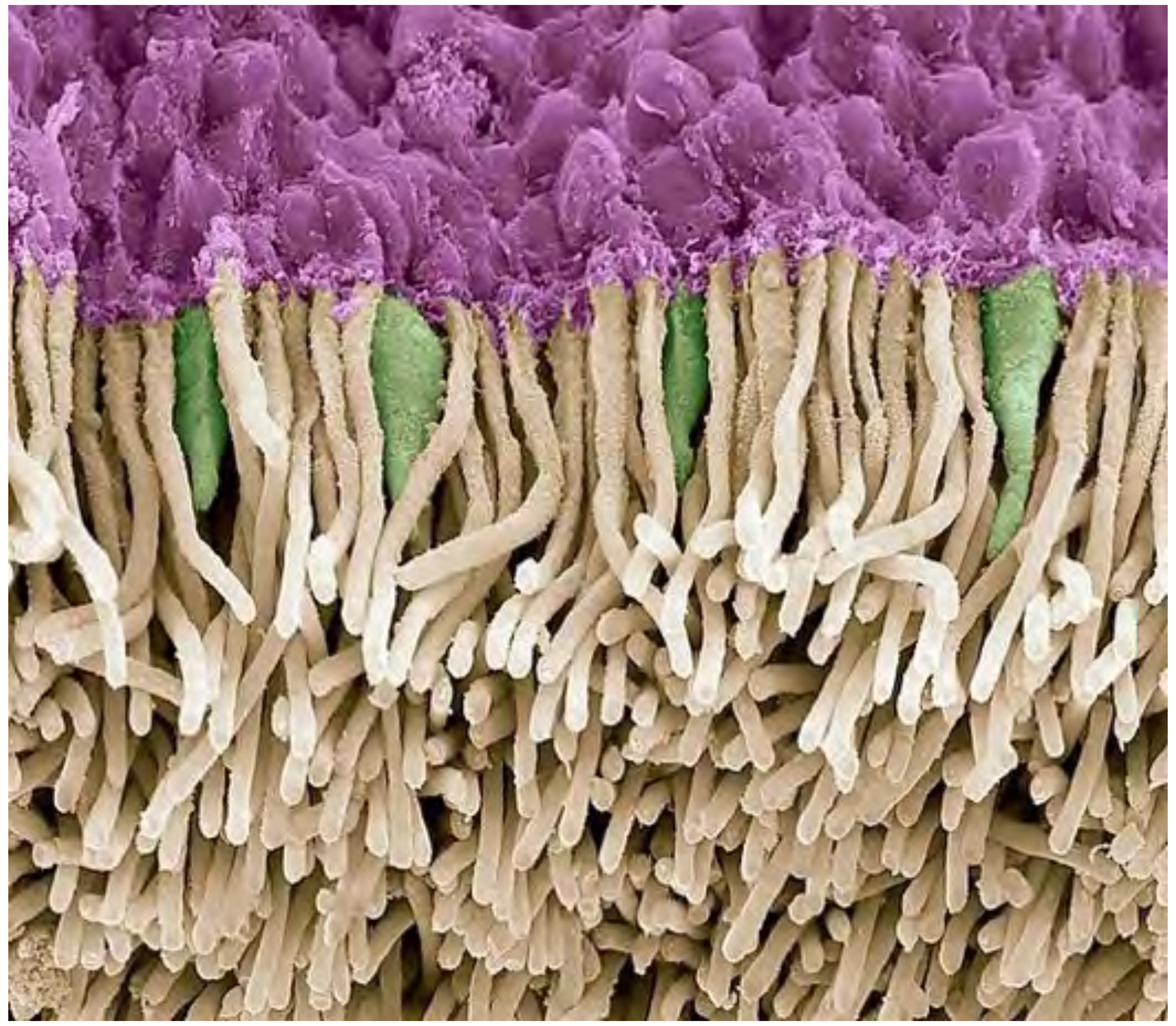
xyY

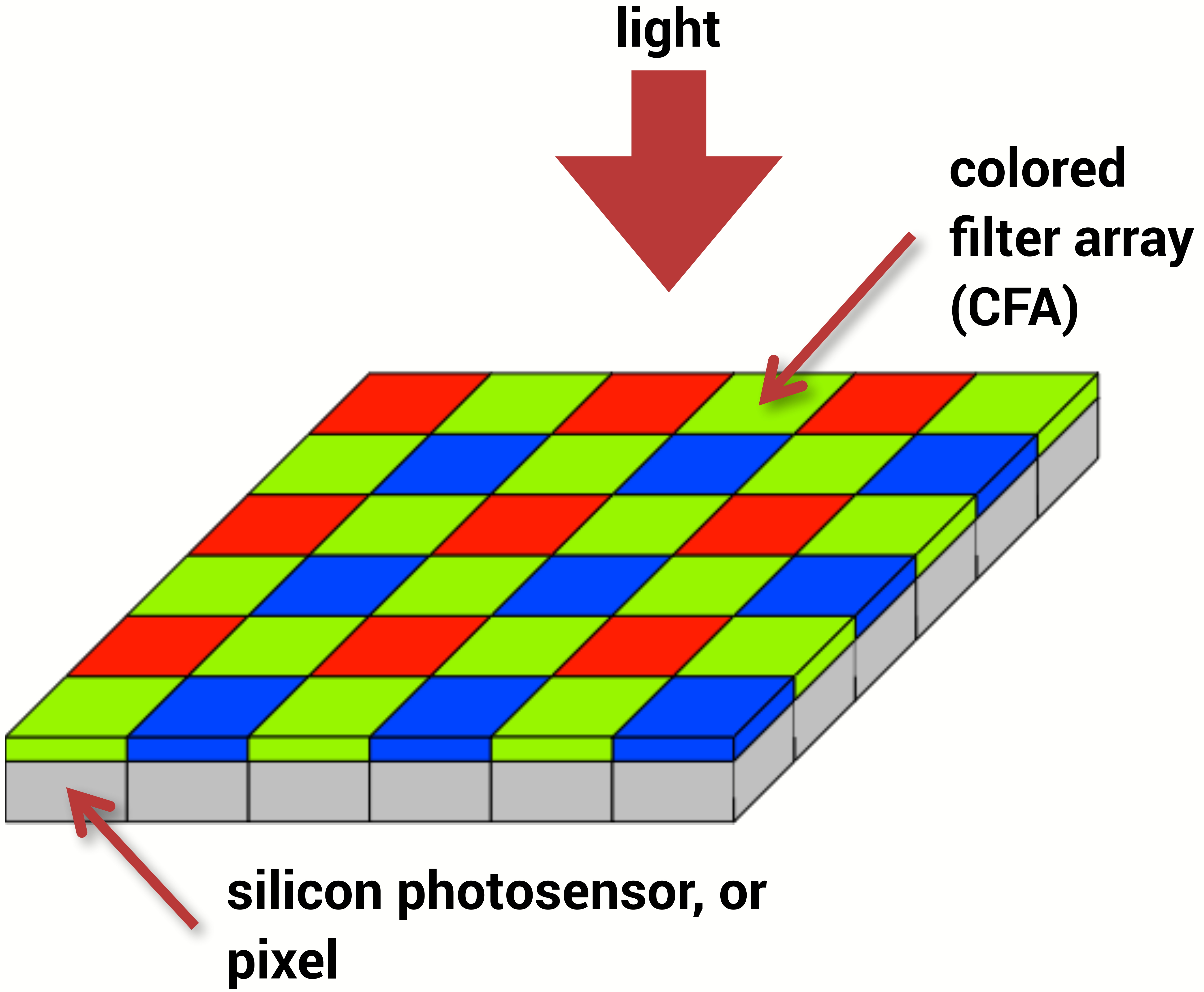
L*a*b*





The silicon equivalent





United States Patent [19]

Bayer

[11] **3,971,065**

[45] **July 20, 1976**

[54] **COLOR IMAGING ARRAY**

[75] Inventor: **Bryce E. Bayer,** Rochester, N.Y.

[73] Assignee: **Eastman Kodak Company,** Rochester, N.Y.

[22] Filed: **Mar. 5, 1975**

[21] Appl. No.: **555,477**

[52] **U.S. Cl.** .... **358/41;** 350/162 SF; 350/317; 358/44

[51] **Int. Cl.²** **H04N 9/24**

[58] **Field of Search** .... 358/44, 45, 46, 47, 358/48; 350/317, 162 SF; 315/169 TV

[56] **References Cited**

UNITED STATES PATENTS

2,446,791	8/1948	Schroeder	358/44
2,508,267	5/1950	Kasperowicz	358/44
2,884,483	4/1959	Ehrenhaft et al.	358/44
3,725,572	4/1973	Kurokawa et al.	358/46

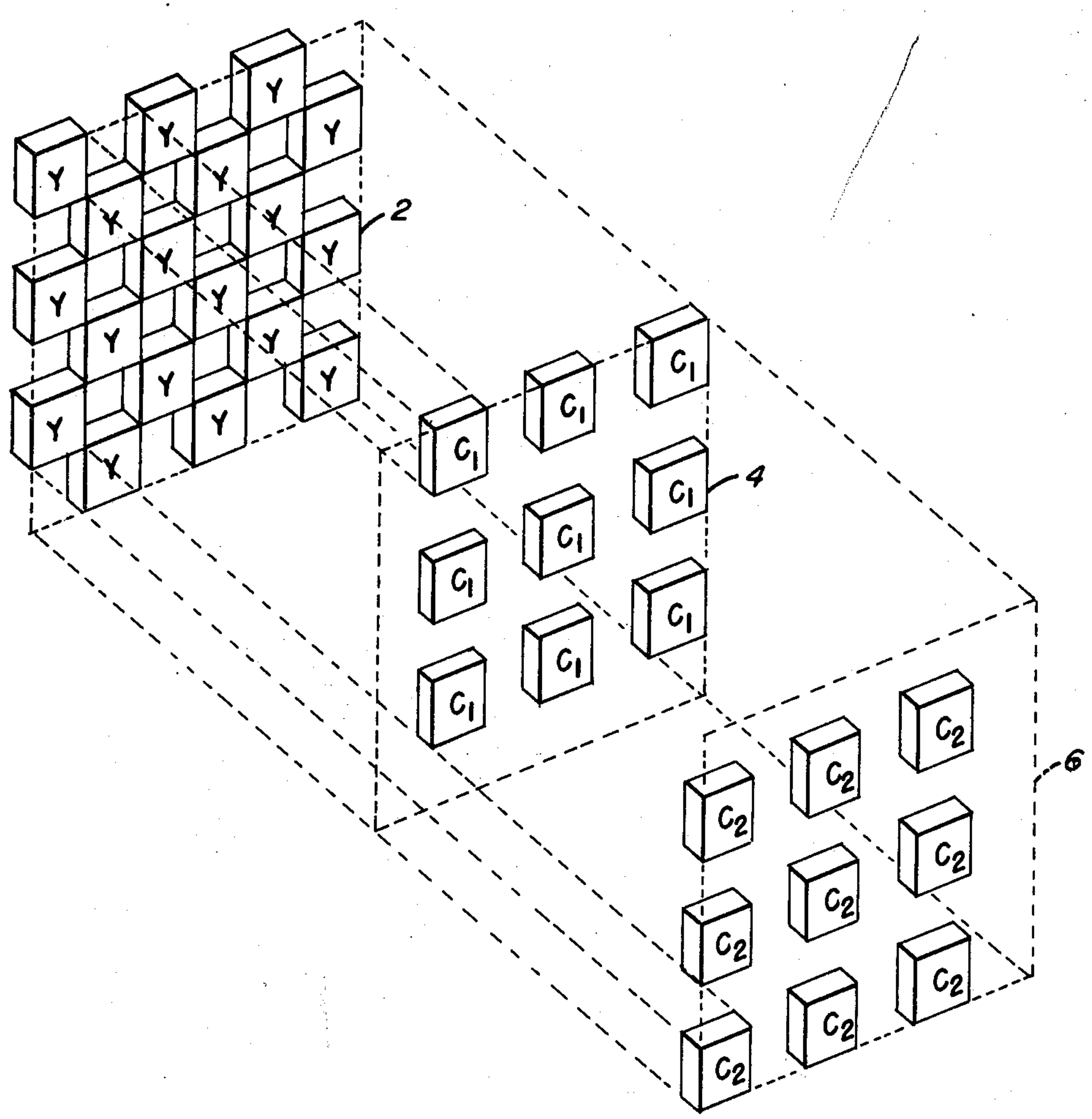
Primary Examiner—George H. Libman
Attorney, Agent, or Firm—George E. Grosser

[57] **ABSTRACT**

A sensing array for color imaging includes individual luminance- and chrominance-sensitive elements that are so intermixed that each type of element (i.e., according to sensitivity characteristics) occurs in a repeated pattern with luminance elements dominating the array. Preferably, luminance elements occur at every other element position to provide a relatively high frequency sampling pattern which is uniform in two perpendicular directions (e.g., horizontal and vertical). The chrominance patterns are interlaid therewith and fill the remaining element positions to provide relatively lower frequencies of sampling.

In a presently preferred implementation, a mosaic of selectively transmissive filters is superposed in registration with a solid state imaging array having a broad range of light sensitivity, the distribution of filter types in the mosaic being in accordance with the above-described patterns.

11 Claims, 10 Drawing Figures



Bayer filter pattern

46	85	153	128	206	144	192	107
78	39	110	85	109	115	129	65
40	73	79	75	120	110	184	110
70	32	78	61	107	50	116	75
48	88	163	123	242	123	149	115
83	66	122	67	116	102	108	73

Bayer filter pattern

46	85	153	128	206	144	192	107
78	39	110	85	109	115	129	65
40	73	79	75	120	110	184	110
70	32	78	61	107	50	116	75
48	88	163	123	242	123	149	115
83	66	122	67	116	102	108	73

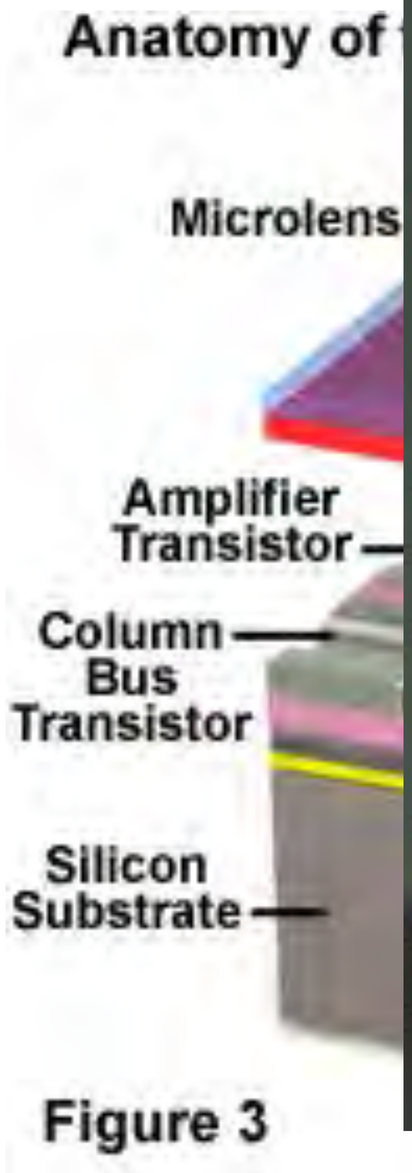
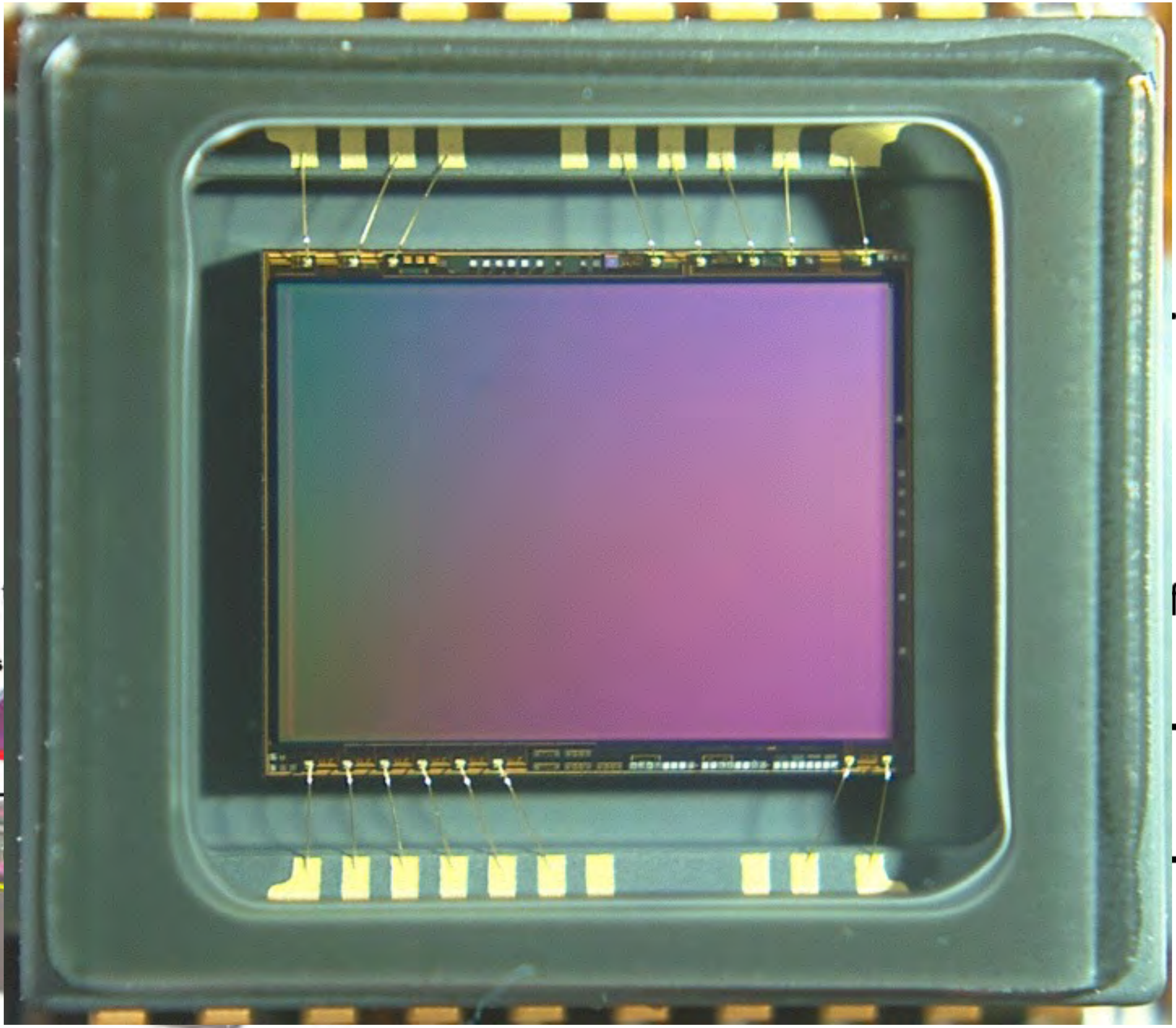
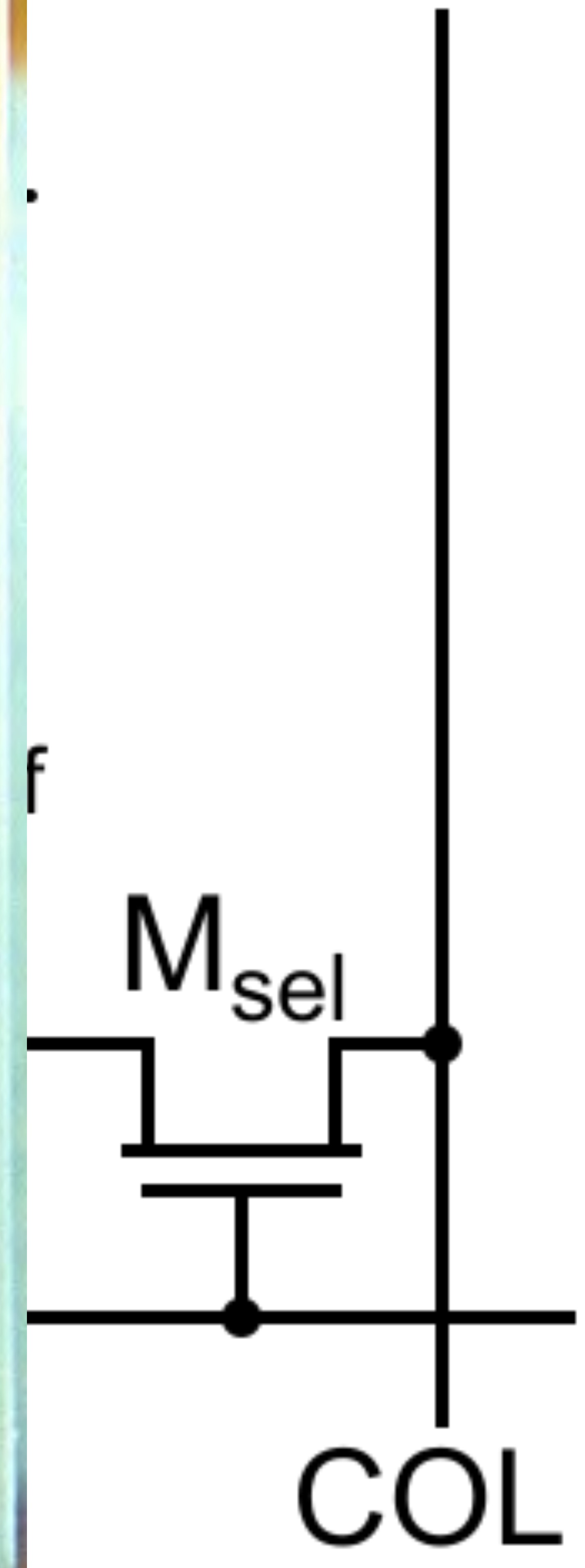
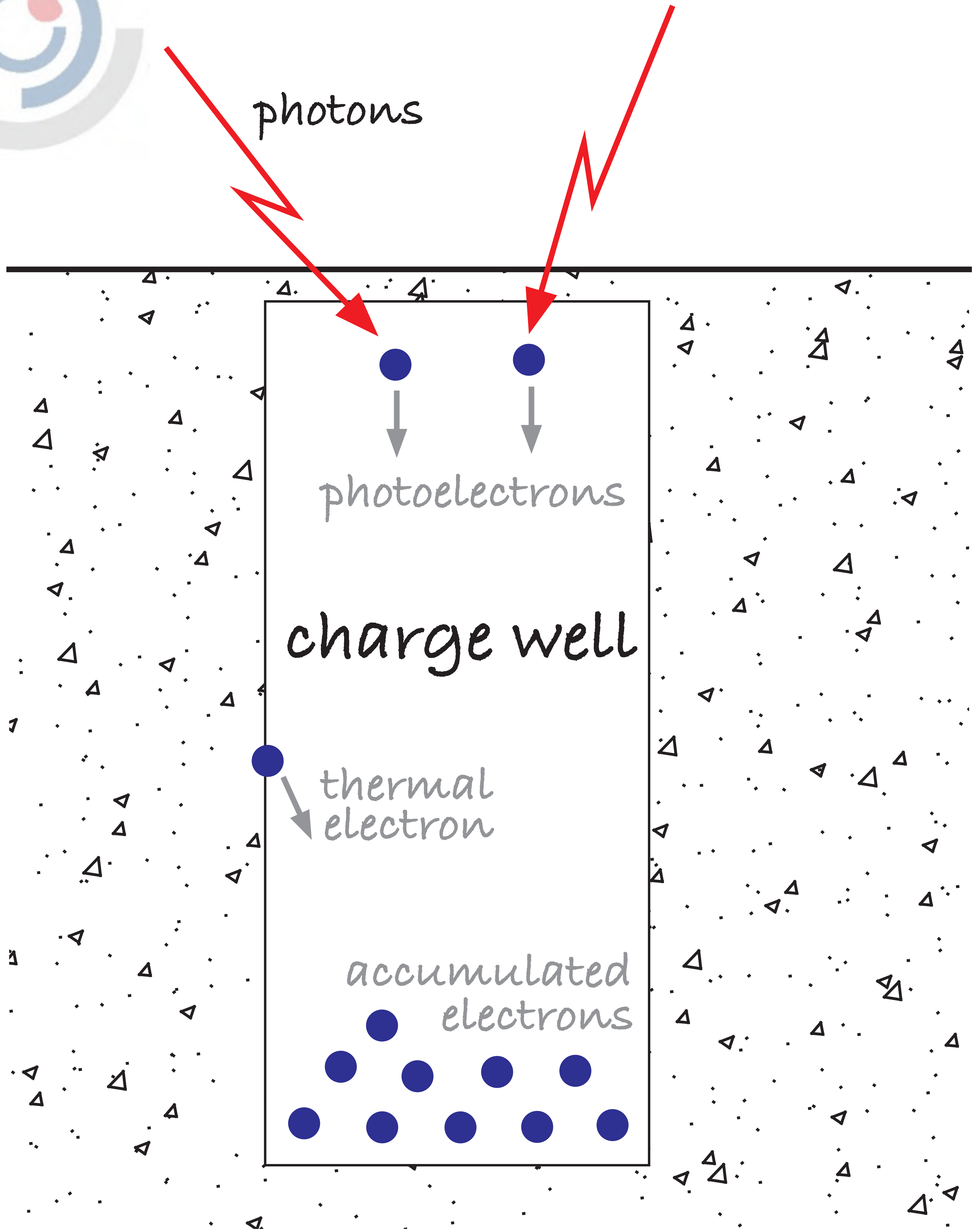

