EFFECT OF STATISTICAL NOISE ON COUPLED PLASMA FLUID – MONTE CARLO KINETIC NEUTRALS SIMULATIONS

Y. Marandet[1], H. Bufferand[1], M. Valentinuzzi[2], G. Ciraolo[2], P. Meliga[3], J. Rosato[1], E. Serre[3], P. Tamain[2]

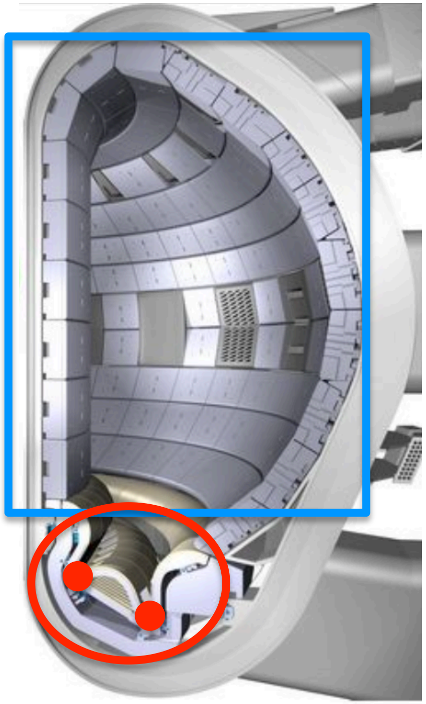
[1] PIIM, CNRS, Aix-Marseille Université, Marseille

[2] IRFM-CEA, Cadarache

[3] M2P2, CNRS, Aix-Marseille Université, Marseille

Introduction – Power exhaust in tokamaks

- ✓ **Plasma wall interactions play an essential role in magnetized fusion devices**
- ✓ **In steady state, the fusion energy has to be extracted**



ITER :
P_{fus} = 500MW → **400 MW neutrons** → **> 500 m²**
→ **100 MW α particles**
+ 50 MW heating (Q=10) → **~ 1 m²**

Spreading the energy on a larger surface area is *mandatory* (B geometry, radiation, …)

- ✓ **Avoid sputtering : lower temperature to a few eV**

Heat flux = 13.6 eV x Γ (particle flux)

- ✓ **On top of that: ELMs (See Marina's talk)**

Introduction

- ✓ **No identified scaling parameters, numerical simulations req.**
- ✓ **2D Transport codes (SOLPS, …, Soledge2D-EIRENE)**

Often: plasma fluid solver + Monte Carlo kinetic for neutrals

- ✓ **Simulations in relevant regimes are *challenging (~ months)***

R_{eff}~1, neutrals reach fluid limit at some places
(= large number of collisions/particle), …

- ✓ **Role of MC noise ? (making things worse, but how exactly ?)**

Introduction

What was the common practice so far ?

- ✓ **Use as many particle as « needed » (decision based on each user's judgement)**
- ✓ **Use last time step as solution**
- ✓ **Monitor global balances (according to a more or less well defined metric) to asses convergence**

No « theoretical » look at this issue (this talk)

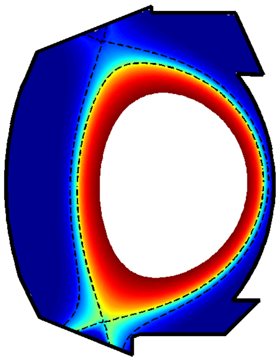
and no error assessment (KUL group, Baelmans et al.)