Anatomy of
Microlens
Amplifier Transistor
Column Bus Transistor
Silicon Substrate

Figure 3




M_{sel}
COL

Exposure



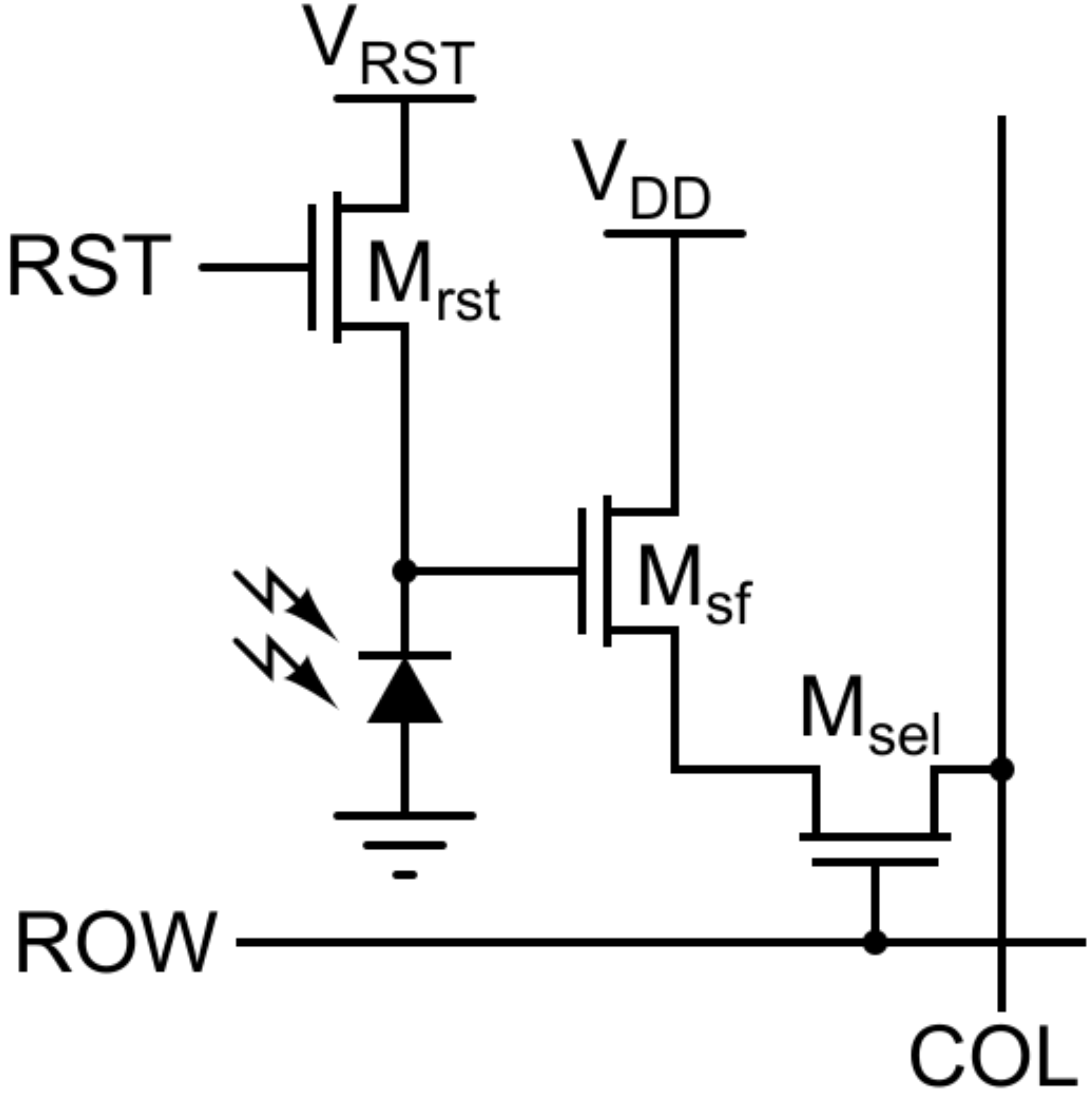
- Total exposure

$$H = qL\frac{T}{N^2}\,\mathrm{lx.s}$$

- Resulting pixel value

$$x = kAH$$

$$k = \frac{118}{10}\mathrm{ISO}$$


V_{RST}
RST
M_{rst}
V_{DD}
M_{sf}
M_{sel}
ROW
COL

Anton River: https://www.youtube.com/watch?v=17PSgsRIO9Q

Display non-linearity

$L \rightarrow$

$L^{\frac{1}{\gamma}}$

gamma encoded images

gamma $\gamma \approx 2.2$

gamma encoding V

$\rightarrow L' = V^{\gamma}$

$L' = L^{\gamma}$

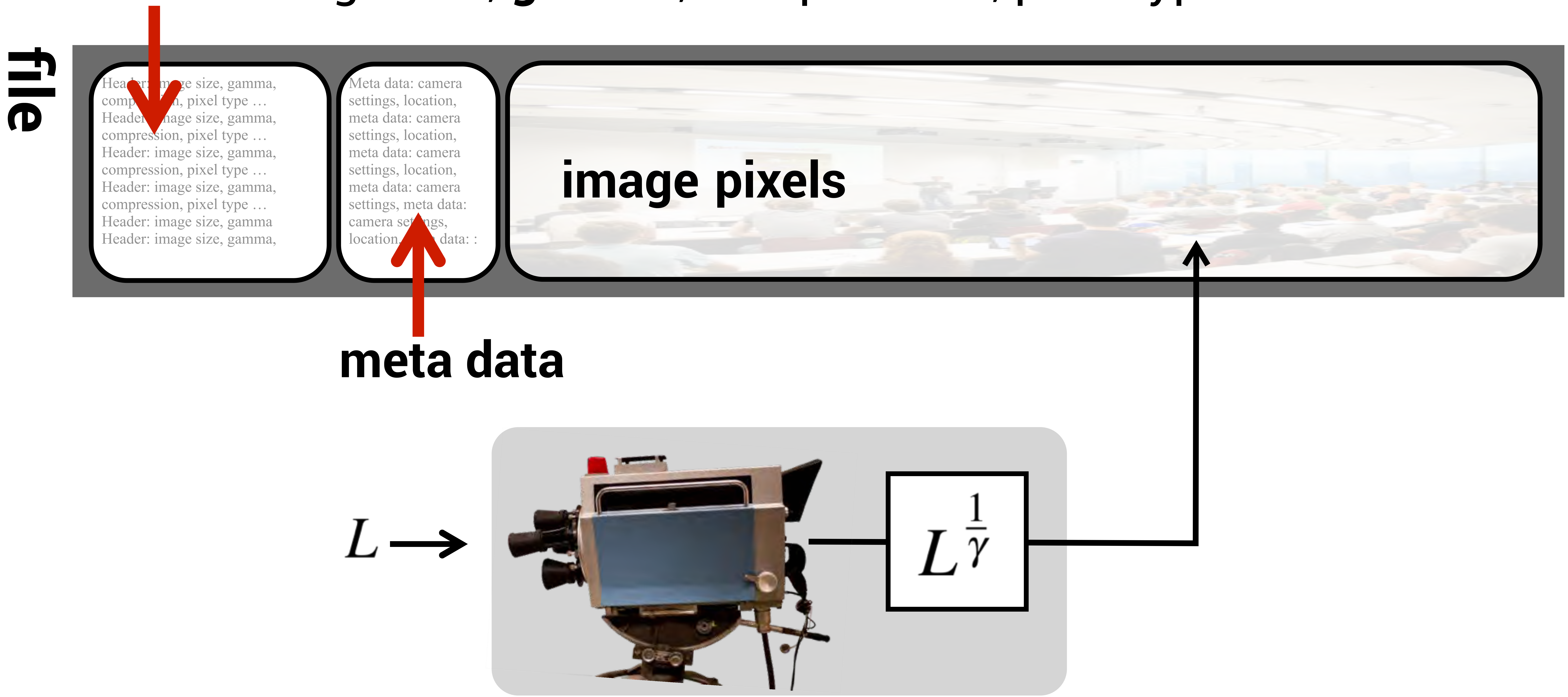
The system is linear end-to-end

gamma decoding
gamma correction

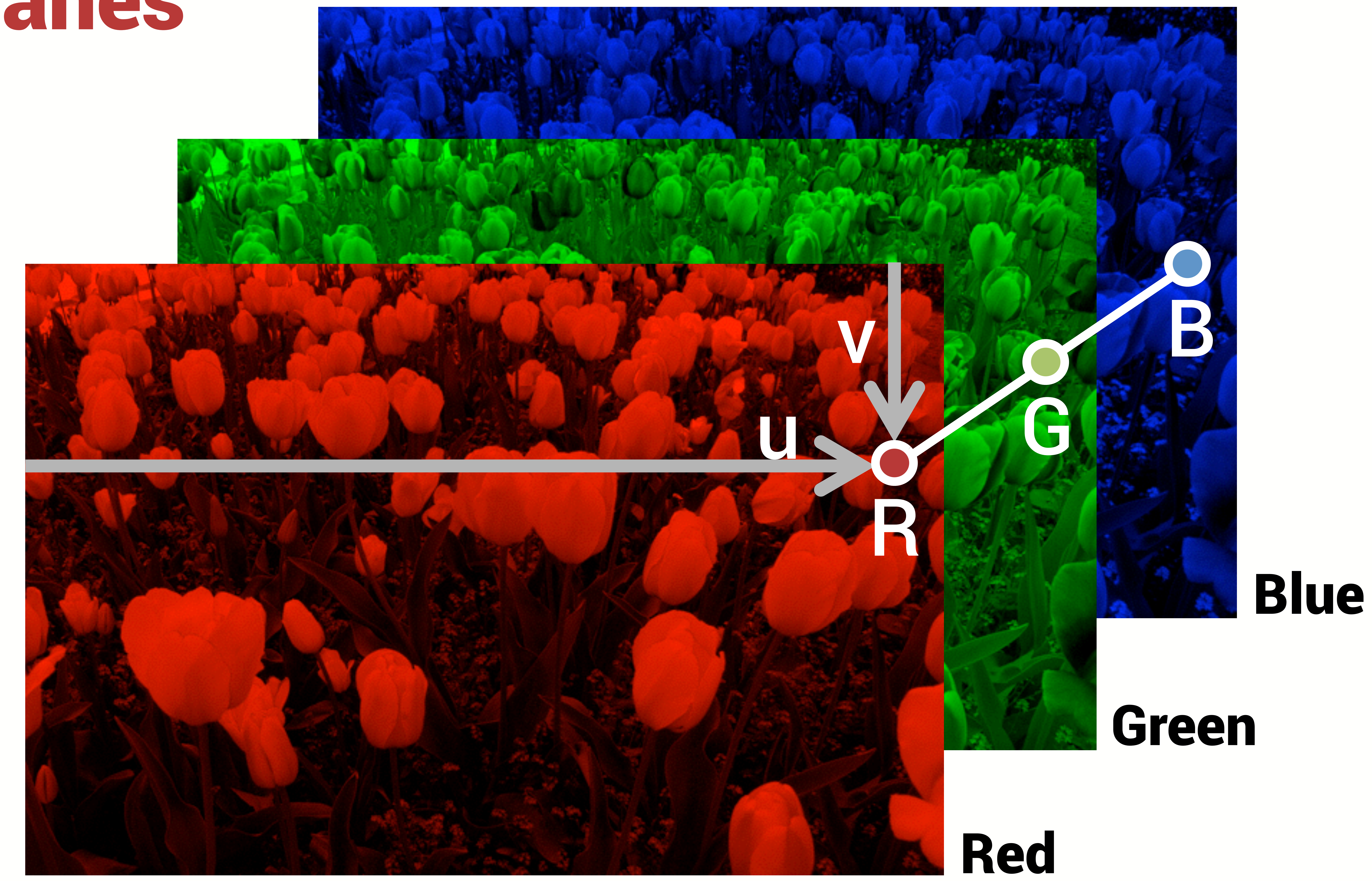
Color **planes**

Image file formats

header: image size, **gamma**, compression, pixel type...



Color **planes**



Cave Paintings ~40,000 years ago

Ideal City (1470)

Piero della Francesca (1415–1492)

Figure 28 (Jan Vredeman de Vries, 1604).
Used with permission from Perspective, Dover Publications,1964.

trompe l'oeil **|ˌtrômp ˈloi|**

noun (pl. **trompe l'oeils** pronunc. **same**)

visual illusion in art, esp. as used to trick the eye into perceiving a painted detail as a three-dimensional object.

Trompe L'oeil Tuscan Window Mural 2009

Kristin Plansky | Used with permission.

New York City, Lower Manhattan, Front St.: **Richard Haas** *Trompe l'oeil* 1975

People are actually avoiding walking in the "hole" 2007

Joe Beever | CC A2.0

Stunning 3D chalk drawing from Zebit stops Liverpool shoppers in their tracks on Bold Street. *2012*

Edgar Meuller http://www.metanamorph.com

Forced perspective
_2011_
Seongbin Im | CC A2.0

On hands _2013_
Kenzie Saunders |
CC A2.0

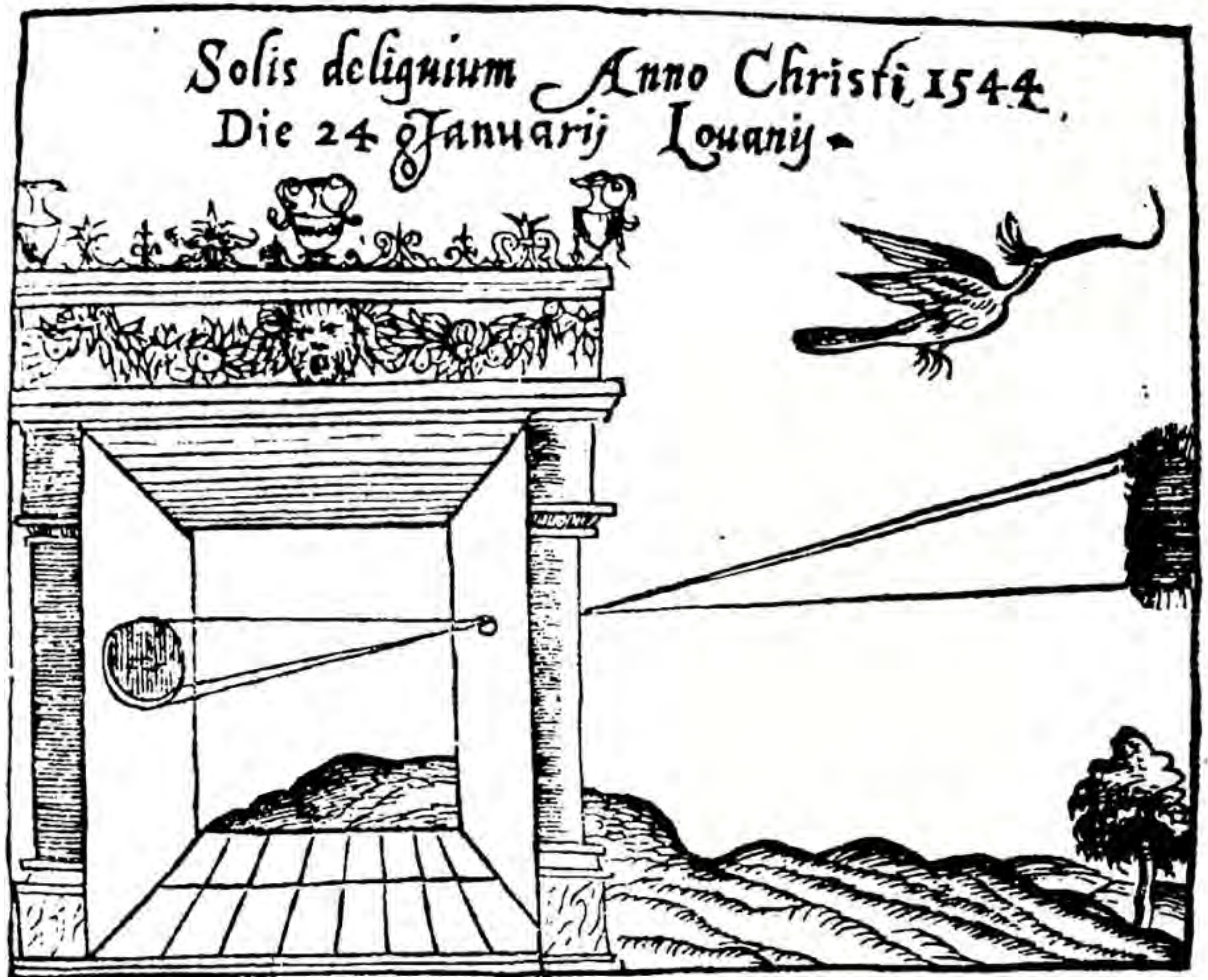



Points in the world

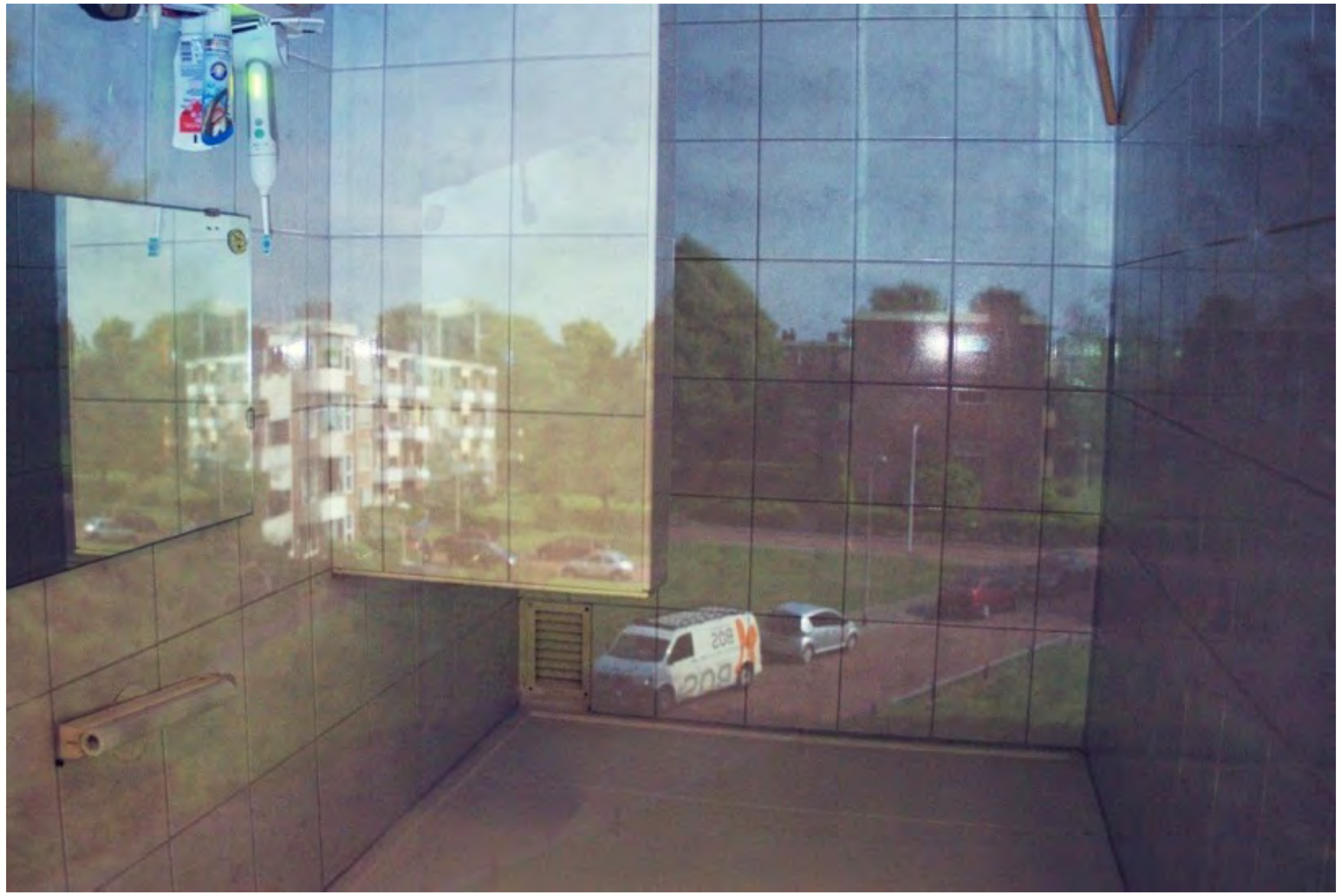

Image plane
Points in
the world


Image plane

The pinhole camera



Pinhole images



Camera obscura 2011

1banaan | CC A2.0

Camera obscura! 2011

half alive - soo zzzz | CC A2.0

The world's largest pinhole camera

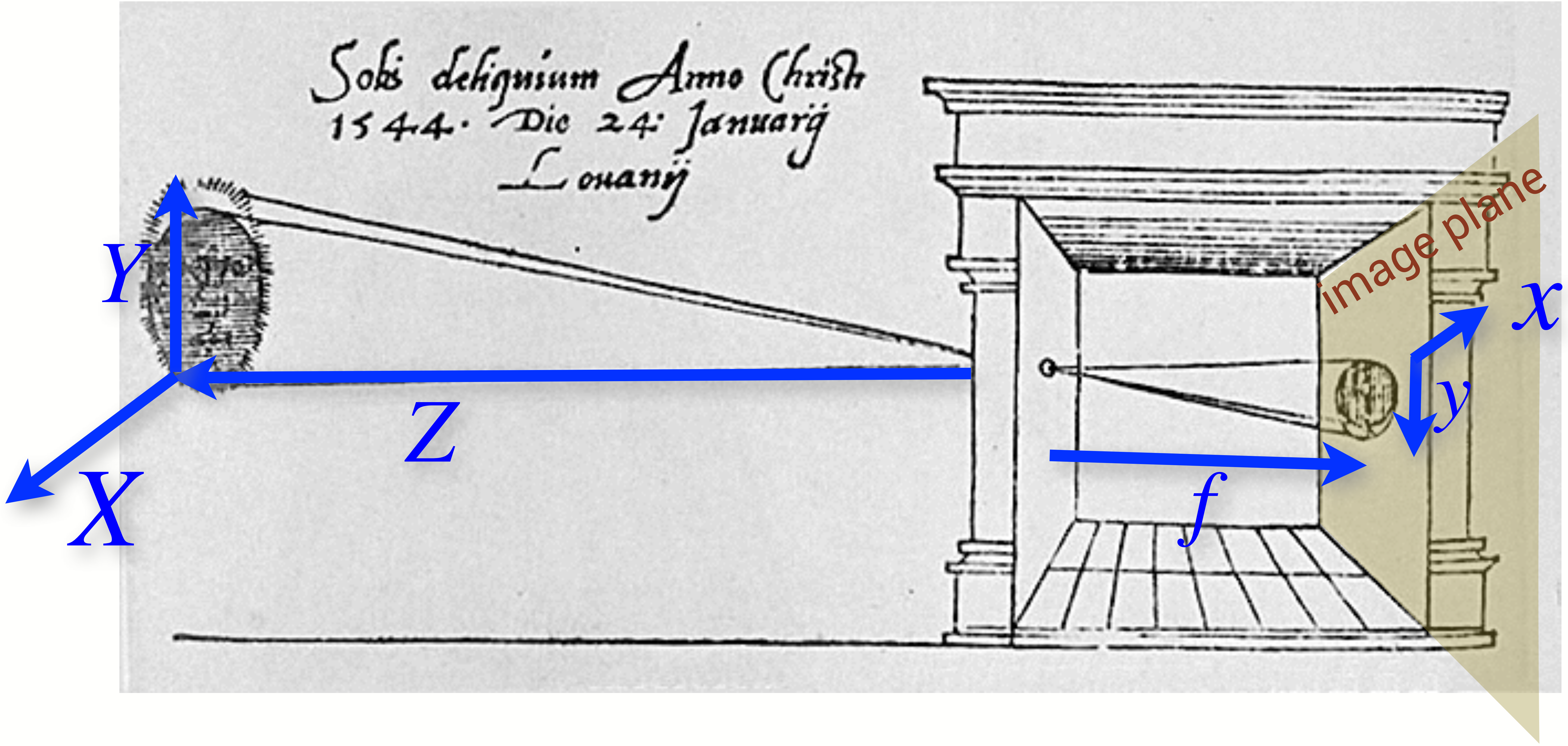
Members of The Legacy Project Collective 2008

Jerry Keane | used with permission

Douglas McCulloh | CC-BY-SA-3.0-2.5-2.0-1.0, via Wikimedia Commons


Image plane

Simple imaging

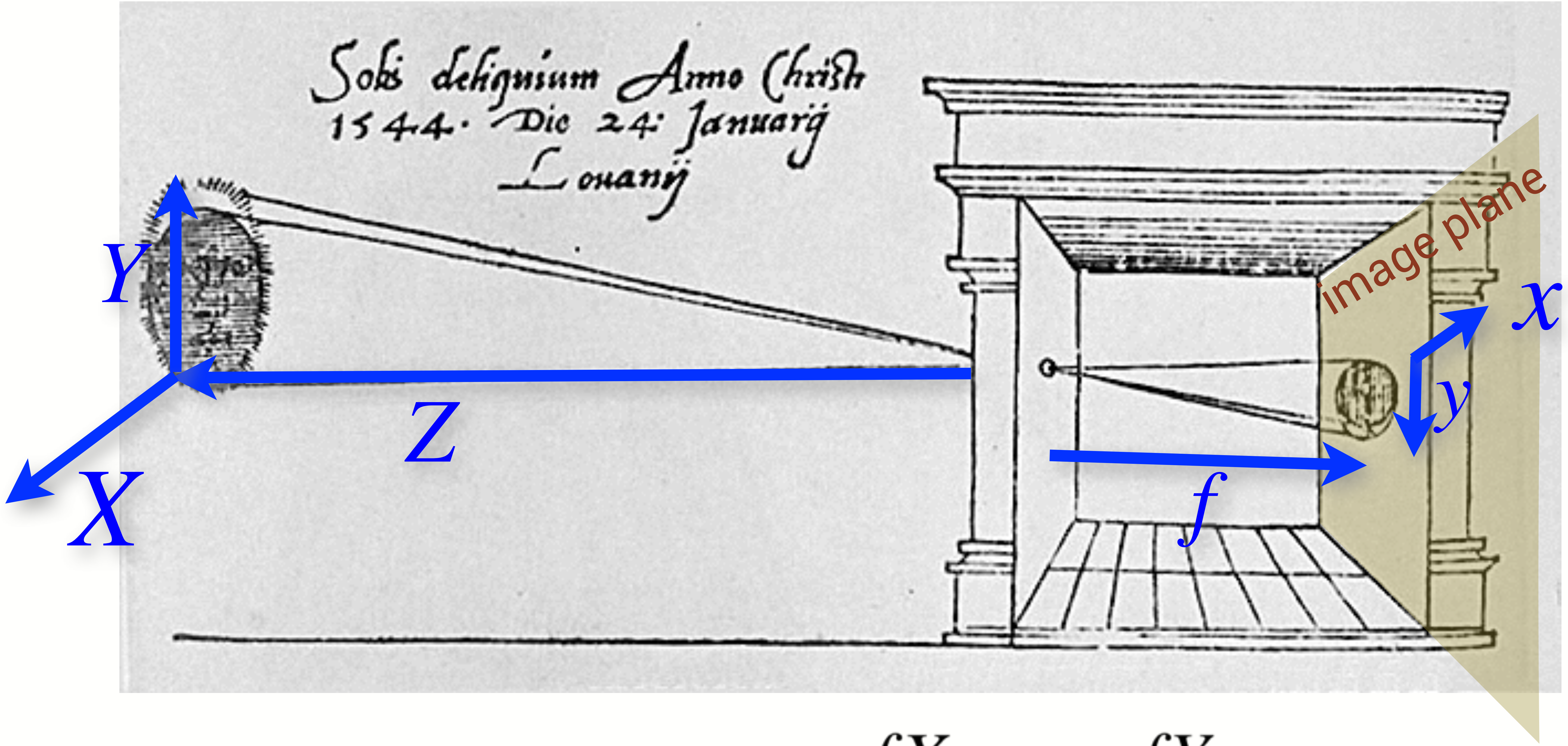


$$\frac{Y}{Z} = \frac{y}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$

- Similar triangles
- Image formation is the mapping of scene points (X,Y,Z) to the image plane (x,y)

Simple imaging



$$\frac{Y}{Z} = \frac{y}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$

$$x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

$$(X, Y, Z) \mapsto (x, y)$$

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

- 3D to 2D
- Perspective projection

With kind permission of

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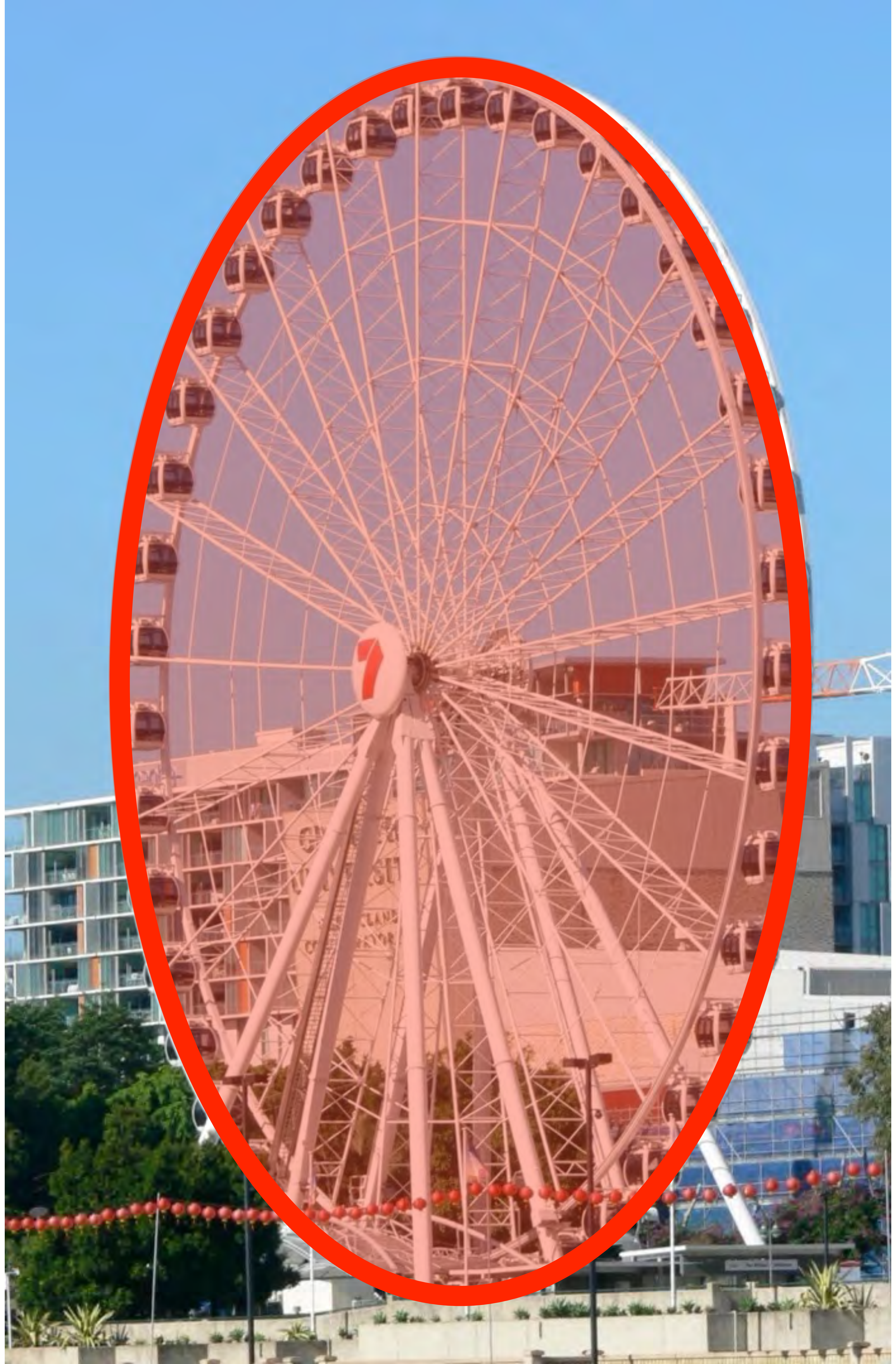
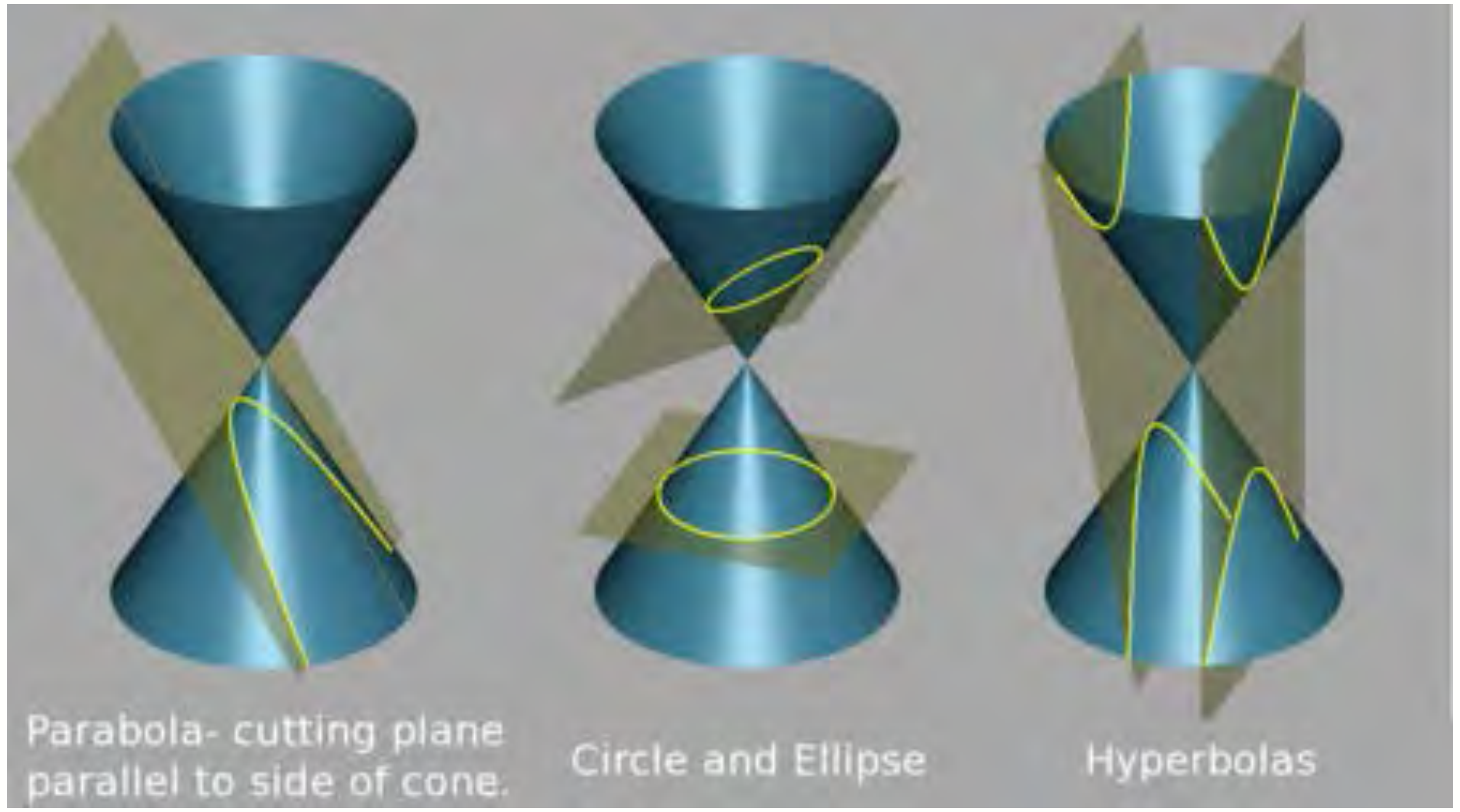


Perspective projection

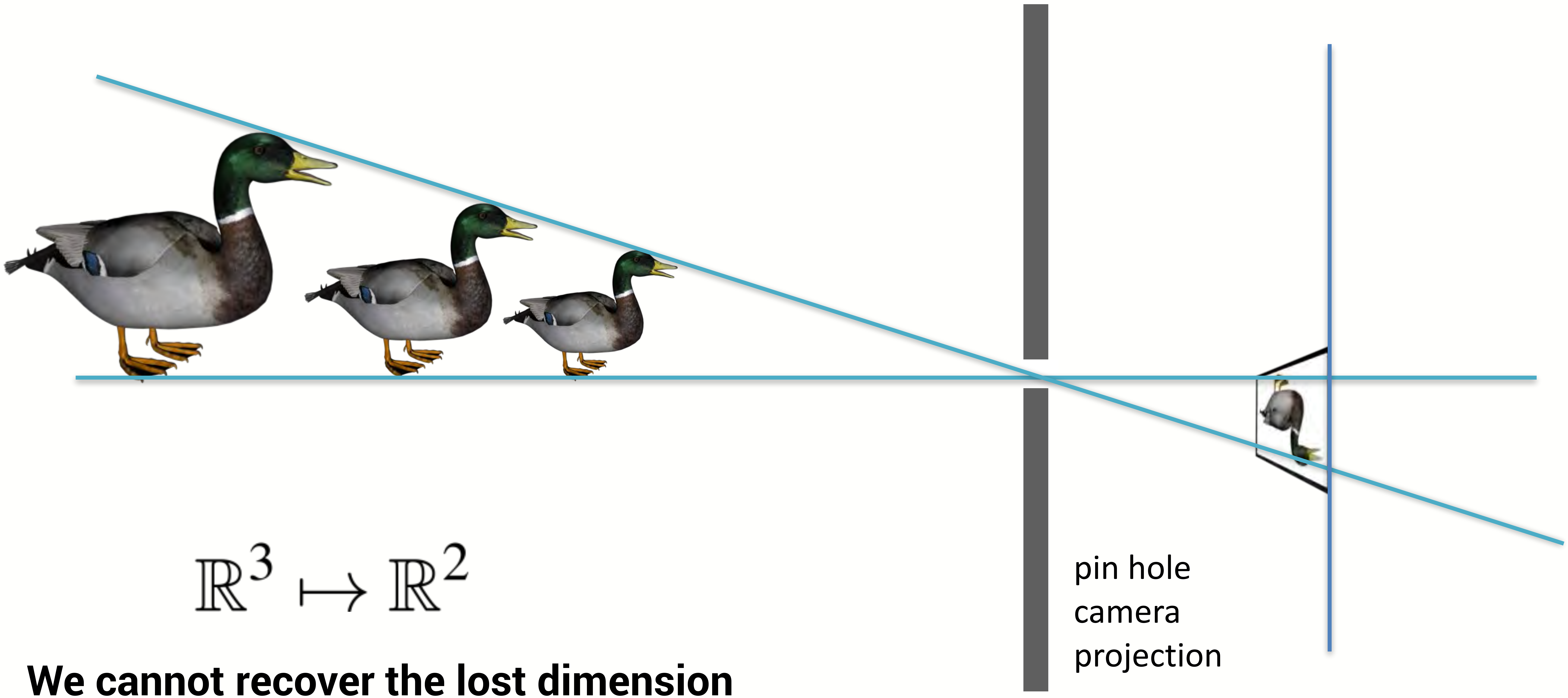
Maps

- Lines → lines
  - ↪ parallel lines not necessarily parallel
  - ↪ angles are not preserved
- Conics → conics



Duk at the English language Wikipedia

No unique inverse



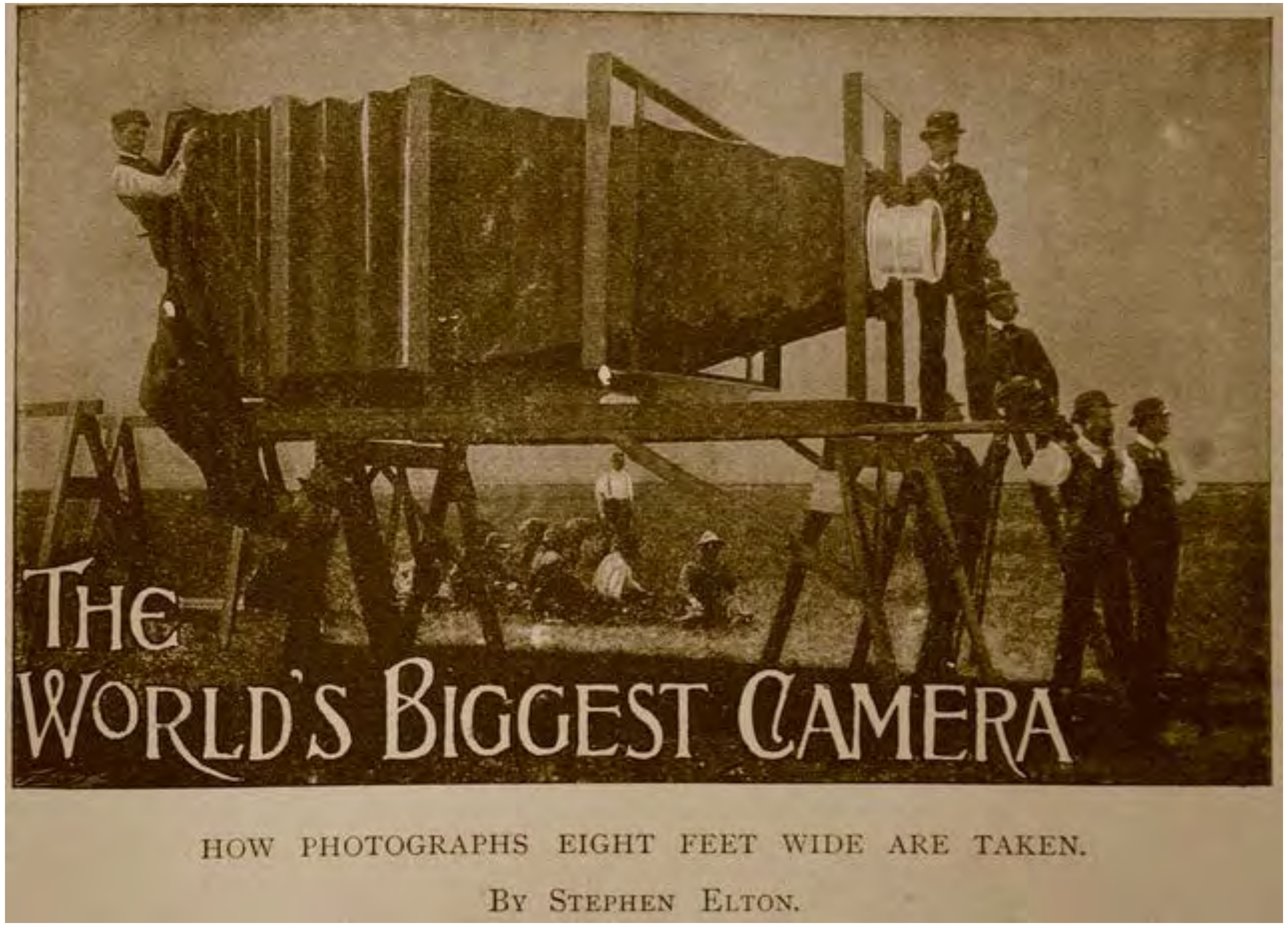
$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

We cannot recover the lost dimension

- Any 2D image could be generated by one of an infinite number of possible 3D worlds


Image plane
inherit cha

Use a lens to gather more light

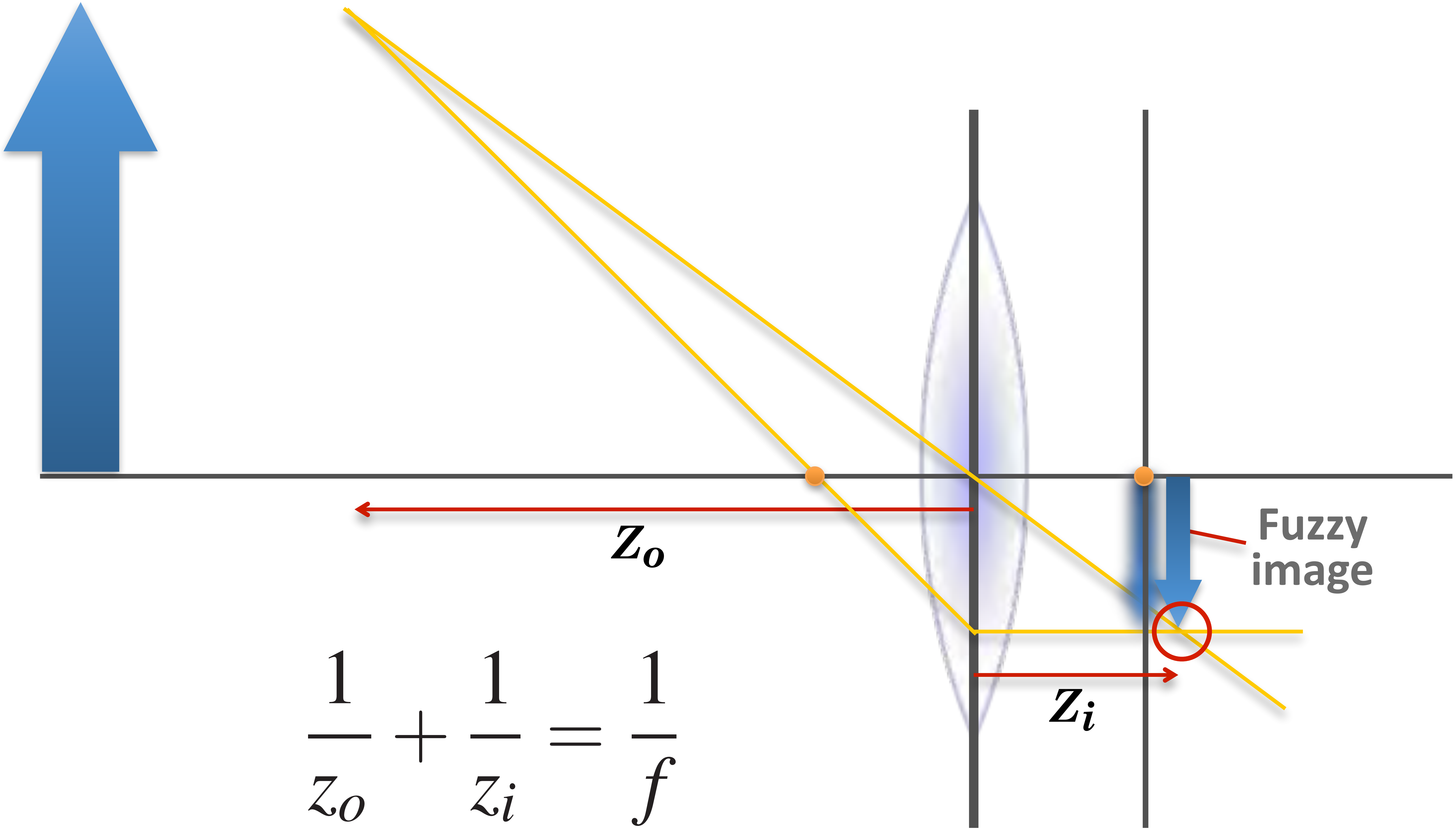


George R. Lawrence 1900


f
Ø

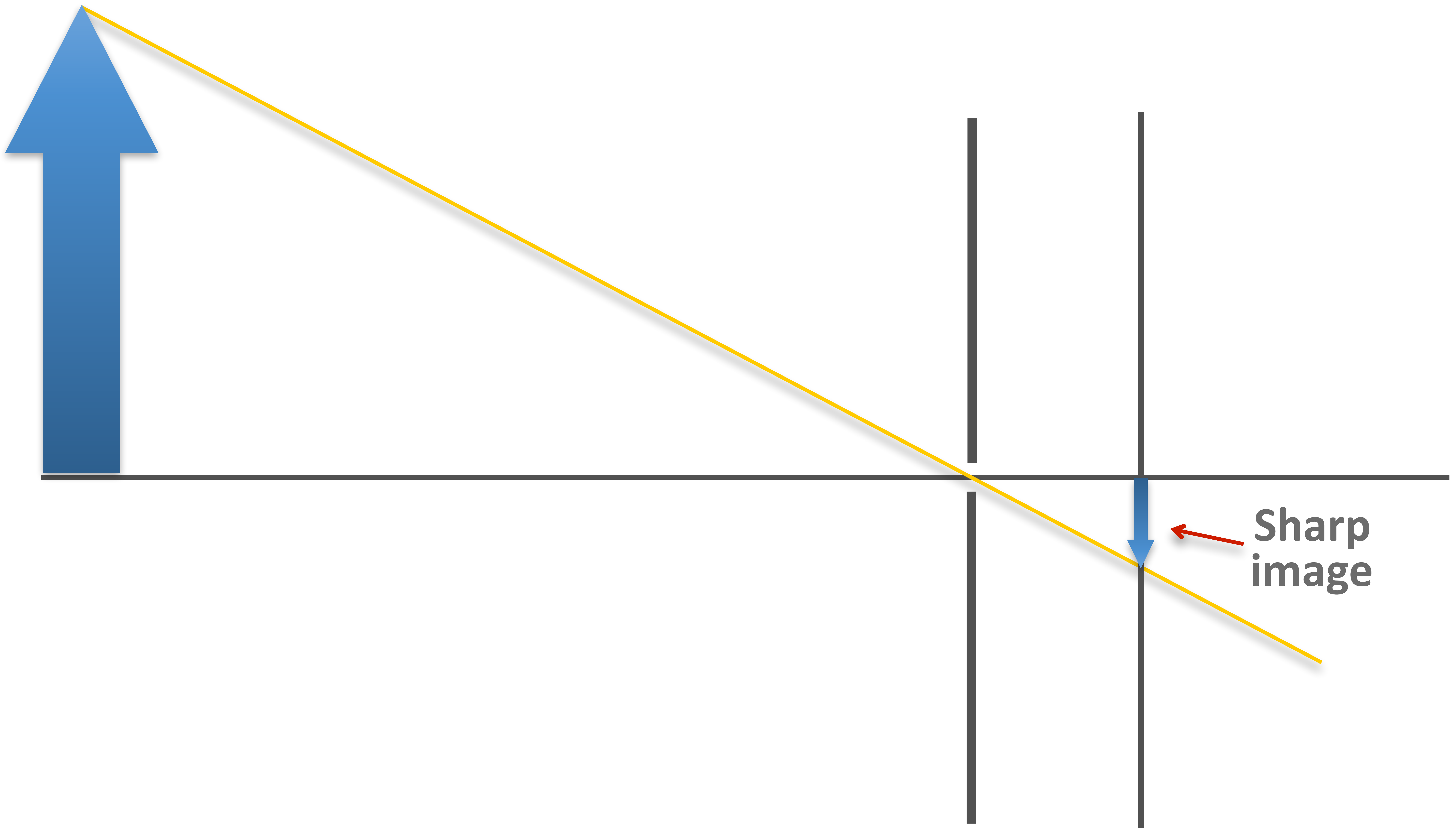
$F = \frac{f}{\phi}$
f
ø
F/1.4 lens
m
1 : 1.4
5 7
NIKKOR
50mm
1:1.4
5755899

$$\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$$

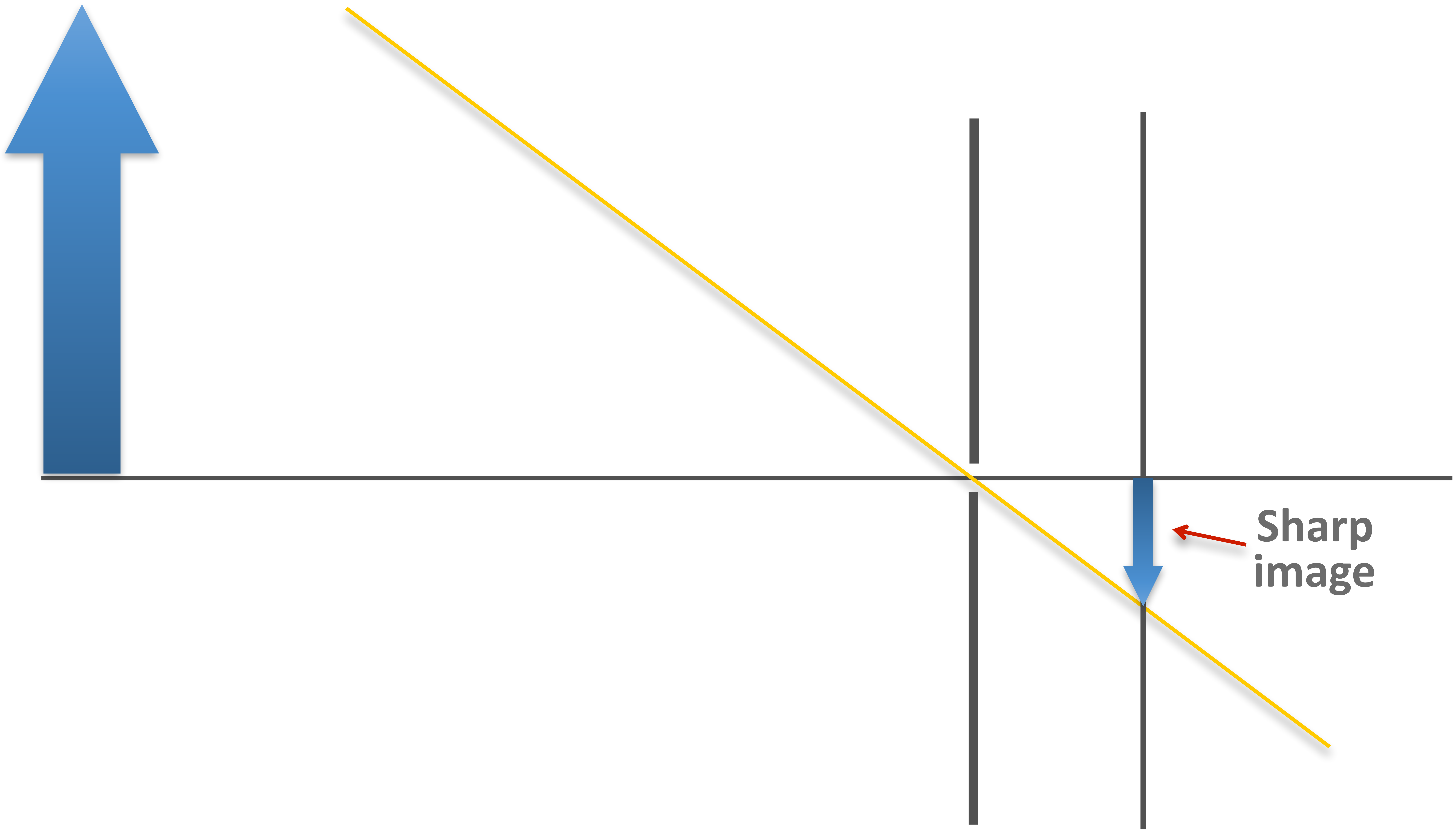


$$\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$$

Pinhole camera doesn't need focus

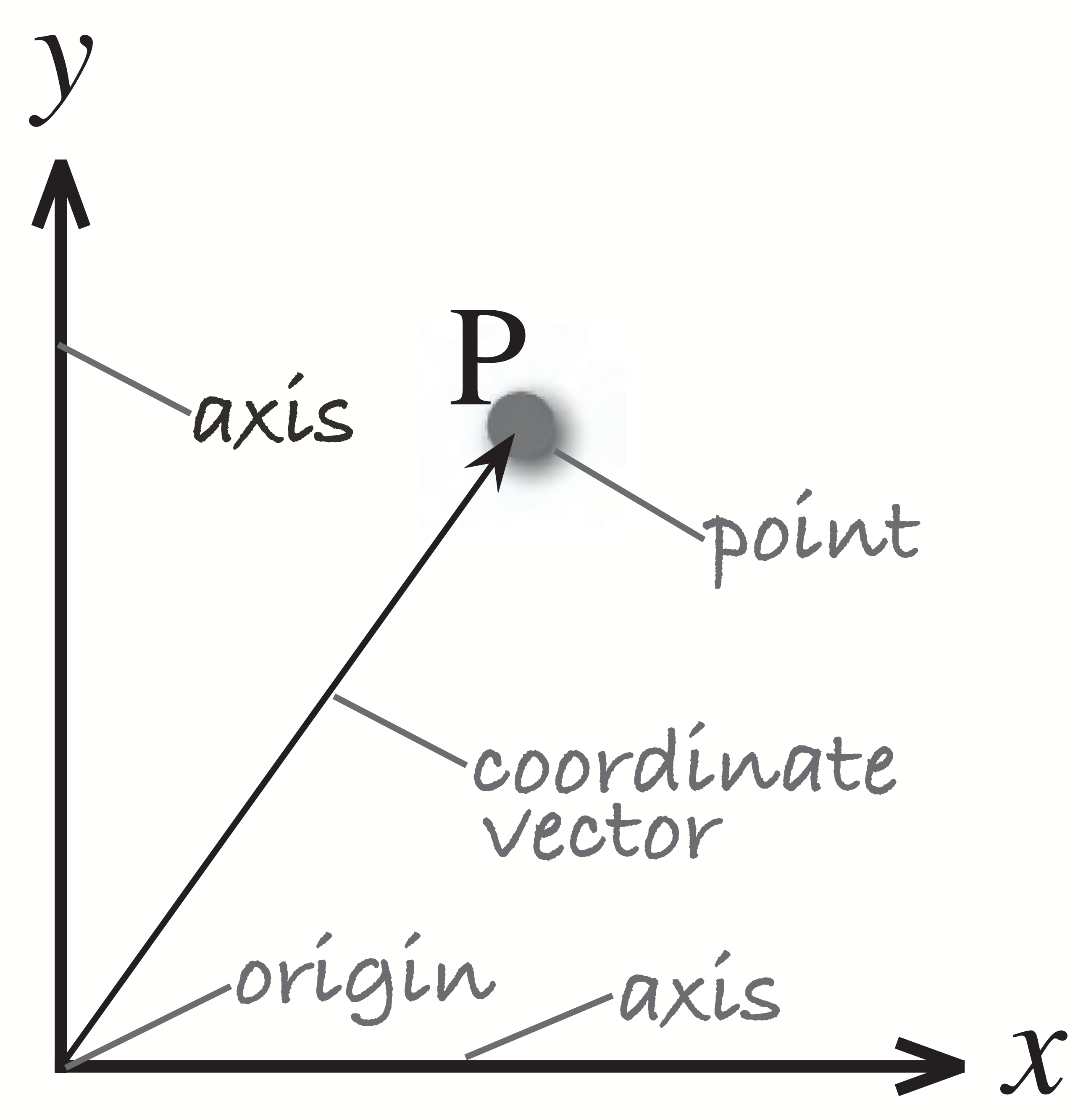


Pinhole camera doesn't need focus



Quick **geometry** recap

- Some familiar concepts from geometry:
  - ➥Euclidean plane
    - – a non-curved space where the rules of Euclidean geometry apply
  - ➥Cartesian coordinates
    - – distances to a point with respect to the origin and measured along orthogonal axes



Homogeneous coordinates

- Cartesian → homogeneous

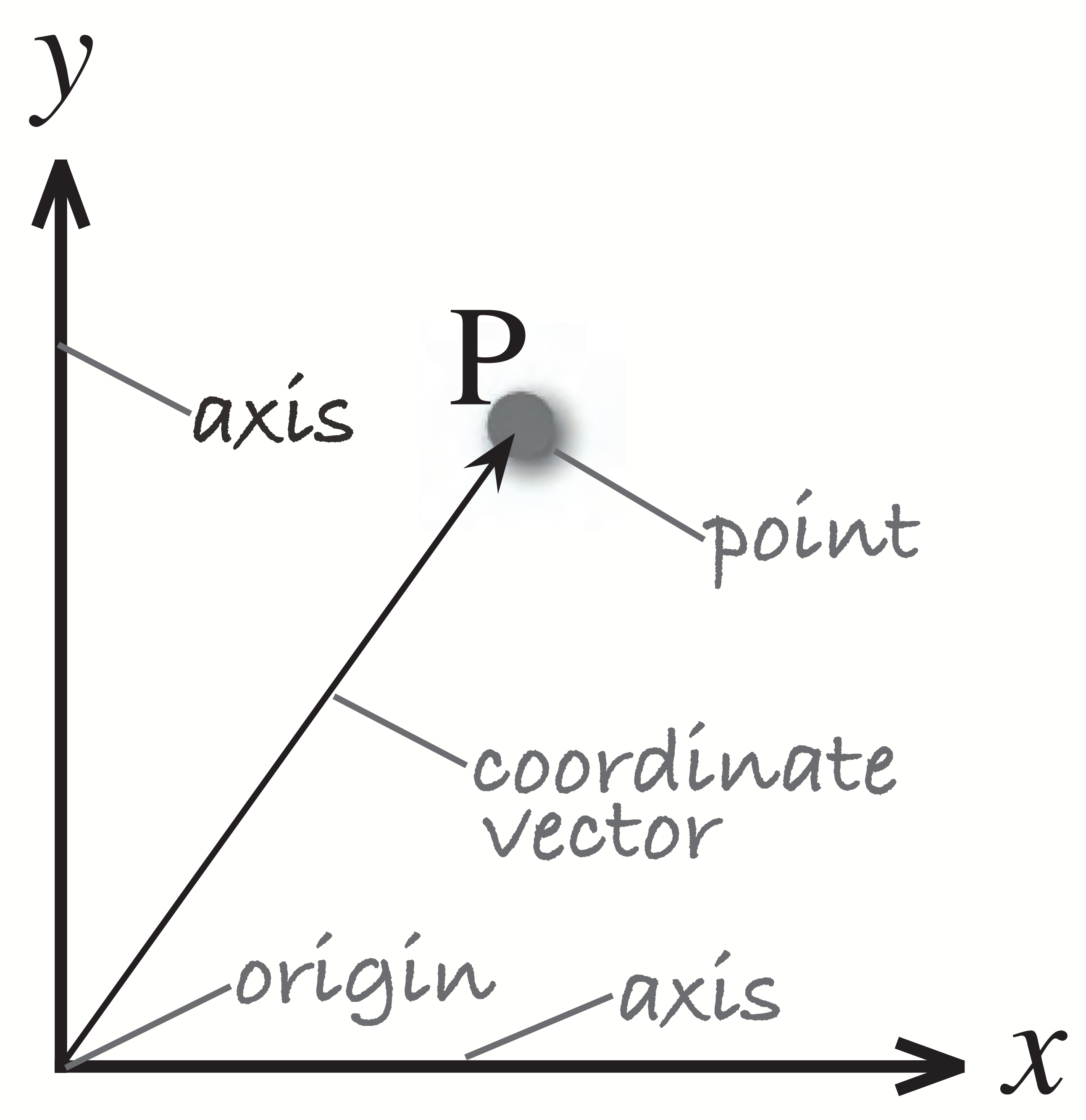
$$P = (x, y) \qquad \tilde{P} = (x, y, 1)$$

$$P \in \mathbb{R}^2 \qquad \tilde{P} \in \mathbb{P}^2$$

- homogeneous → Cartesian

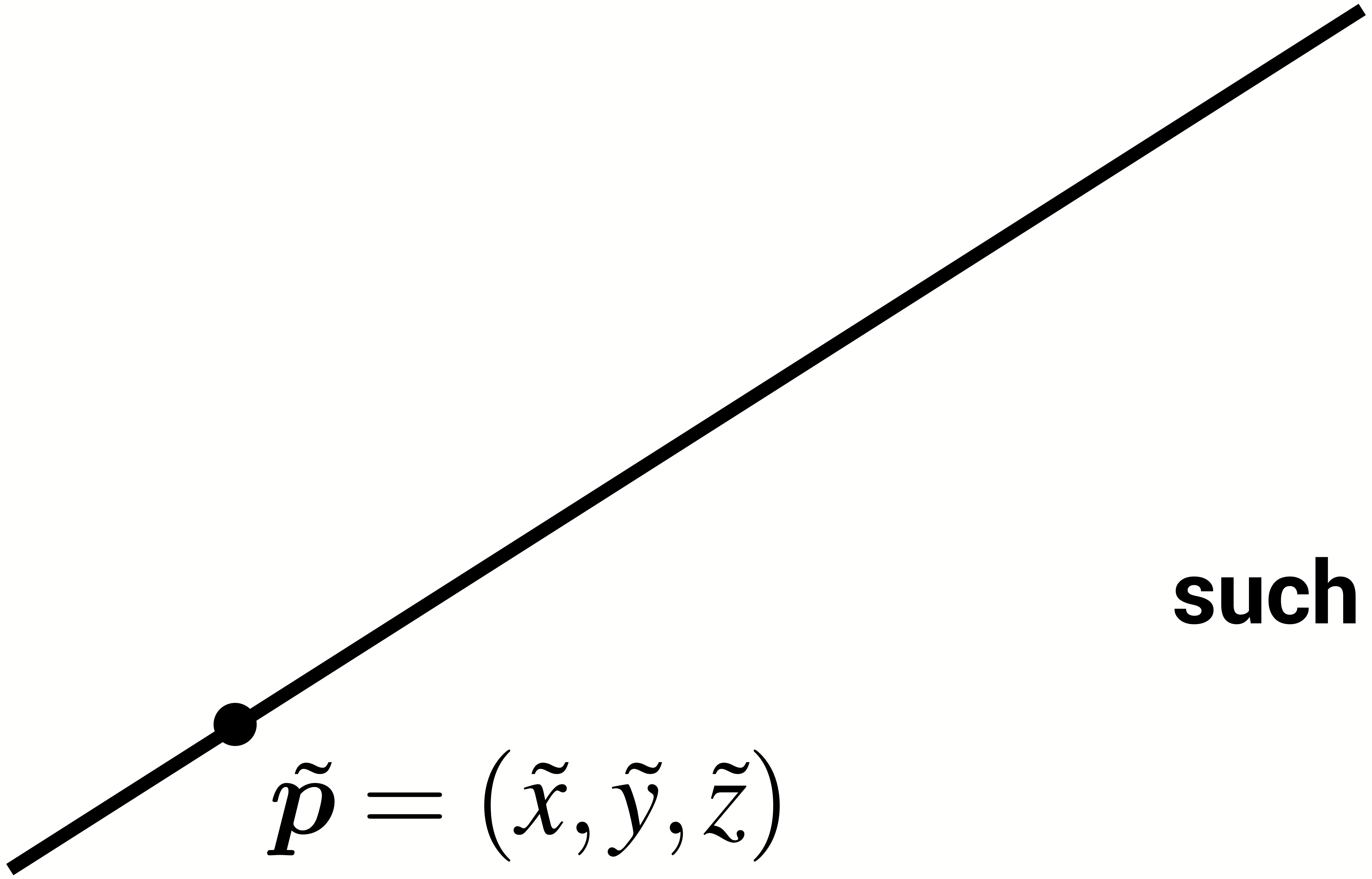
$$\tilde{P} = (\tilde{x}, \tilde{y}, \tilde{z}) \qquad P = (x, y)$$

$$x = \frac{\tilde{x}}{\tilde{z}}, \; y = \frac{\tilde{y}}{\tilde{z}}$$



Lines and points are duals

A line in homogeneous form



$$\tilde{\boldsymbol{p}} = (\tilde{x}, \tilde{y}, \tilde{z})$$

$$\tilde{\ell} = (l_1, l_2, l_3)$$

Point equation of a line

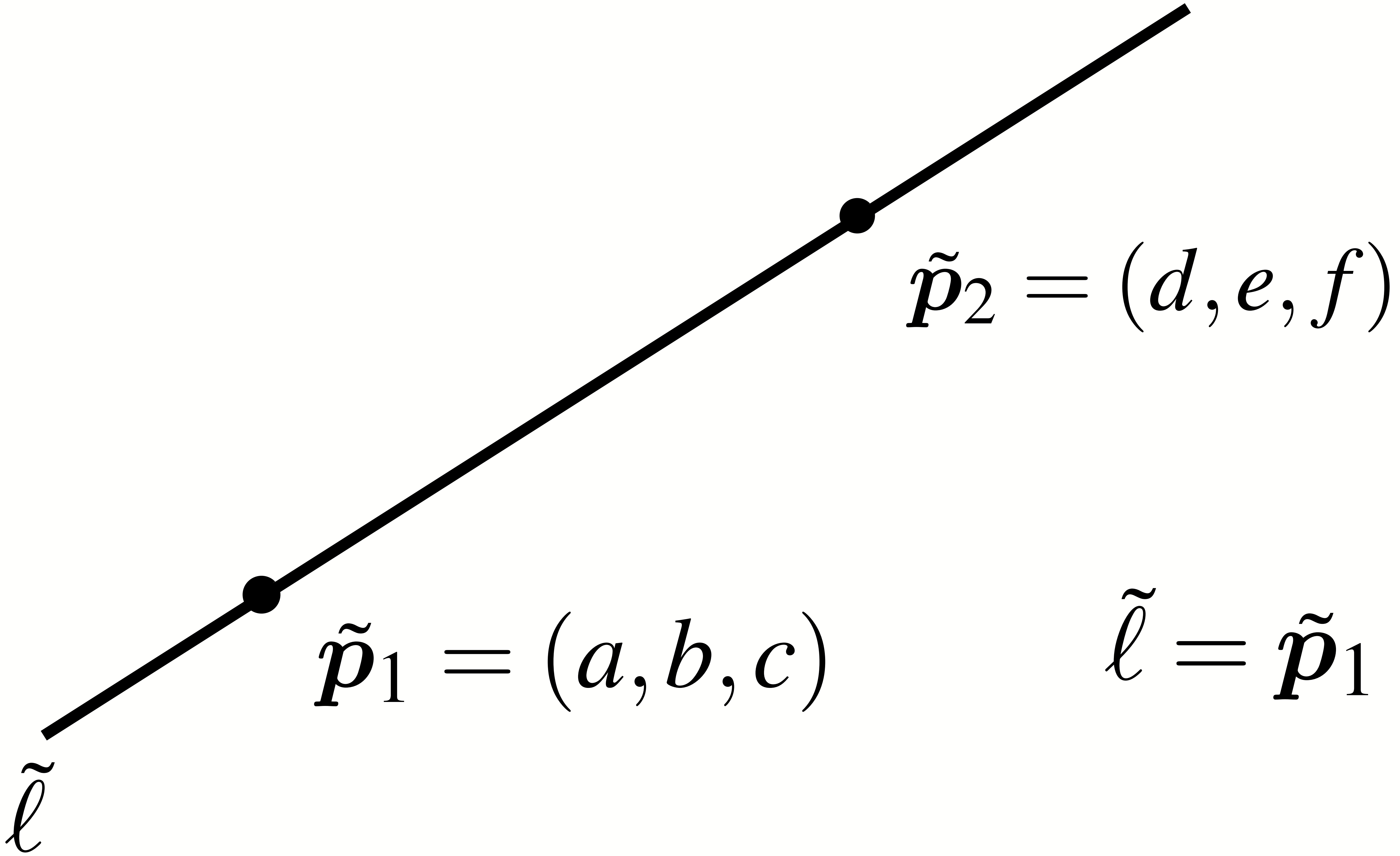
such that $\rightarrow$ $\tilde{\ell}^T \tilde{\boldsymbol{p}} = 0$

$$\ell_1 \tilde{x} + \ell_2 \tilde{y} + \ell_3 \tilde{z} = 0$$

contrast to

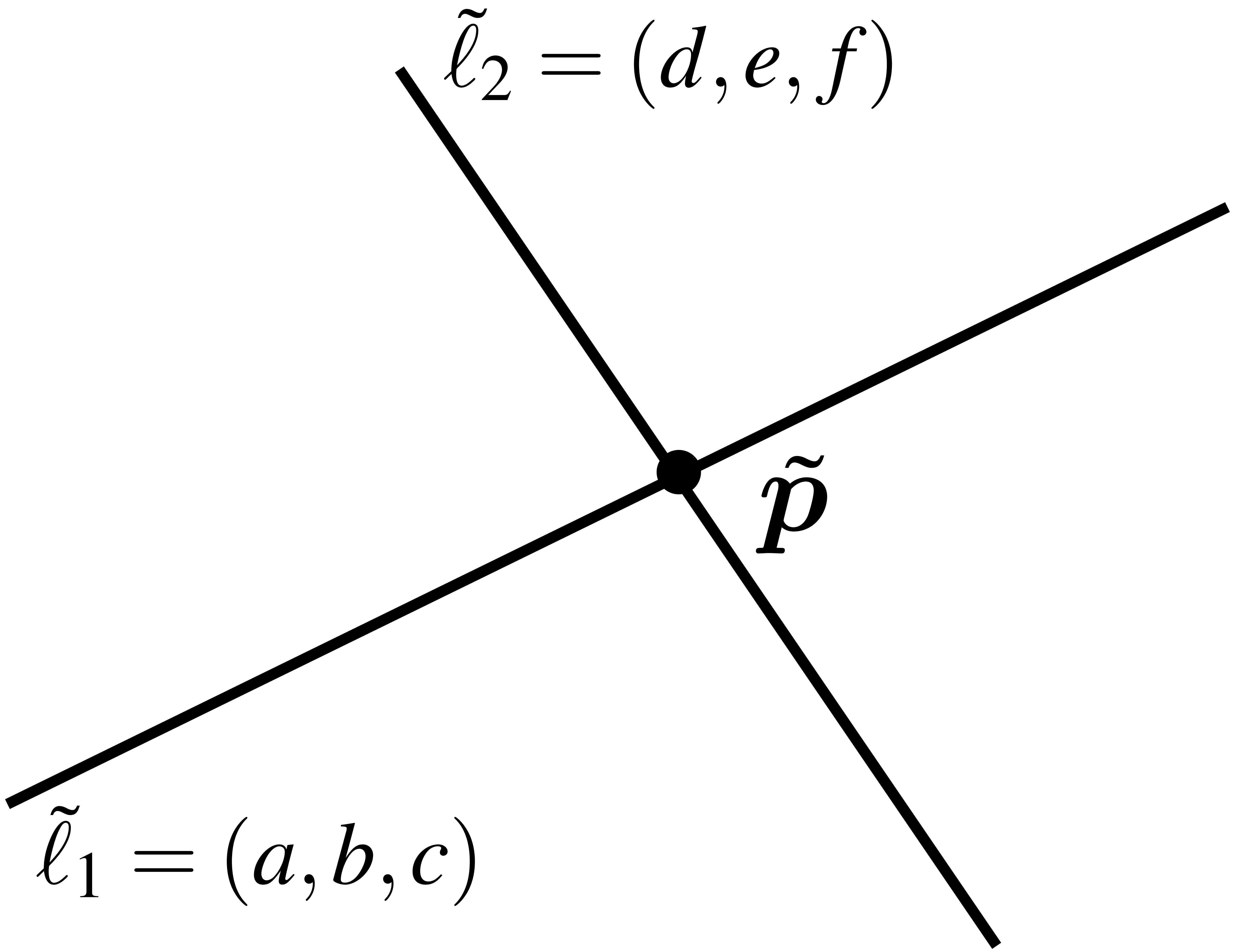
$$y = mx + c$$

Line joining points



$$\tilde{\ell} = \tilde{\boldsymbol{p}}_1 \times \tilde{\boldsymbol{p}}_2$$

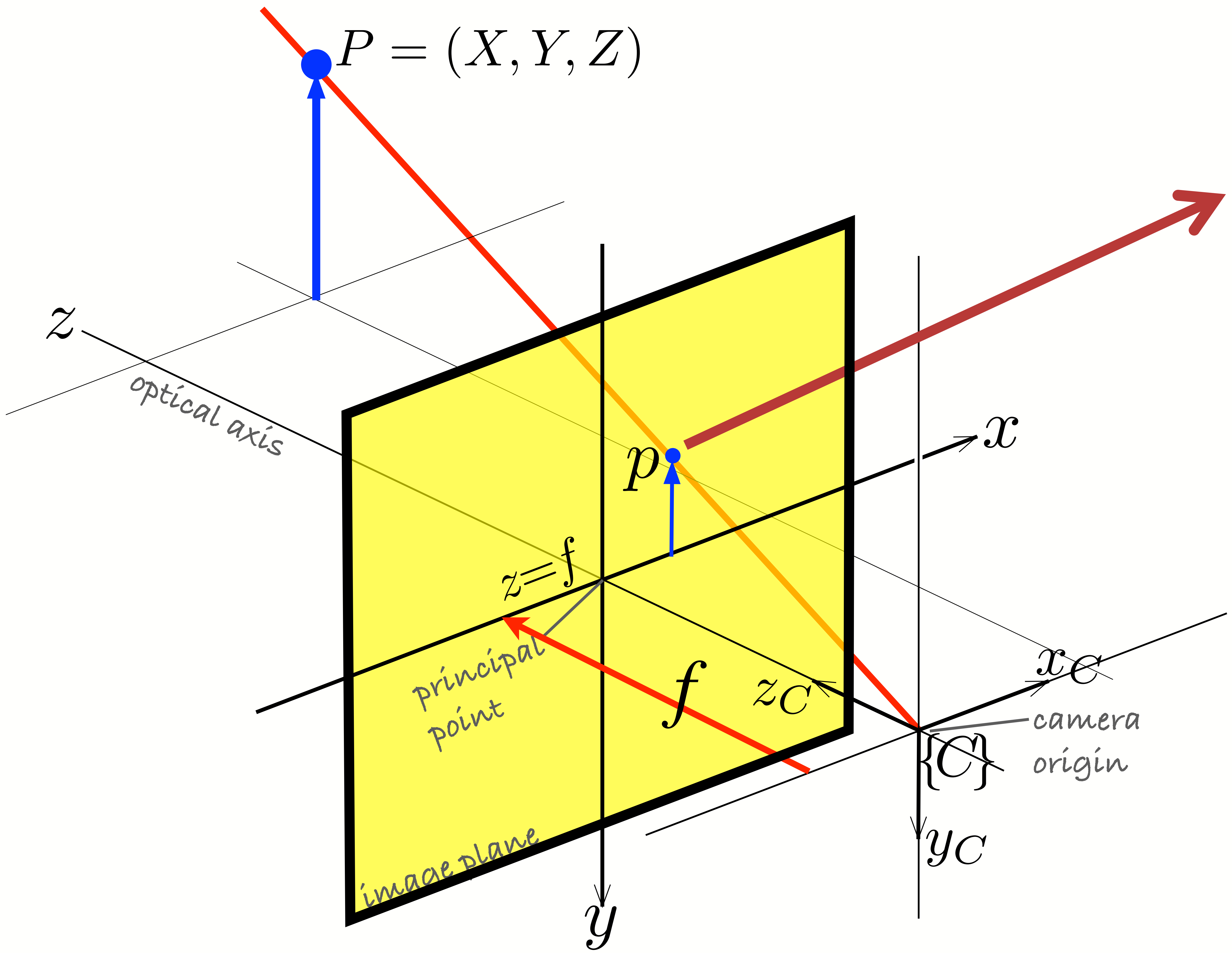
Intersecting lines



$$\tilde{\boldsymbol{p}} = \tilde{\ell}_1 \times \tilde{\ell}_2$$

line equation of a point

Central projection model



$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

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Pin-hole model in homogeneous form

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\tilde{x} = fX, \tilde{y} = fY, \tilde{z} = Z$$

$$x = \frac{\tilde{x}}{\tilde{z}}, \; y = \frac{\tilde{y}}{\tilde{z}}$$

$$\Rightarrow x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

- Perspective transformation, with the pesky divide by Z, is **linear** in homogeneous coordinate form.

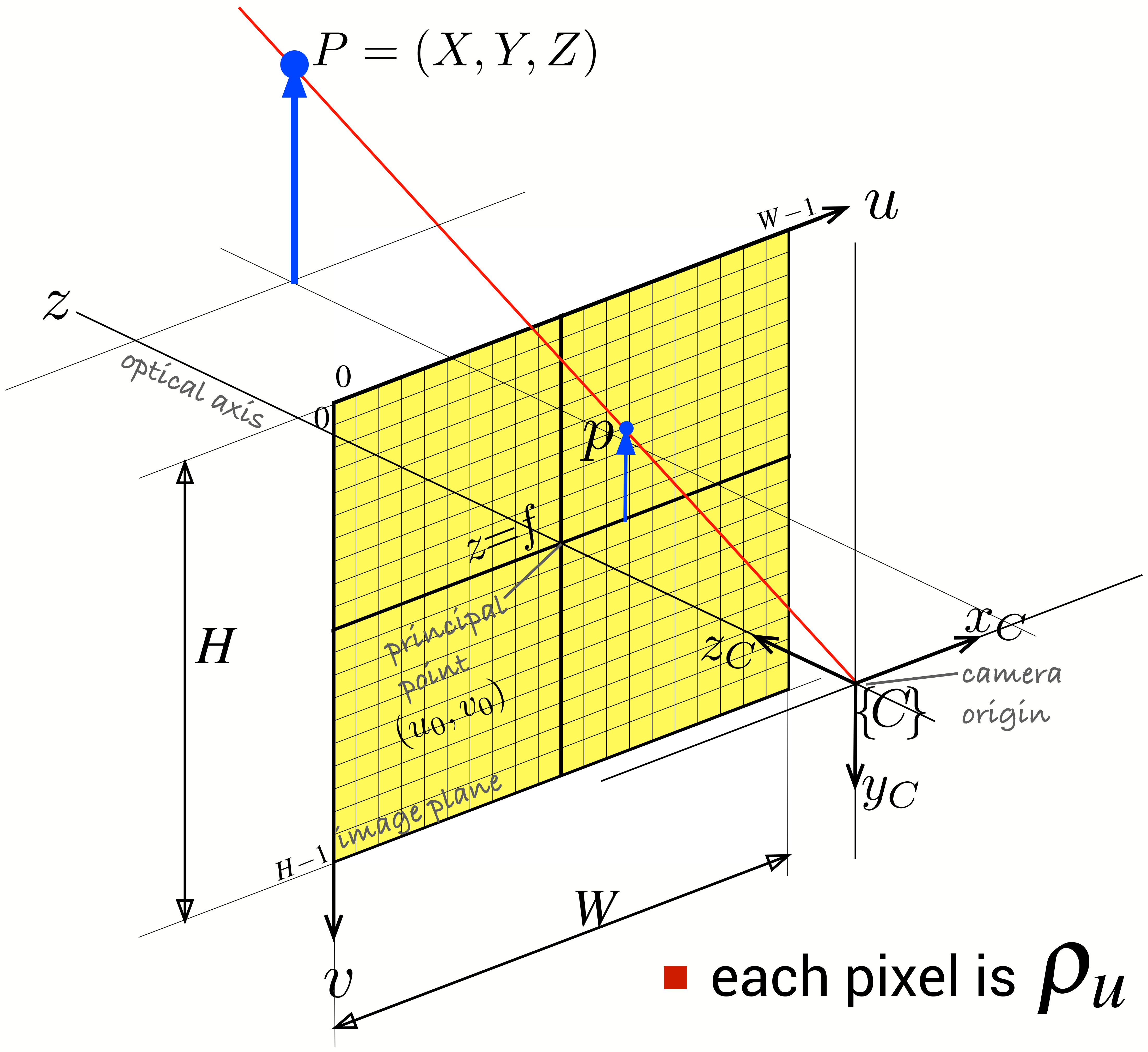
$$\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0\\ 0 & f & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\\ 1\end{pmatrix}$$

3D to 2D

$$\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix} f & 0 & 0 & 0\\ 0 & f & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

scaling/ zooming

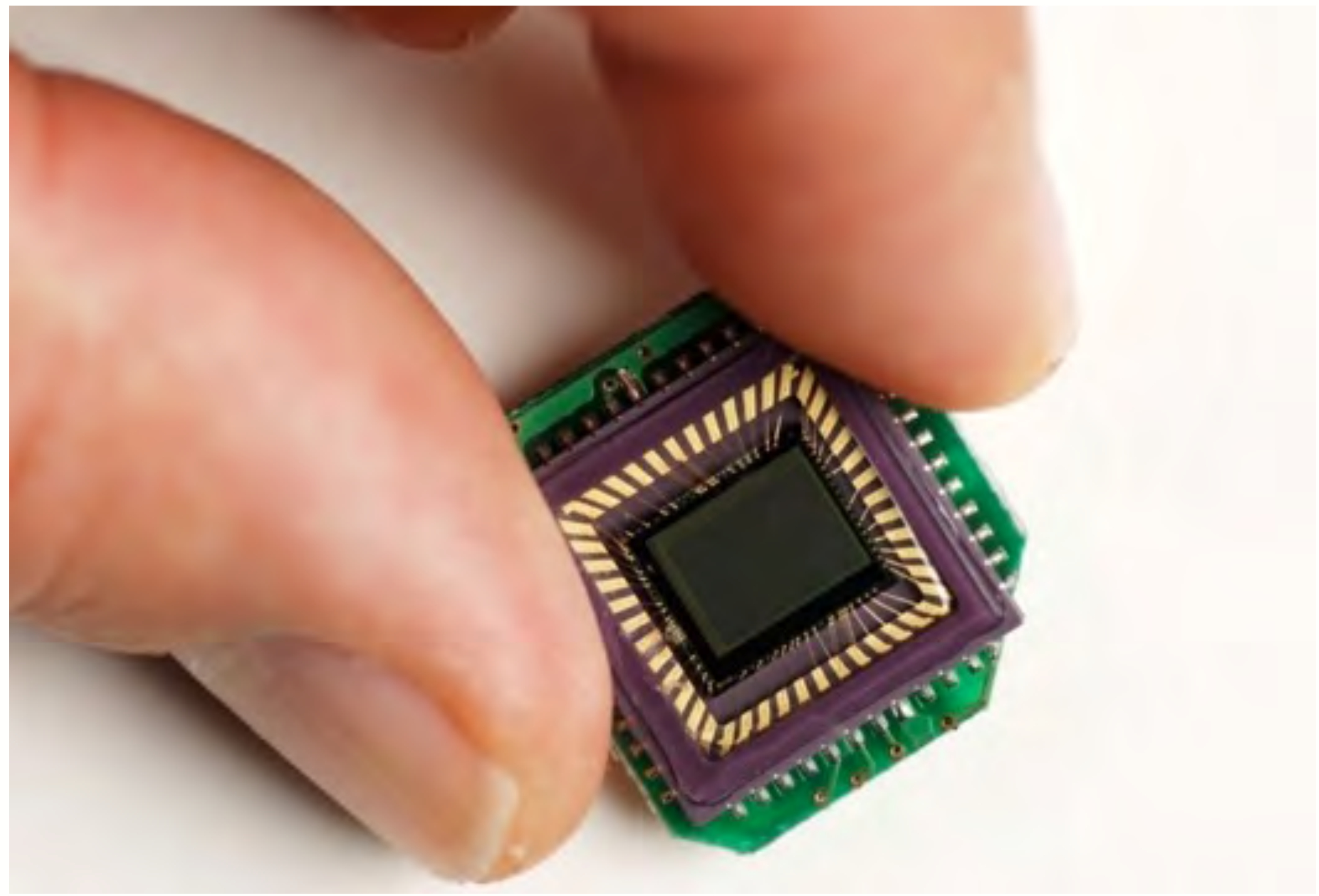
Change of coordinates



- scale point from metres to pixels
- shift the origin to top left corner

$$u = \frac{x}{\rho_u} + u_0$$

$$v = \frac{y}{\rho_v} + v_0$$



- each pixel is $\rho_u \times \rho_v$

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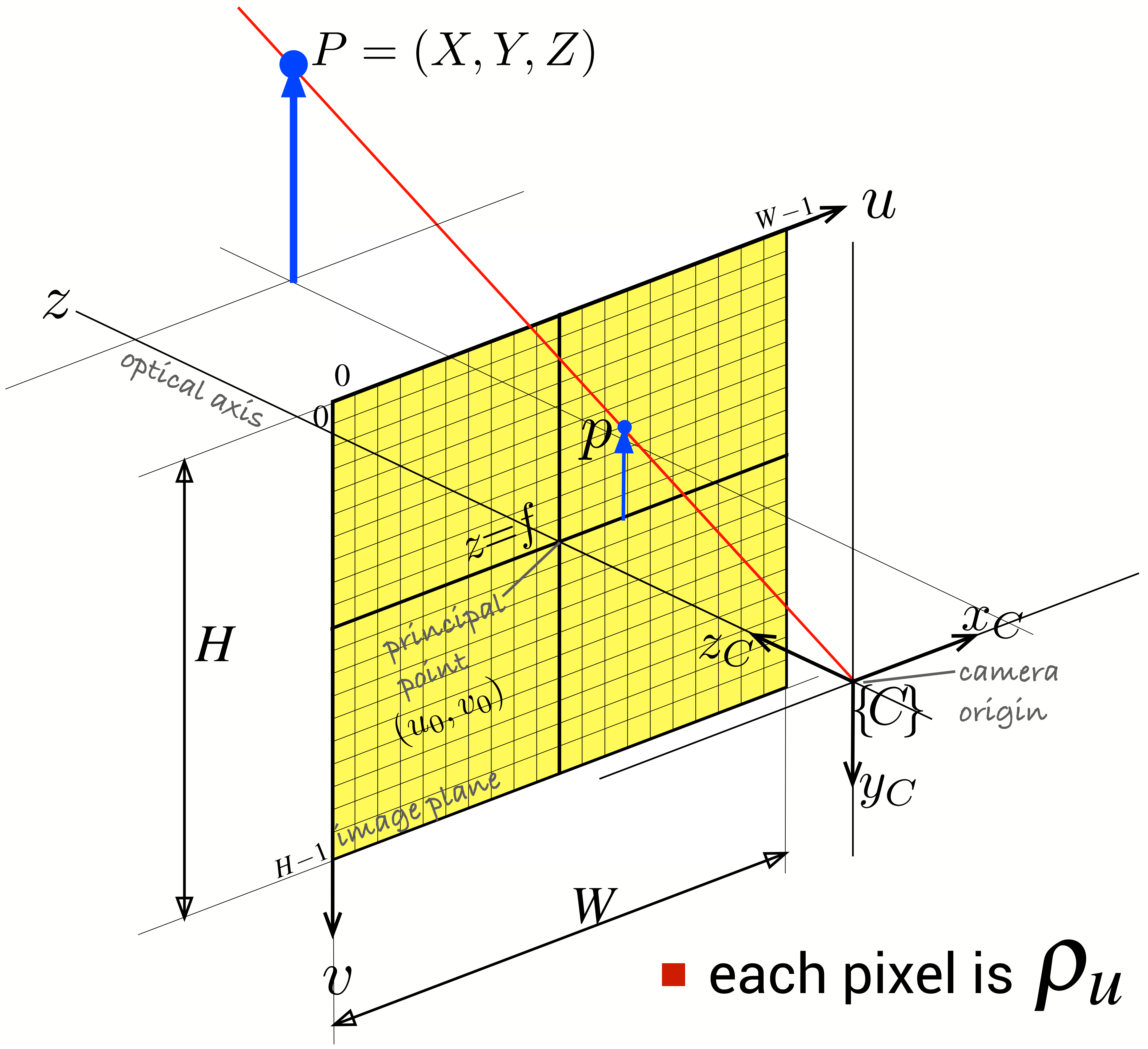
Change of **coordinates**



- scale point from metres to pixels
- shift the origin to top left corner

$$u = \frac{x}{\rho_u} + u_0$$

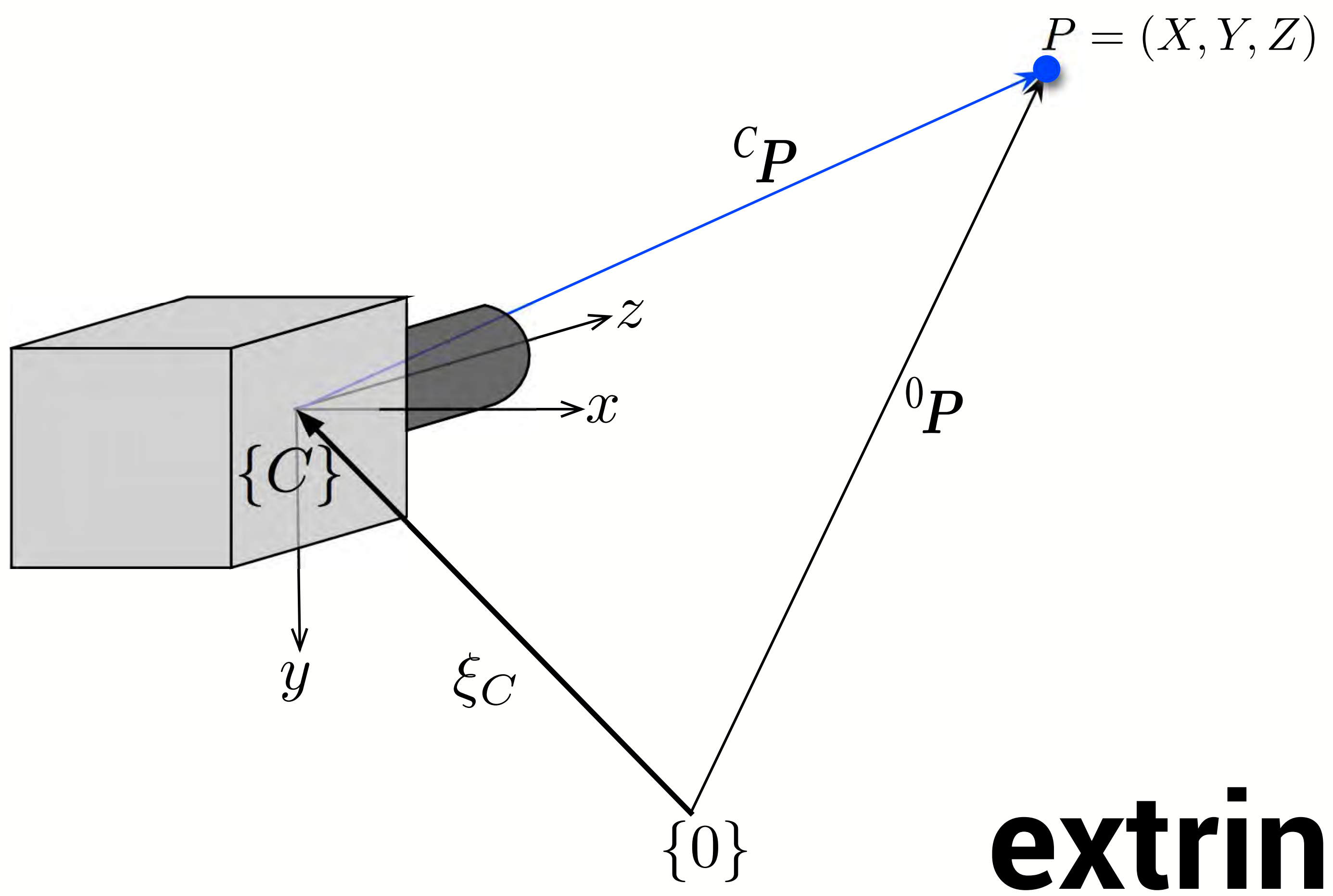
$$v = \frac{y}{\rho_v} + v_0$$

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix}$$

- each pixel is $\rho_u \times \rho_v$

$$p = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \tilde{u}/\tilde{w} \\ \tilde{v}/\tilde{w} \end{pmatrix}$$

Complete **camera model**



$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

extrinsic parameters

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \underbrace{\underbrace{\begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\mathbf{K}} \begin{pmatrix} \mathbf{R} & t \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1}}_{\mathbf{C}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \quad \ominus \xi_C$$

intrinsic parameters

camera matrix

Camera **matrix**

- Mapping points from **the world** to an **image (pixel) coordinate** is simply a **matrix multiplication** using **homogeneous coordinates**

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$u = \frac{\tilde{u}}{\tilde{w}}, v = \frac{\tilde{v}}{\tilde{w}}$$

Scale invariance

- Consider an arbitrary scalar scale factor

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \lambda \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- $\tilde{u}, \tilde{v}, \tilde{w}$ will all be scaled by λ
- but $u = \dfrac{\tilde{u}}{\tilde{w}}, v = \dfrac{\tilde{v}}{\tilde{w}}$
- so the result is unchanged

Normalized camera matrix

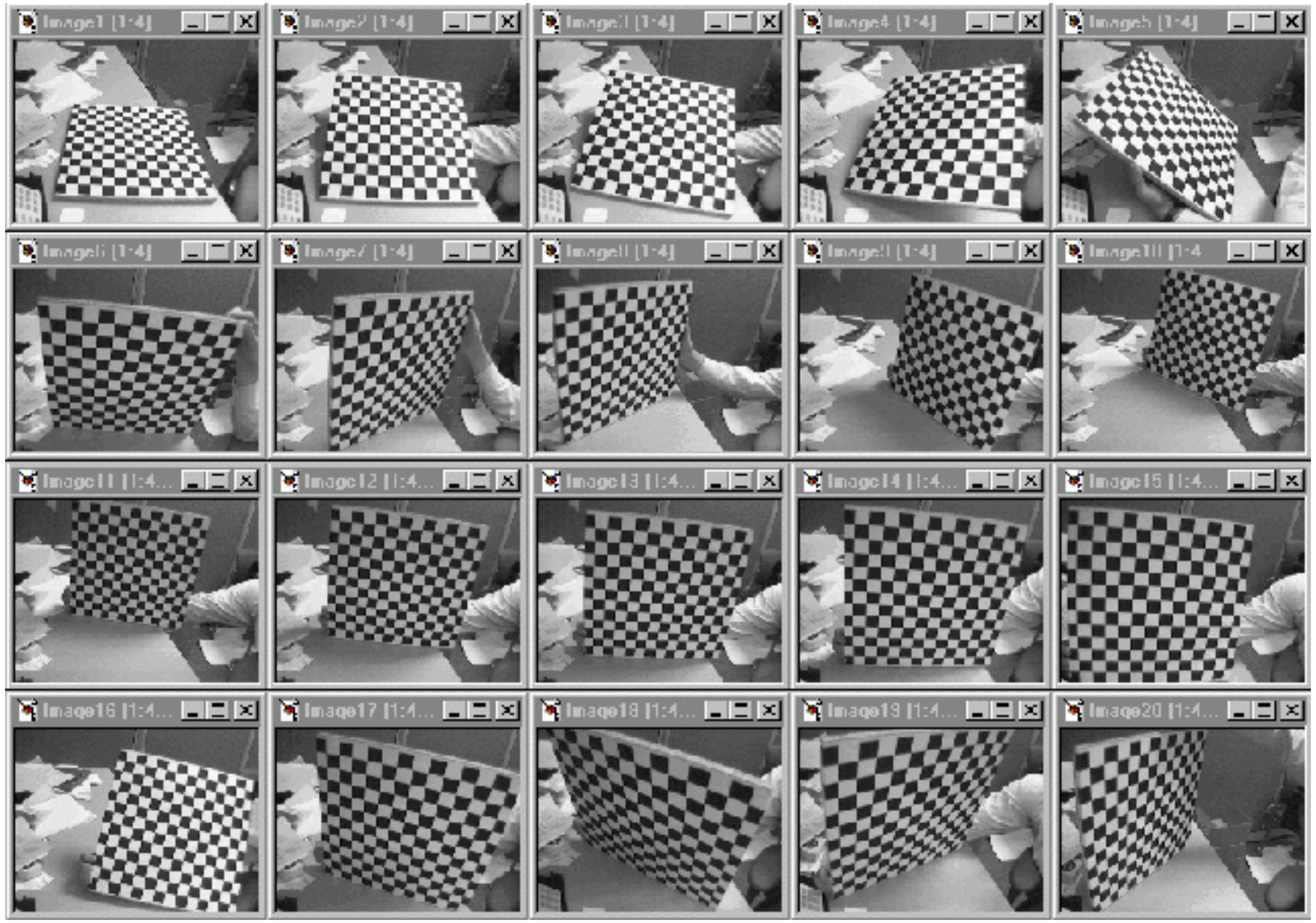
- Since scale factor is arbitrary we can fix the value of one element, typically C(3,4) to one.

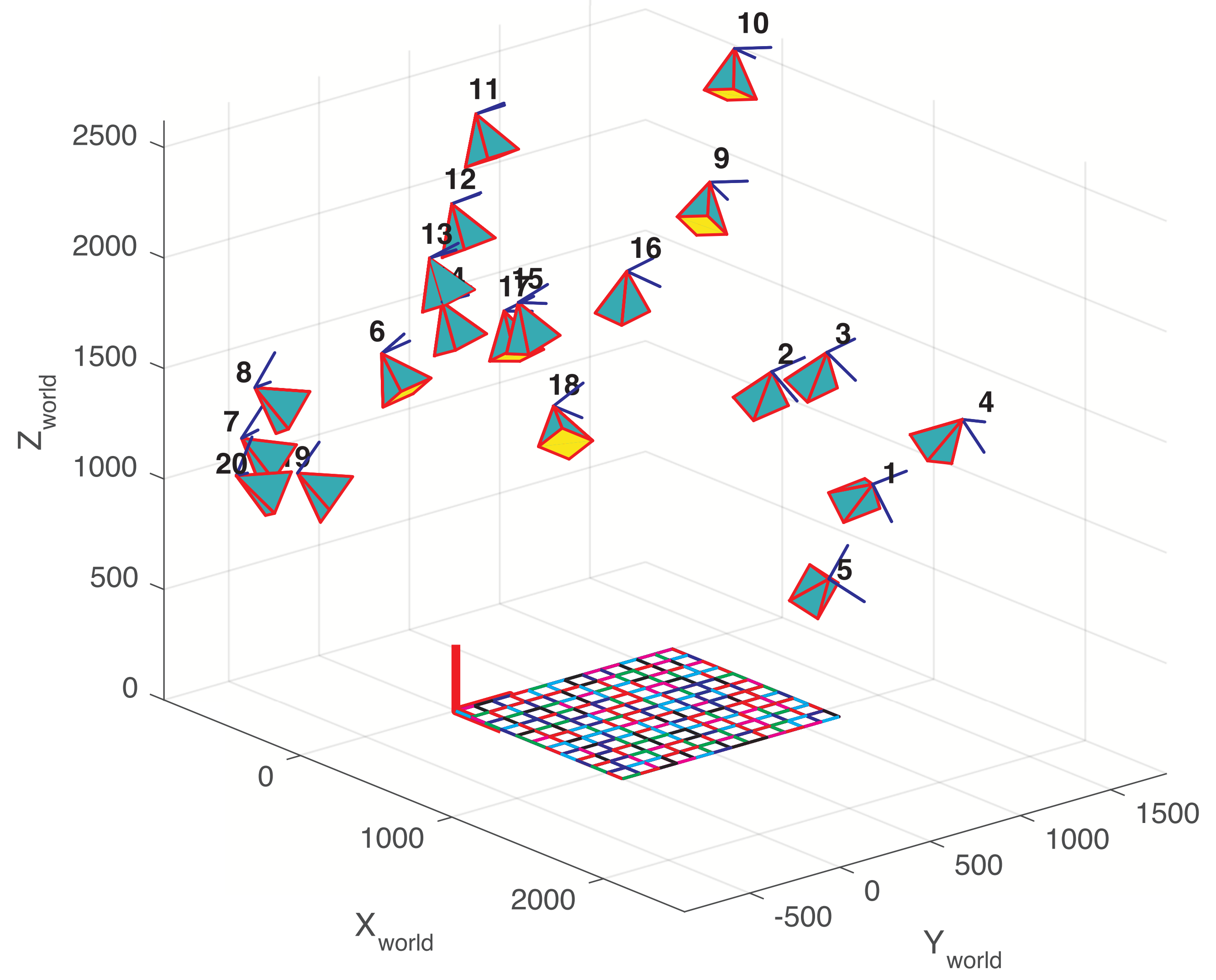
- focal length
- pixel size
- camera position
- & orientation

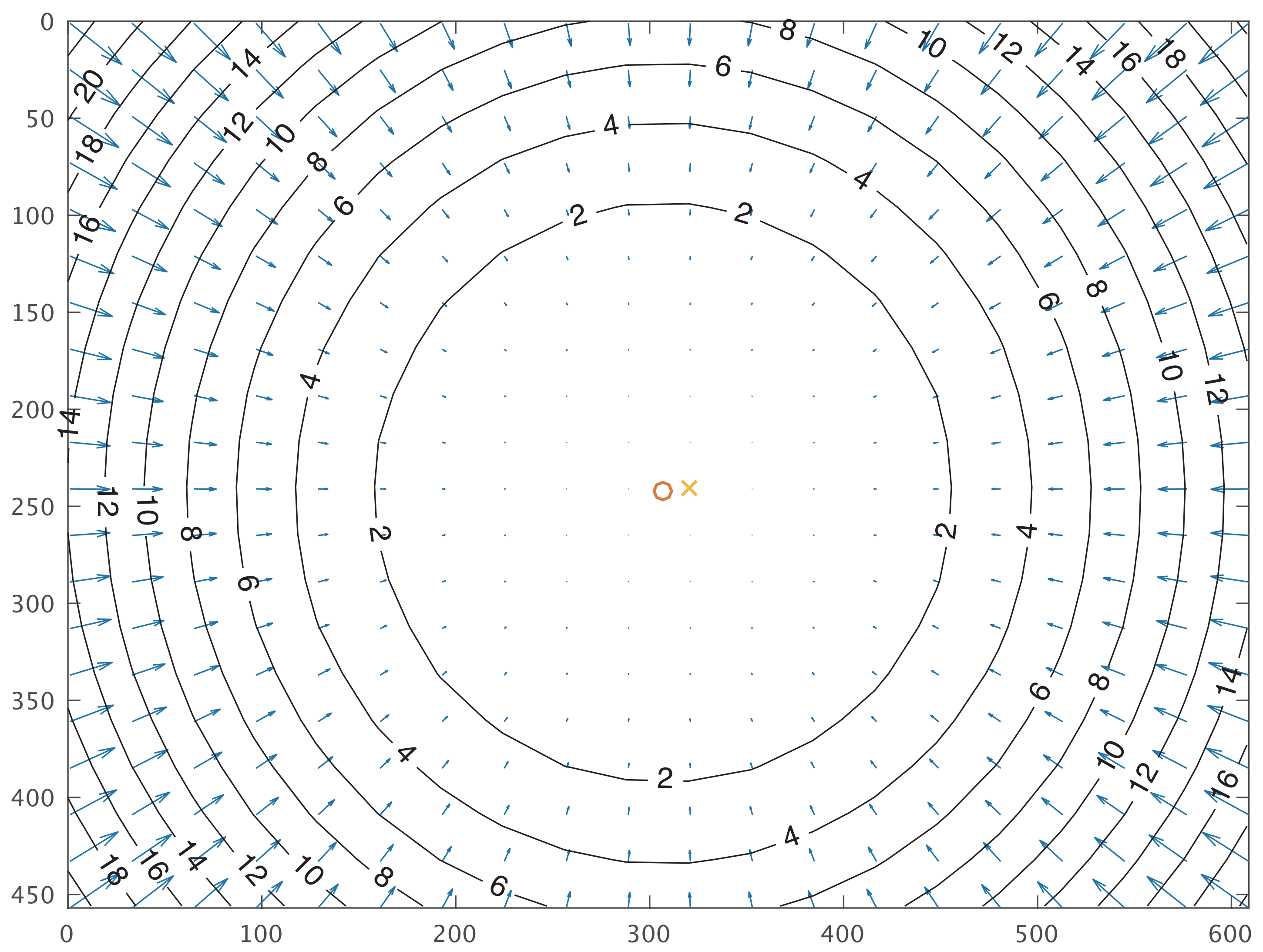
$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$u = \frac{\tilde{u}}{\tilde{w}}, v = \frac{\tilde{v}}{\tilde{w}}$$

Camera calibration

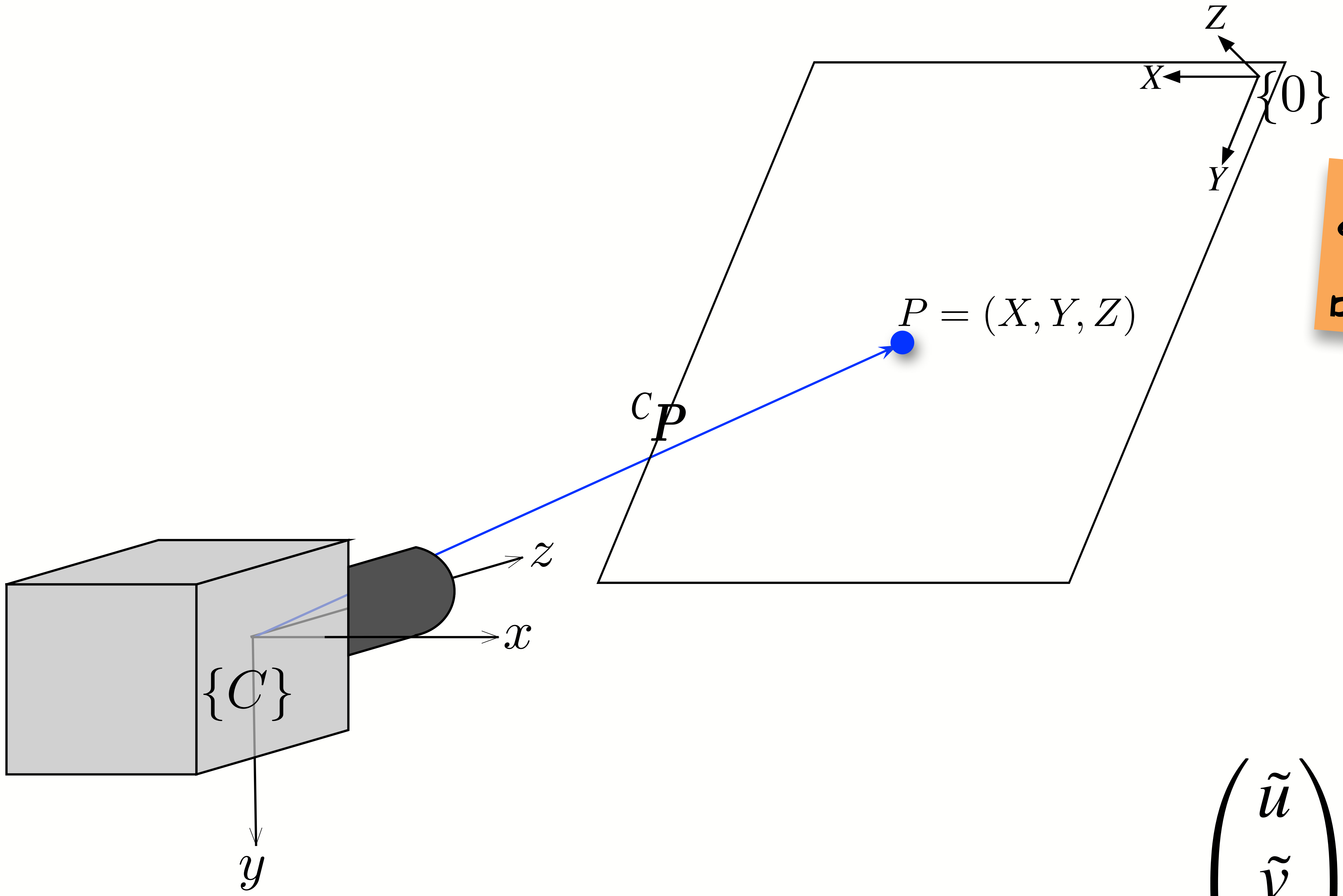






- Process to determine intrinsic and extrinsic camera parameters

Points on a **plane**

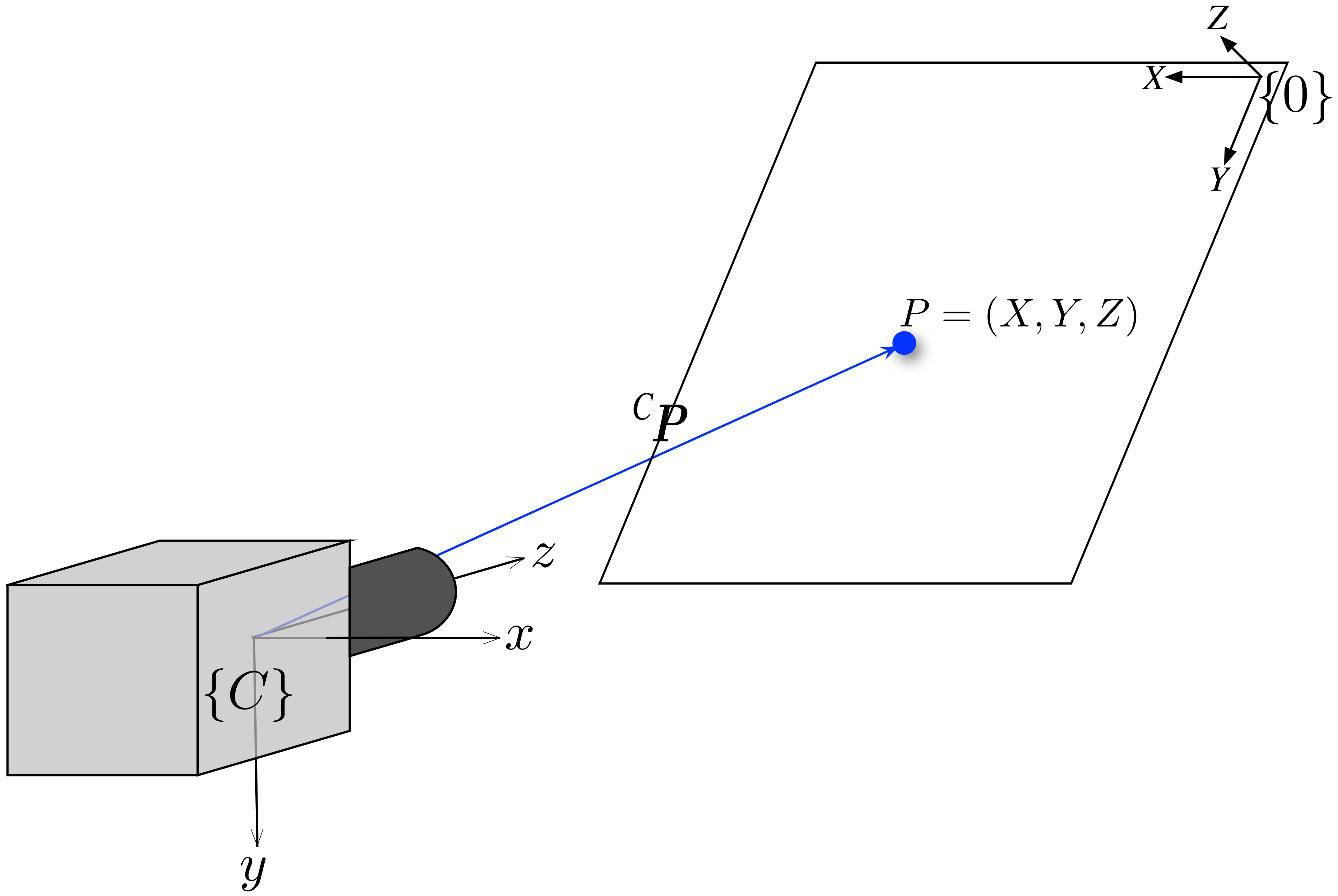


all points on the plane have z=0

3 x 3 matrix

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & & C_{14} \\ C_{21} & C_{22} & & C_{24} \\ C_{31} & C_{32} & & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ \\ 1 \end{pmatrix}$$

Planar **homography**



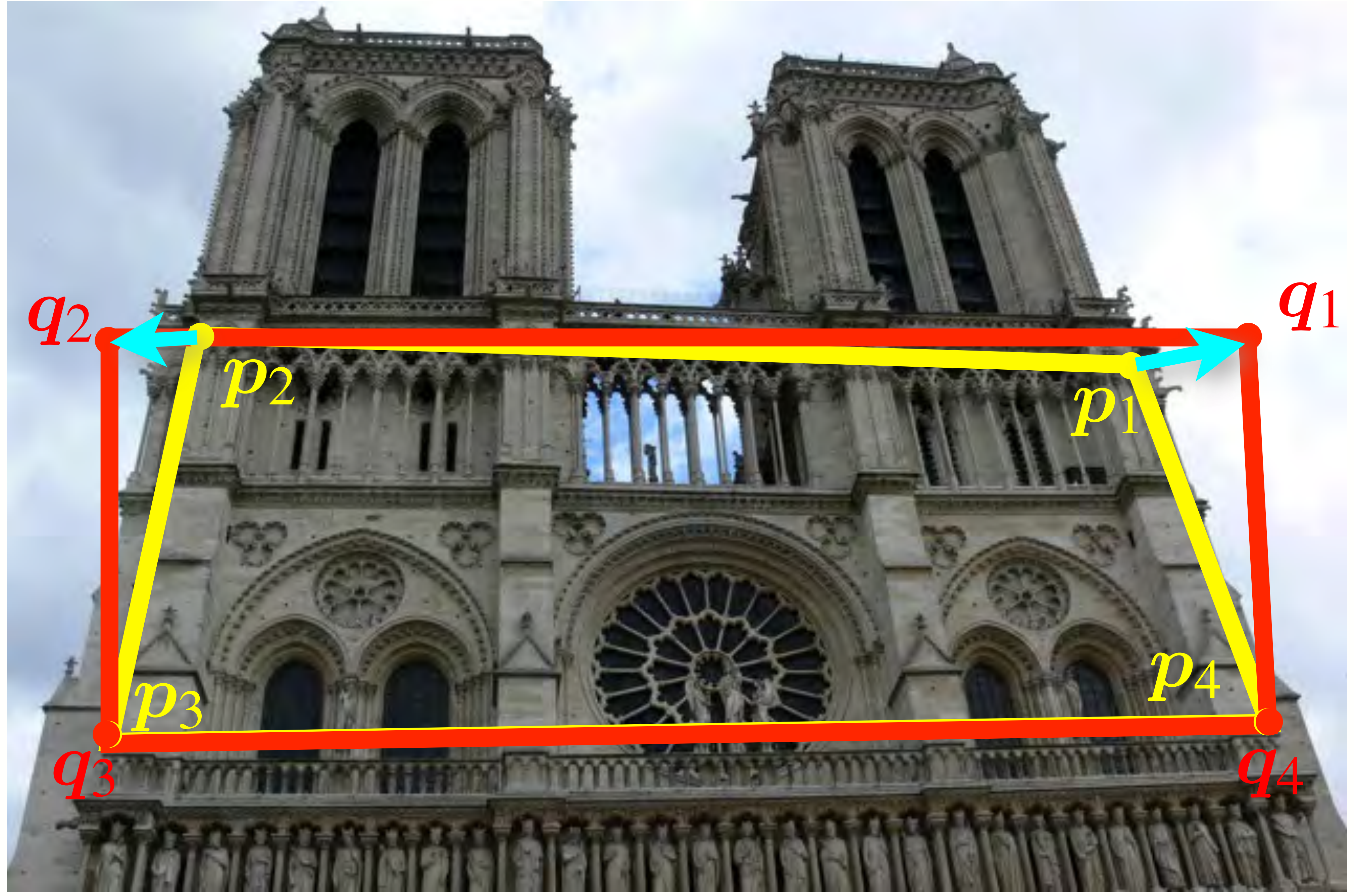
homography matrix

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

- Once again the scale factor is arbitrary
- 8 unique numbers in the homography matrix
- Can be estimated from 4 world points and their corresponding image points

$$\mathbf{H} = \mathbf{R} + \frac{\boldsymbol{t}}{d}\boldsymbol{n}^T$$

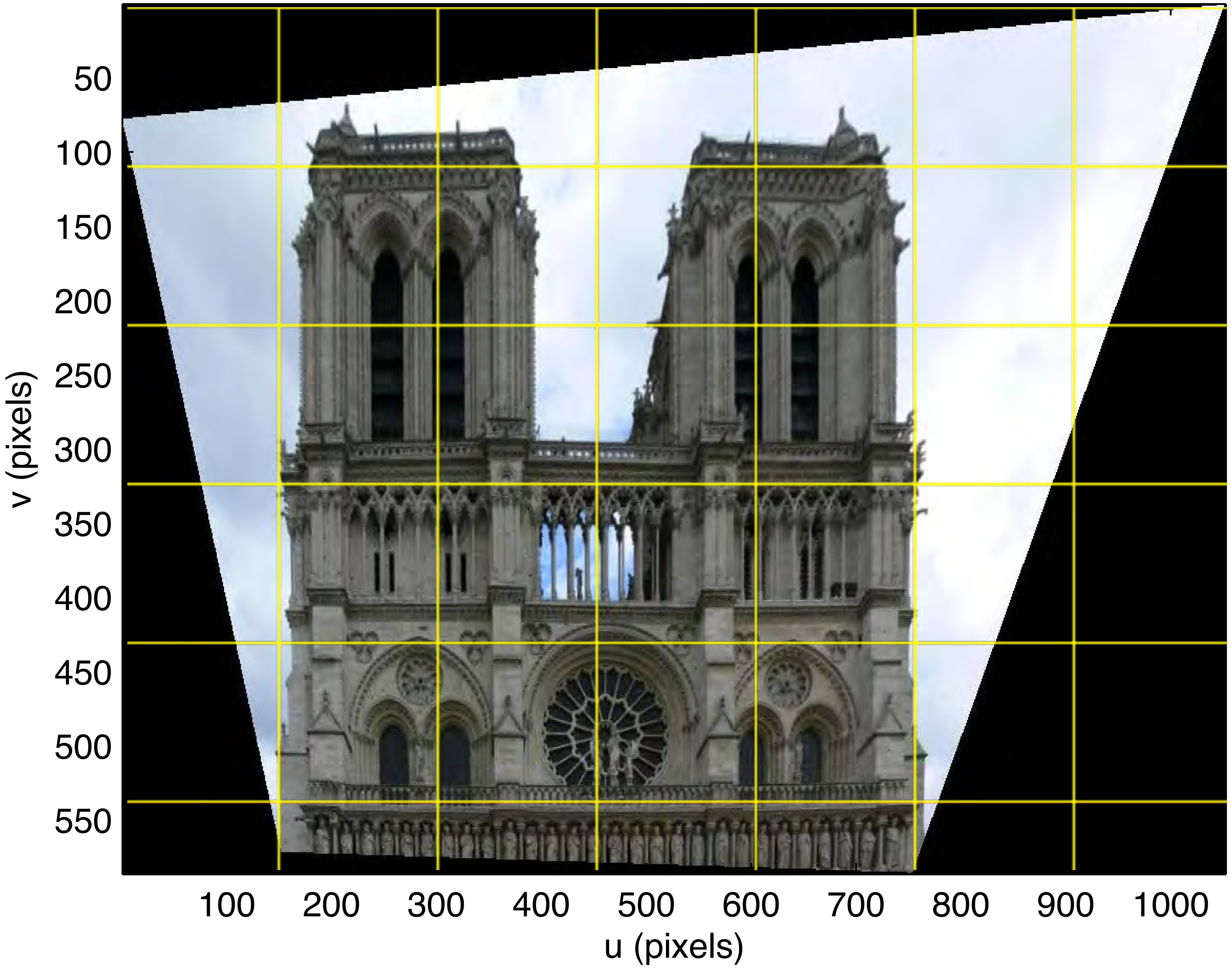
Perspective rectification



$$q = \mathbf{H}p$$

```
>> H = homography(P, Q)
H=
 1.4003   0.3827 -136.5900
-0.0785   1.8049 -83.1054
-0.0003   0.0016 1.0000
```

Perspective rectification



$$q = \mathbf{H}p$$

```
>> homwarp(H, im, 'full')
```

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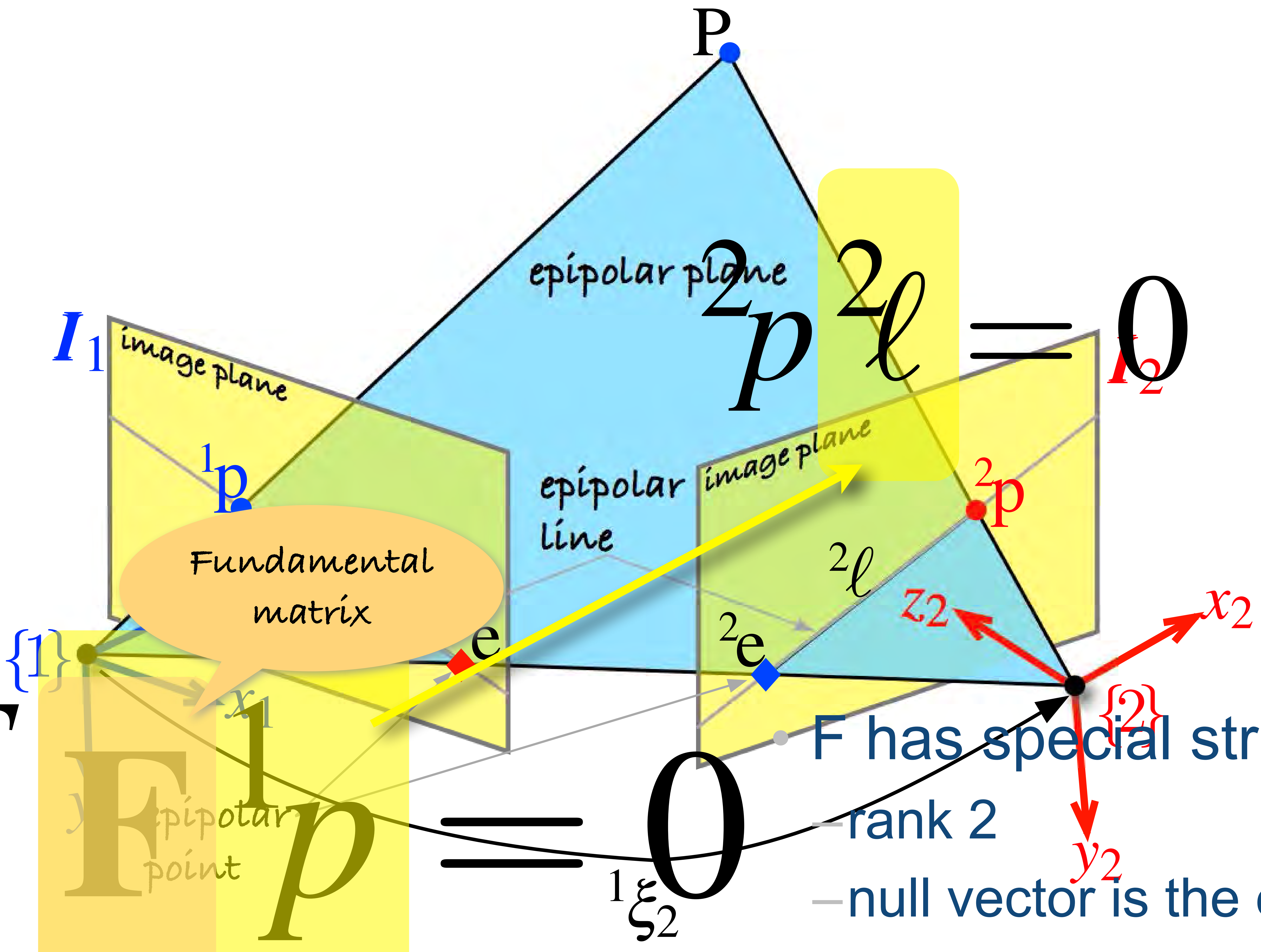
NEVO 1
TANCOK 2
MANAUDOU 3
BOVELL 4
TO 5
MOROZOV 6
DERLYUGOV 7
CHEAH 8
OMEGA
arena
DOHA QATAR

Fundamental matrix

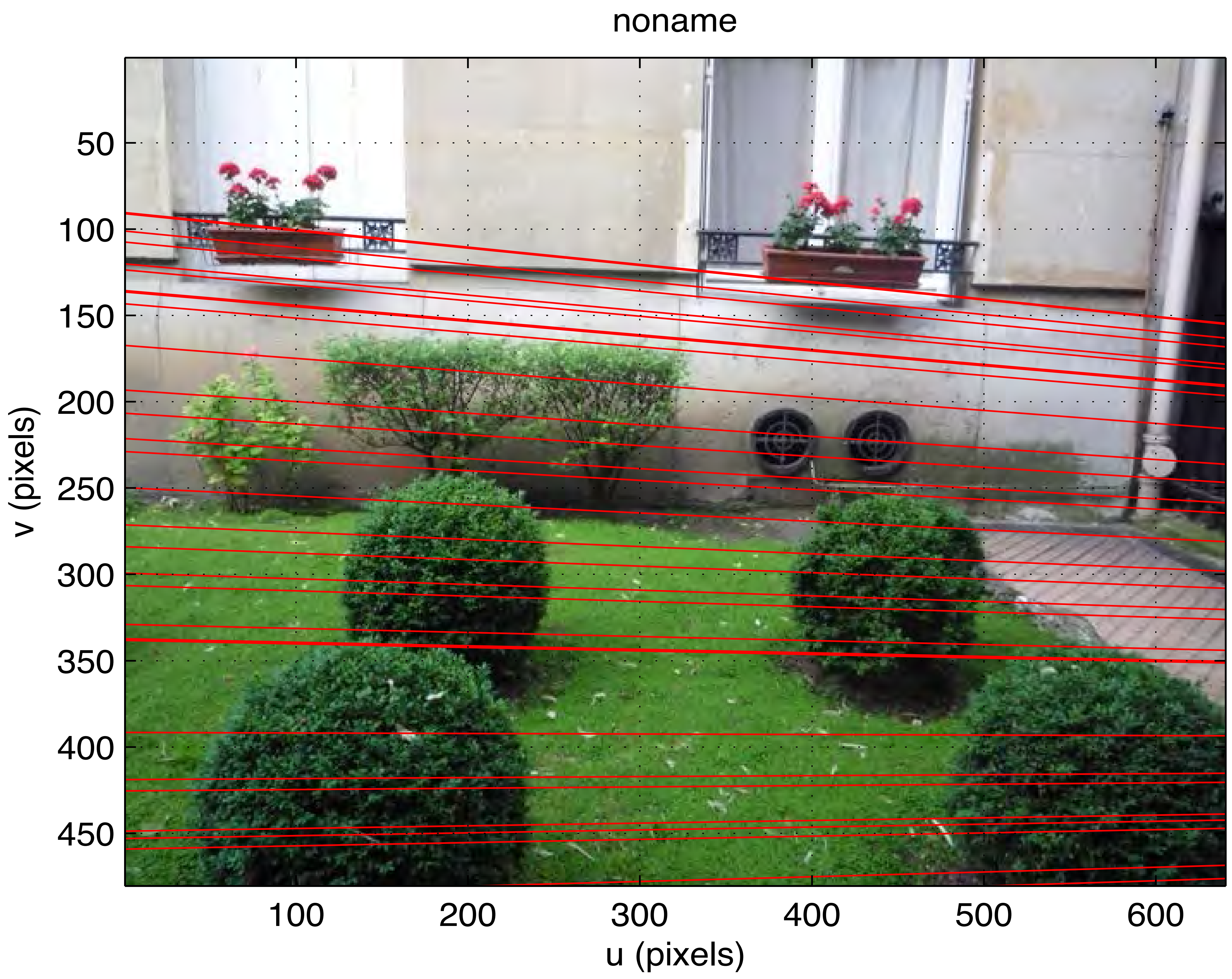
$$^{2}p^{T}\,\mathbf{F}\,{}^{1}p = 0$$

$$^{2}p\,{}^{2}\ell = 0$$



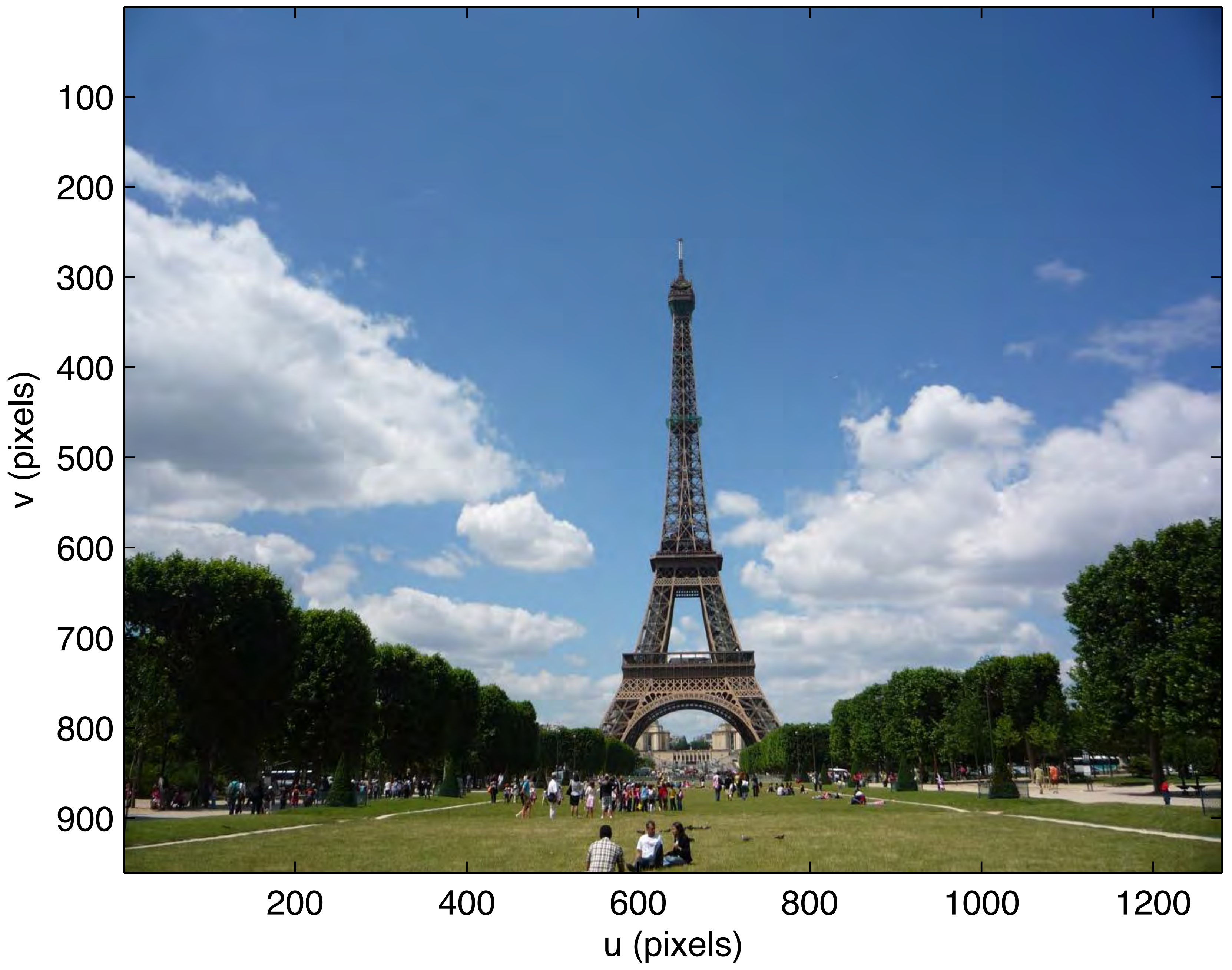
- F has special structure
  - rank 2
  - null vector is the epipole coordinate
  - can be estimated from 4 point pairs

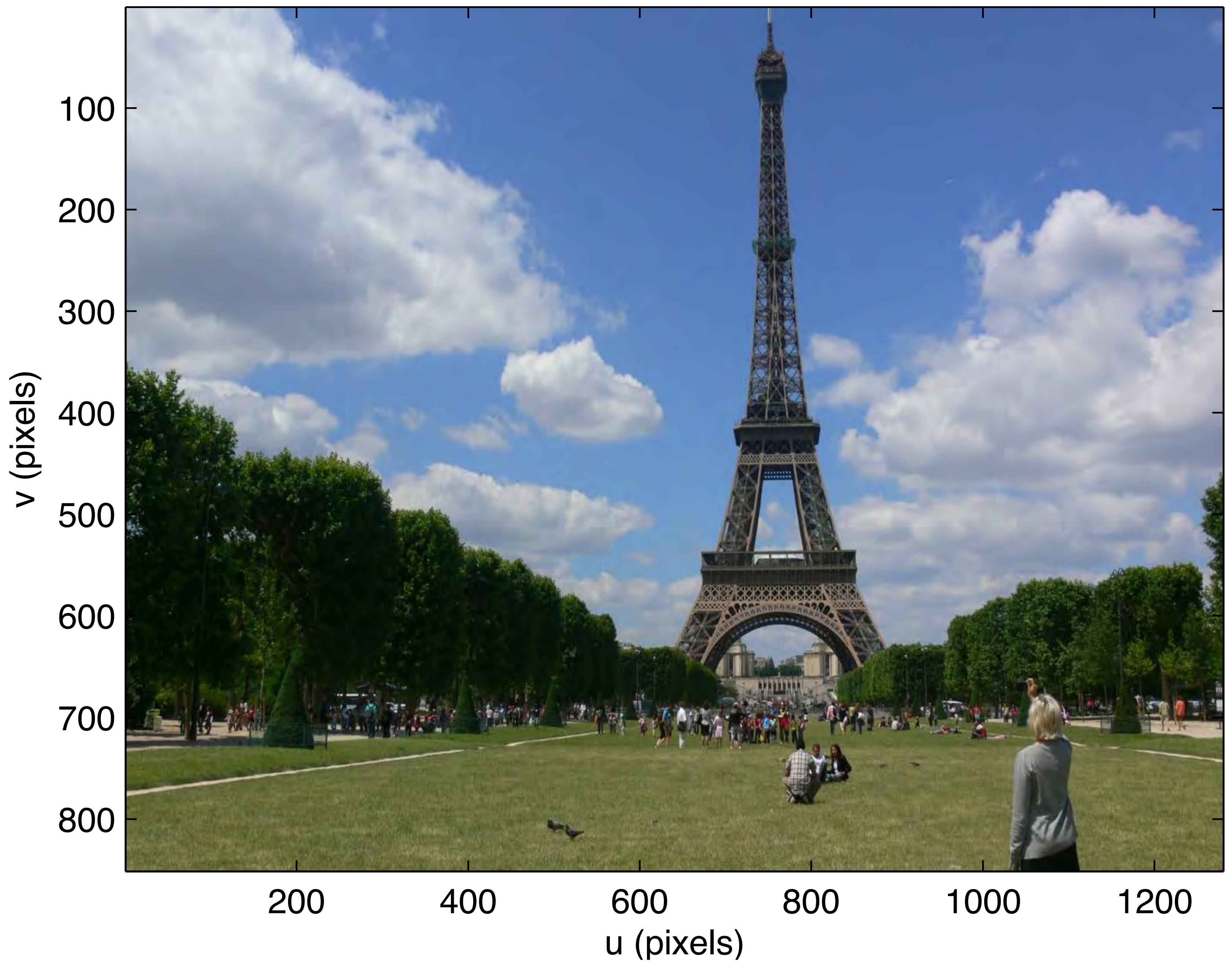
Epipolar lines



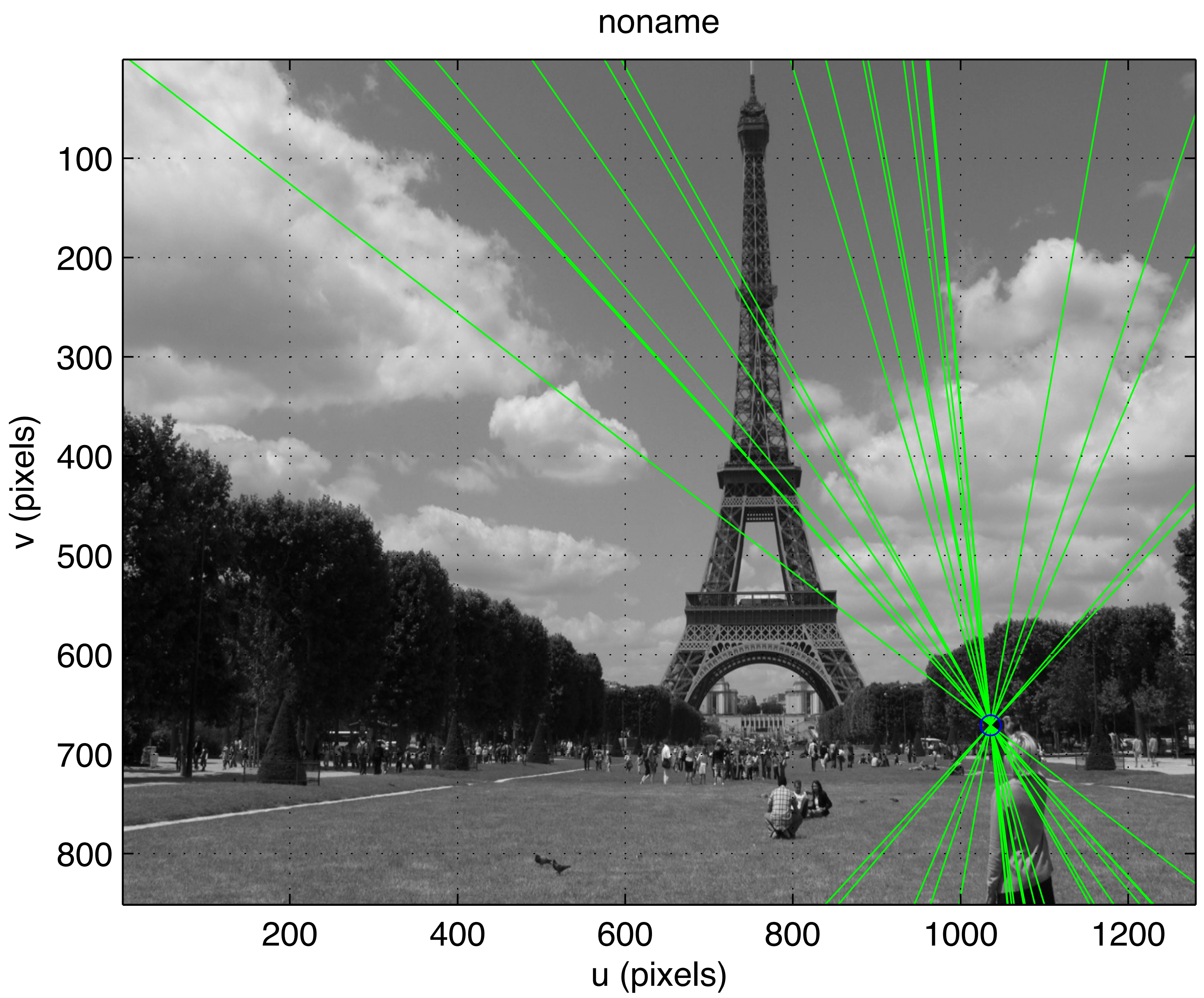
- Epipolar lines intersect at the position of the other camera

```
>> F =
      0.0000   -0.0000   -0.0001
      0.0000    0.0000    0.0097
      0.0003   -0.0098    0.2825
```





- Epipolar lines intersect at the position of the other camera



Essential matrix

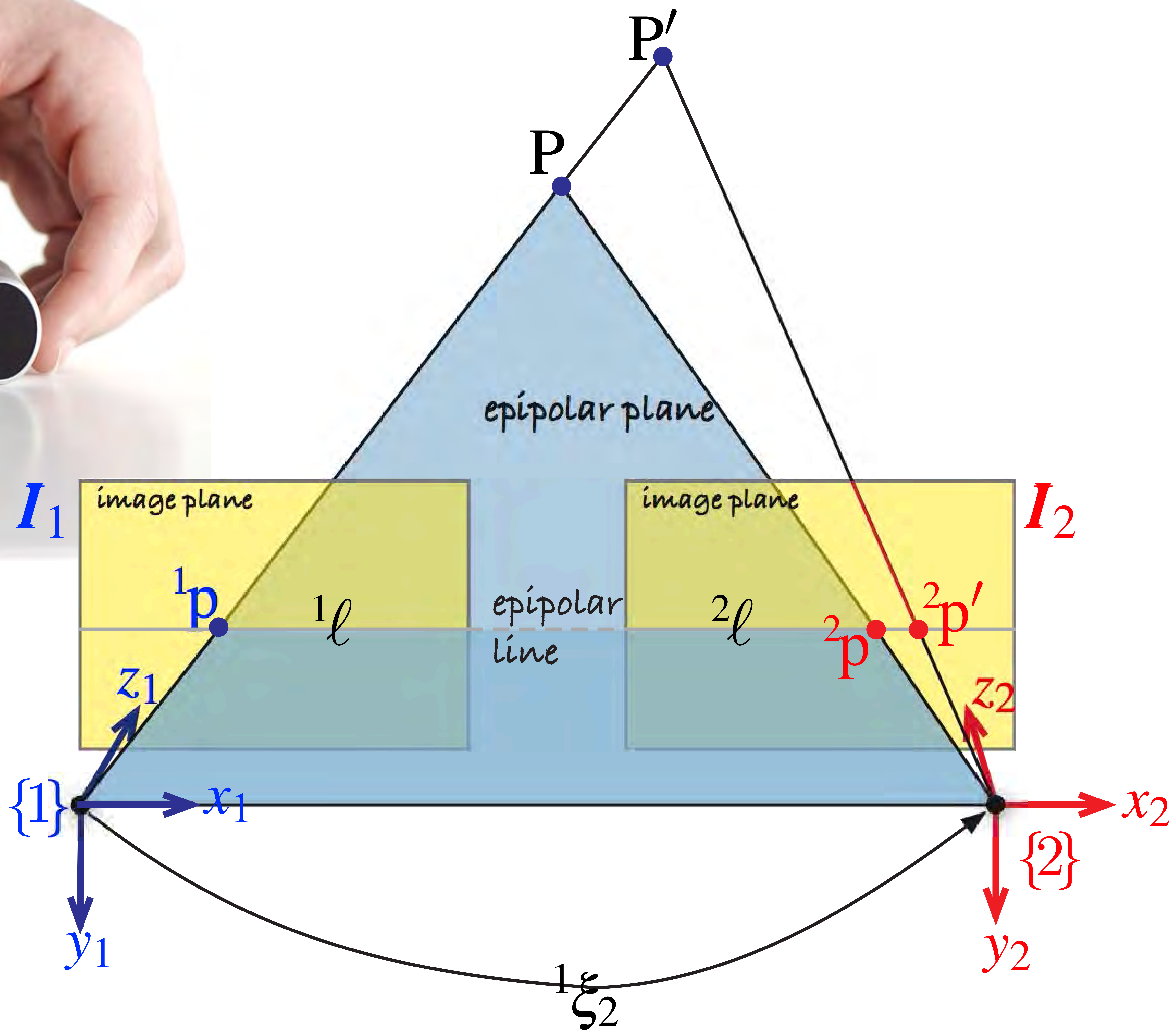
$$ {}^{2}\tilde{\boldsymbol{x}}\ \mathbf{E}\ {}^{1}\tilde{\boldsymbol{x}} = 0 \qquad\qquad \mathbf{E} = \mathbf{K}^{T}\mathbf{F}\mathbf{K} $$

- Only 5DOF
- Is related to the relative camera pose

$$ \mathbf{E} = [\boldsymbol{t}]_{\times}\mathbf{R} $$

- Camera pose can be solved for
  - in general two solutions
  - translation only up to scale

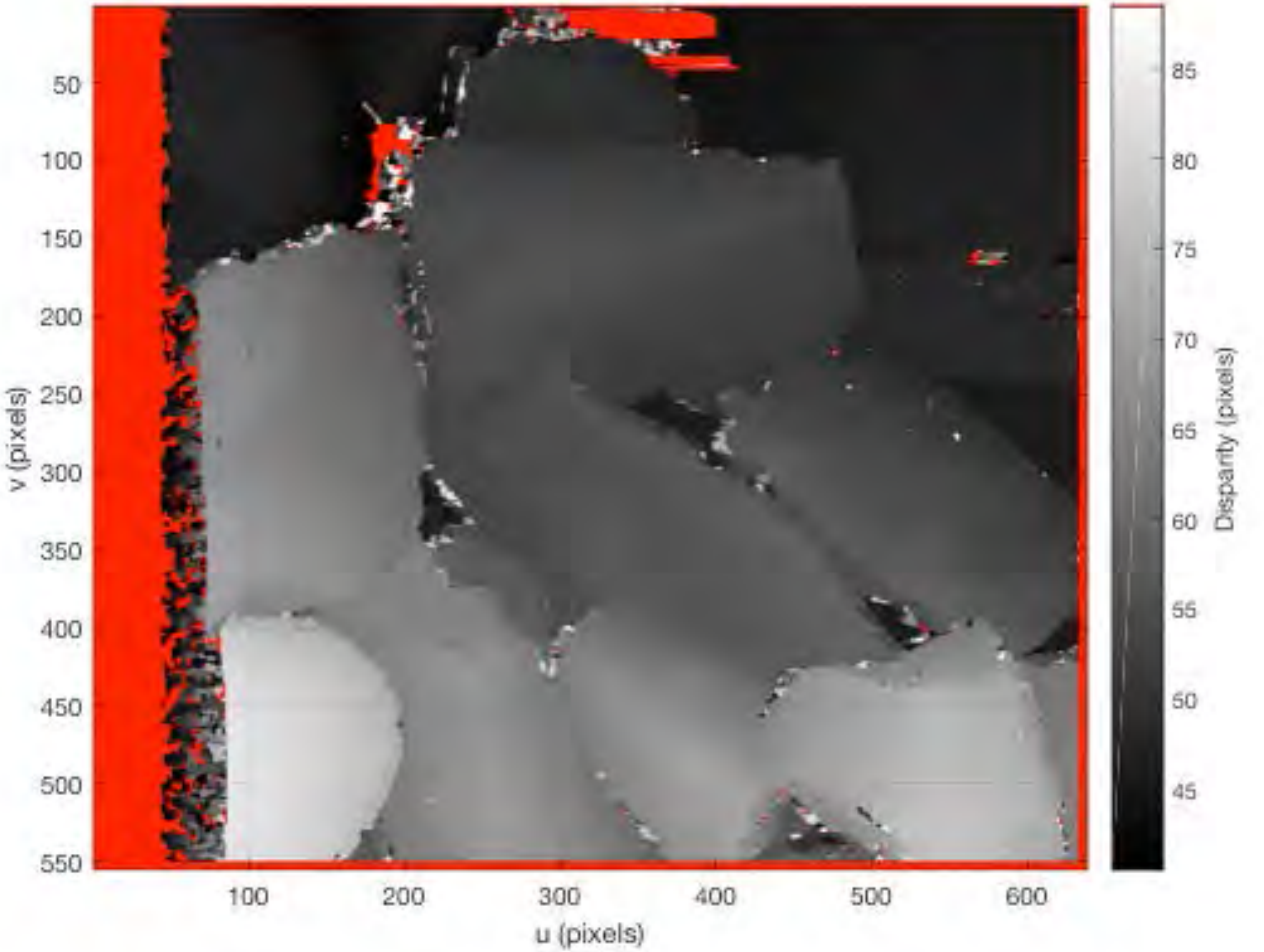

ZED


P′
P
epipolar plane
I_1
image plane
image plane
I_2
^{1}p
$^1\ell$
epipolar
line
$^2\ell$
^{2}p
^{2}p′
z_1
z_2
{1}
x_1
x_2
{2}
y_1
y_2
$^1\xi_2$

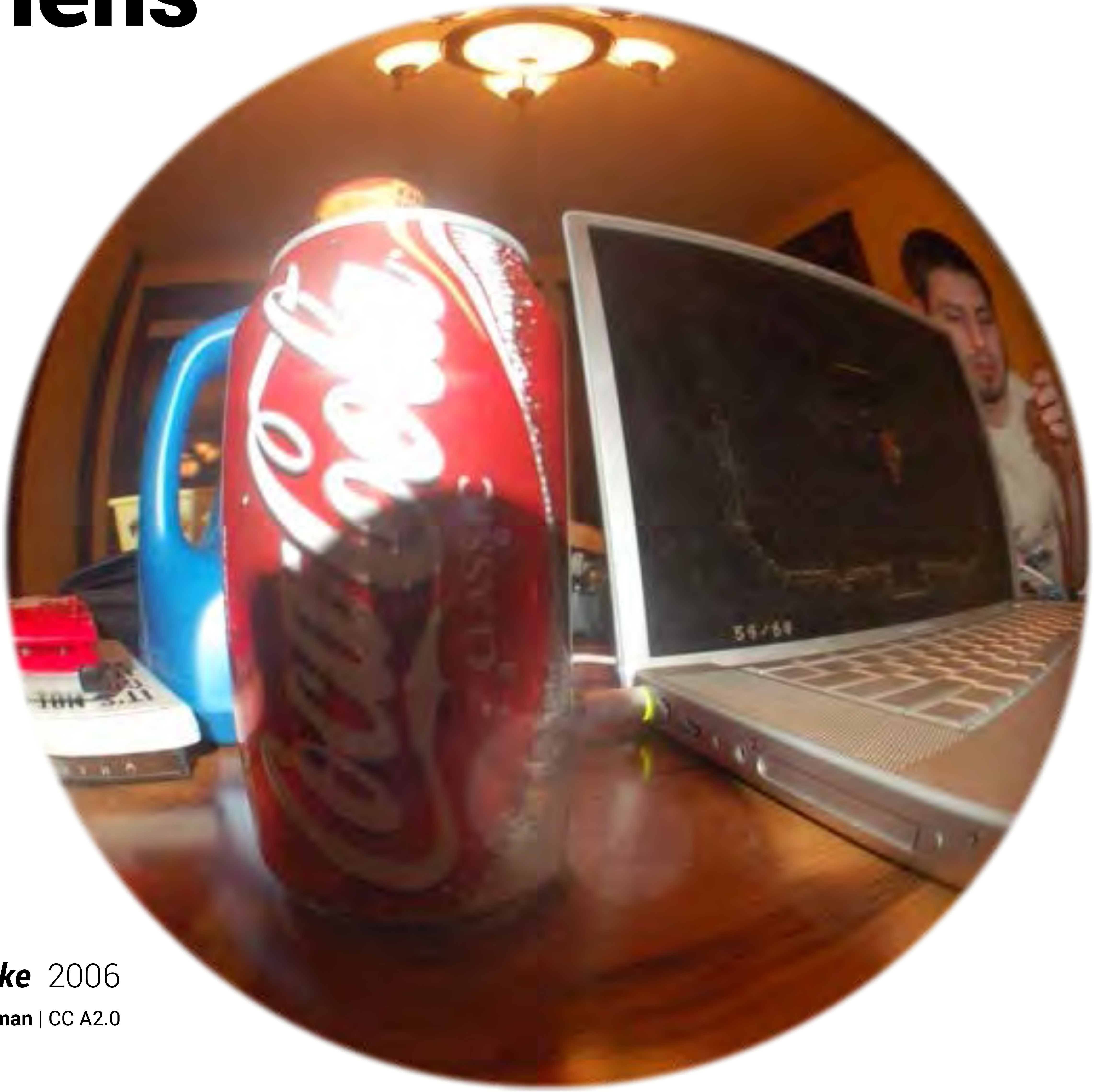
Stereo disparity

$$d \propto \frac{fb}{Z}$$

Stereo disparity



Fisheye lens



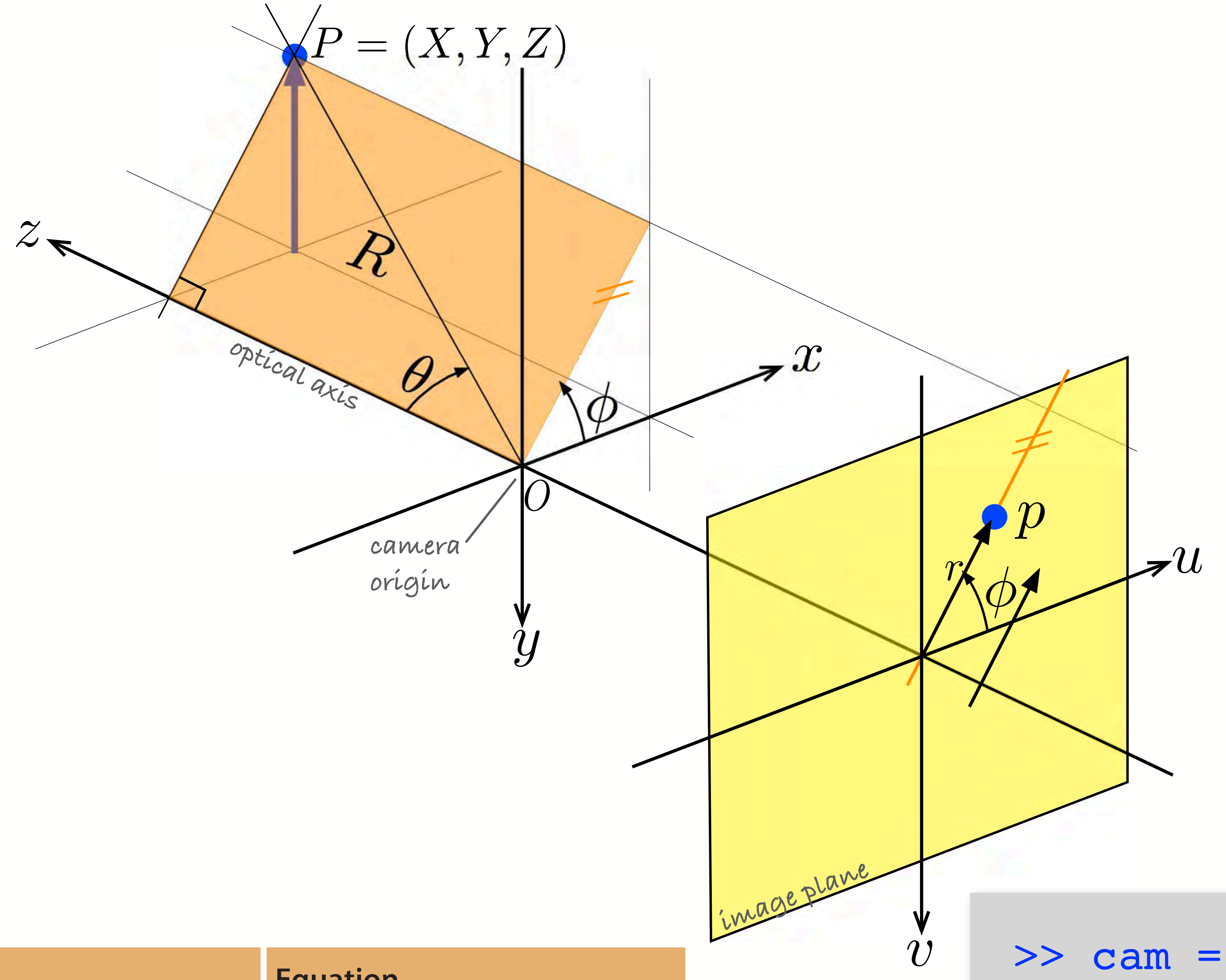
Fisheye coke 2006

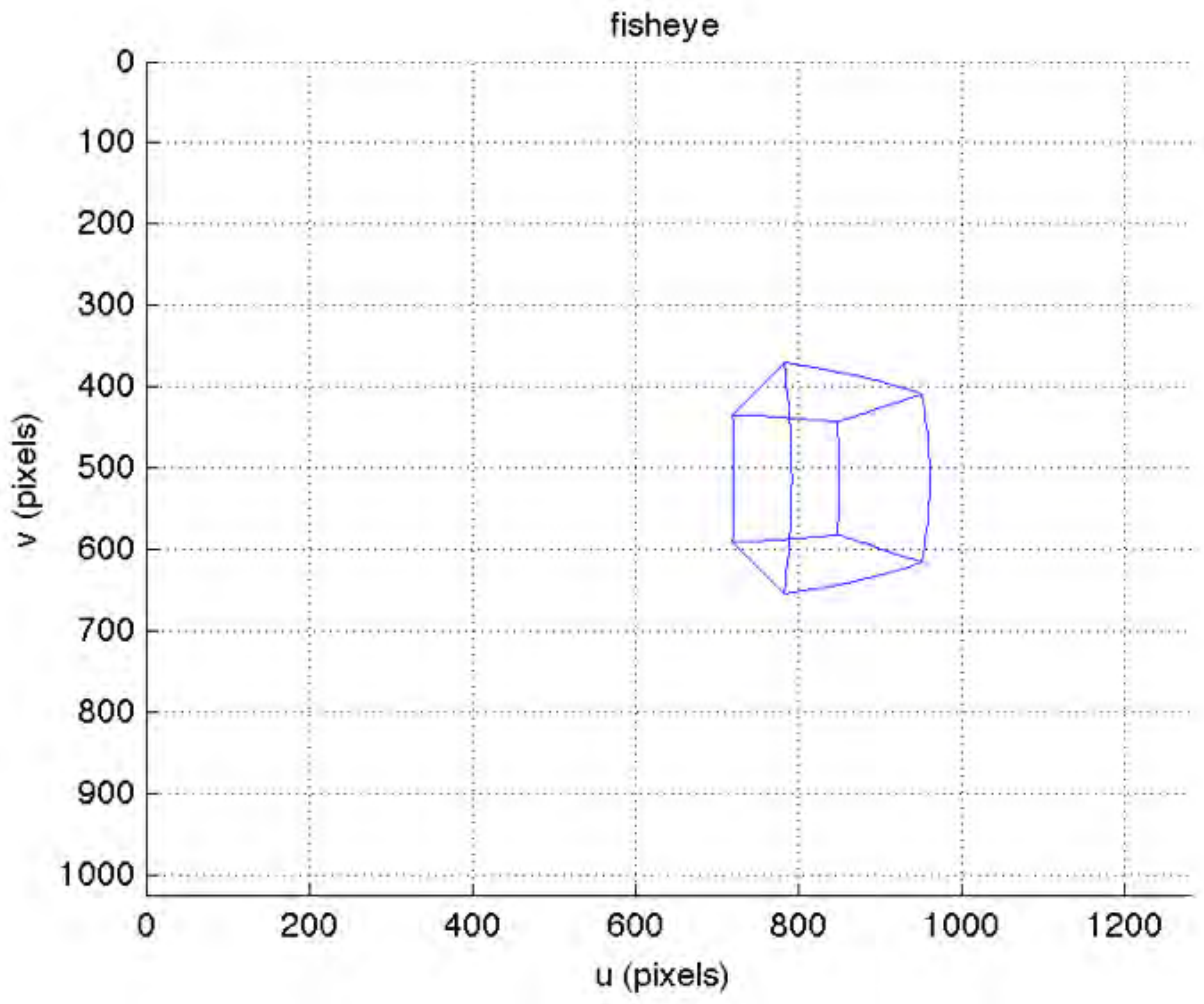
Joel Gillman | CC A2.0

Spiratone fisheye lens 2008

Alessandro Leite | CC A2.0

Fisher imaging model

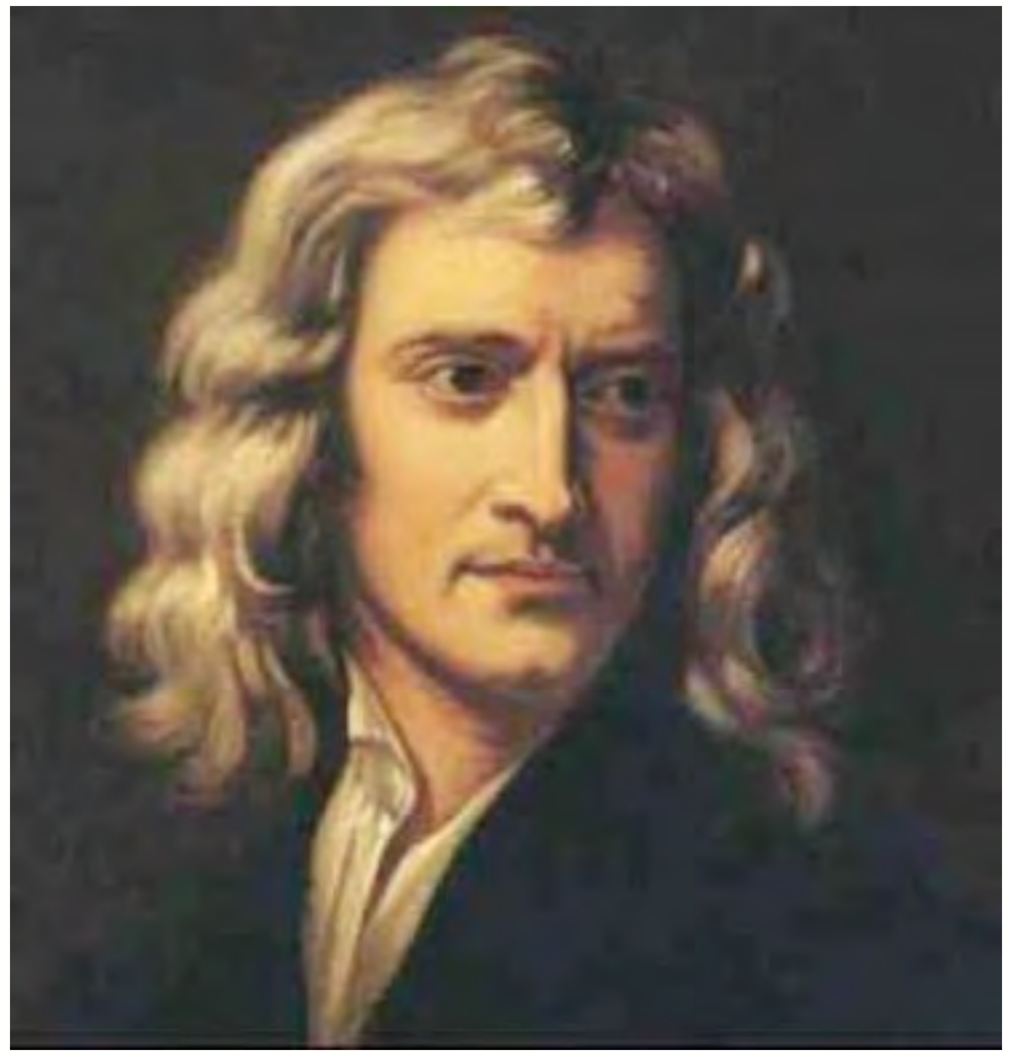




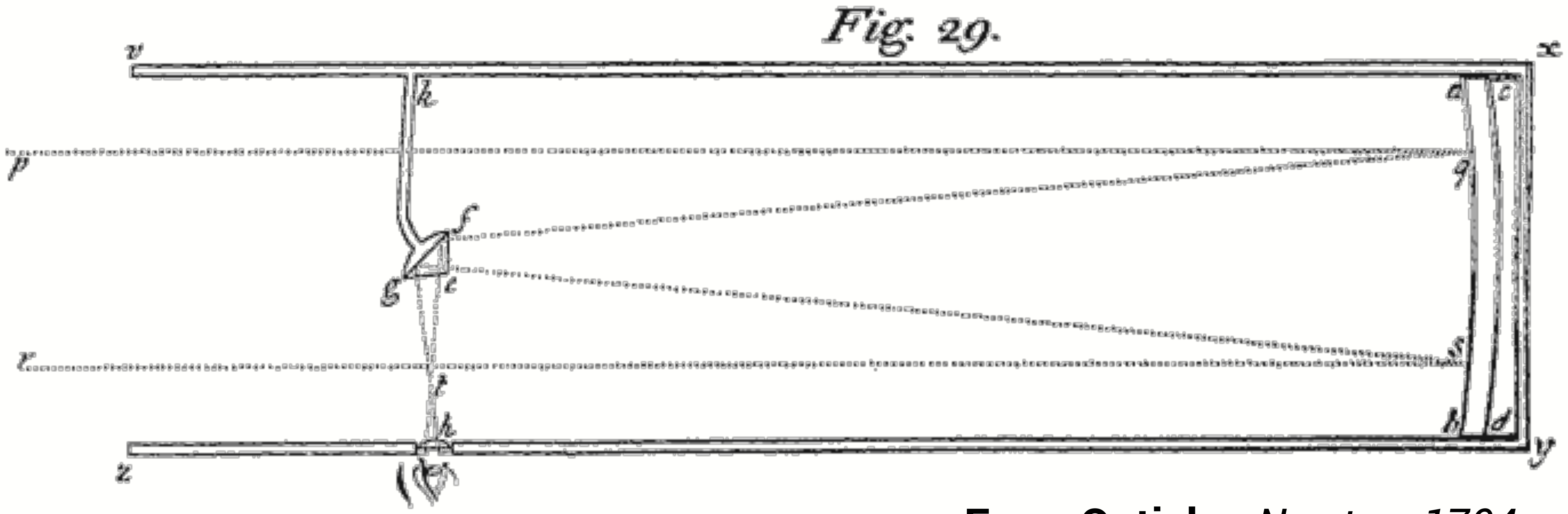
Mapping	Equation
Equiangular	$r = k\theta$
Stereographic	$r = k\tan(\theta/2)$
Equisolid	$r = k\sin(\theta/2)$
Polynomial	$r = k_1\theta + k_2\theta^2 + \cdots$

```
>> cam = FishEyeCamera('name', 'fisheye', ...
   'projection', 'equiangular', ...
   'pixel', 10e-6, ...
   'resolution', [1280 1024])
>> [X,Y,Z] = mkcube(0.2, 'centre', [0.2, 0, 0.3], 'edge');
>> cam.mesh(X, Y, Z)
```

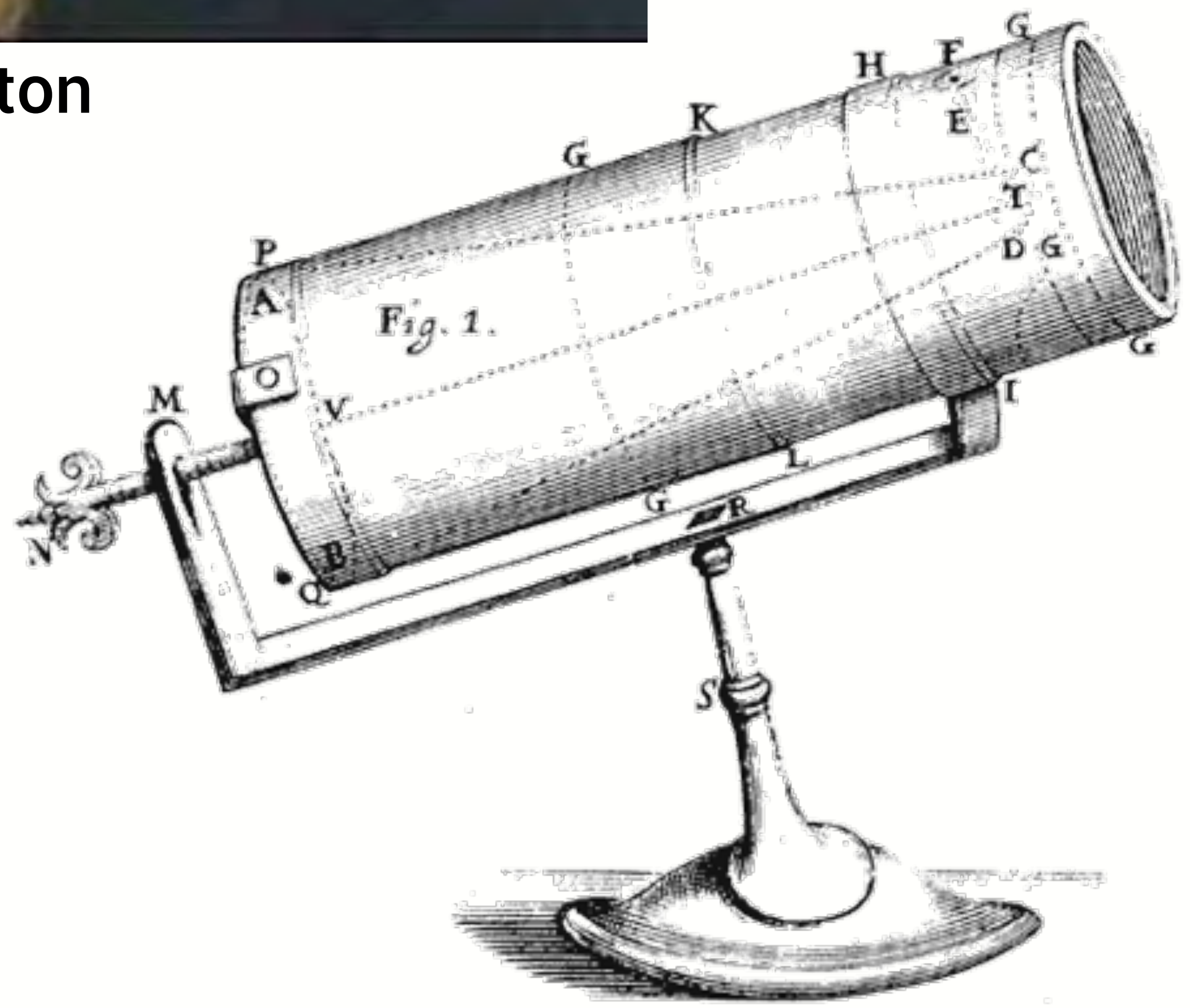
Imaging by reflection



Isaac Newton



From Opticks, *Newton, 1704*.



An Accompt of a New Catadioptrical Telescope invented by Mr. Newton | by Isaac Newton

Philosophical Transactions of the Royal Society, No. 81
(25 March 1672)

Panorama lens

Panorama long 2013

Michael Milford

Panorama round 2013

Michael Milford

Panorama lens