general architecture of Soledge2d-EIRENE

Particle fluxes on the boundary - Recycling

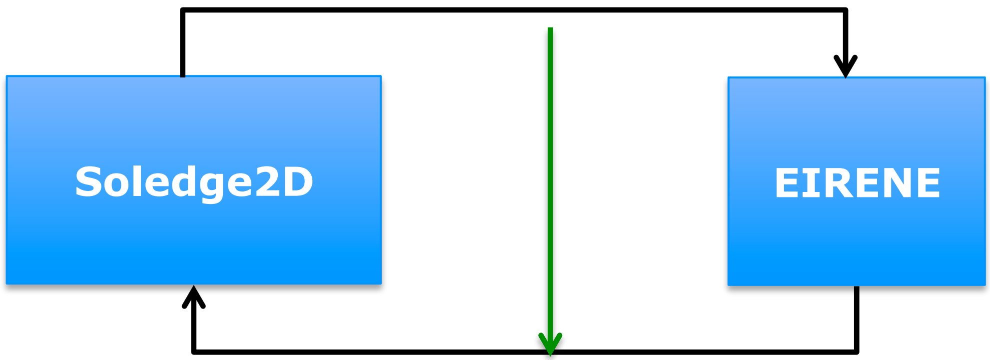
plasma parameters

Plasma



2D fluid
URANS

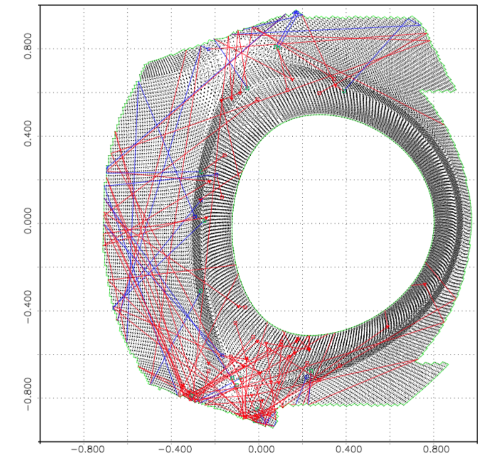
Γ_α, n_α, u_α, T_α



$S_{n\alpha}$, $S_{m\alpha}$, $S_{Ei\alpha}$, $S_{E\varepsilon}$

Volumetric sources

Neutral Gas



3D MC
Linear Boltzmann

✓ **Short cycling scheme essential for code speed-up**

(Soledge2D relies on a mixed implicit/explicit scheme)

Outline

1) **General considerations on transport codes convergence**

2) **Simplified model with synthetic noise**

3) **Effect of noise on the simulations**

4) **Conclusions and Perspectives**

Outline

1) General considerations on transport codes convergence

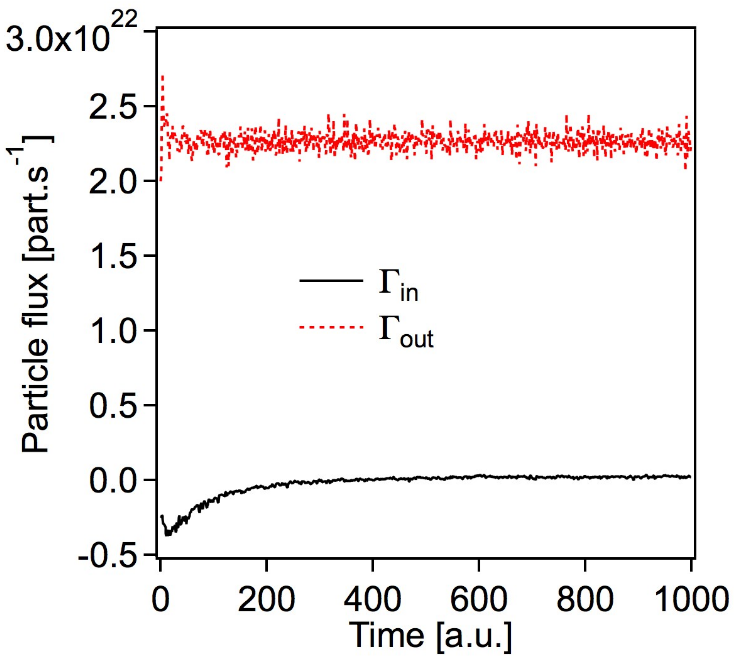
2) Simplified model with synthetic noise

3) Effect of noise on the simulations

4) Conclusions and Perspectives

Convergence of fluid/kinetic MC transport codes

Ex. of convergence of particle balance



✓ **The plasma is fluctuating**

how to define a *stationary solution* ?

✓ **"Looks like my favourite stochastic process …"**

✓ **"Looks like a turbulence code output …"**

Is there a (useful) connection to be made ?

Take the problem for what it is …

With MC statistical noise:

- ✓ ***system of stochastic differential equations forced by multiplicative noise***

$$S_n = n(n_0 + \delta n_0)\overline{\sigma v}$$

- ✓ **Probability average <…>, quantities of interest = moments**

- ✓ **1 run = 1 random seed = 1 realization of the stochastic process**

- ✓ **Estimation by ensemble averaging (N runs, can be painful)**

Estimation of moments by time averaging

- ✓ **Similar situation as in turbulence theory (e.g. Monin&Yaglom)**
- ✓ **ensemble average impractical, so we rely on the ergodic theorem**

$$\left\langle \left(\frac{1}{T} \int_{-T/2}^{T/2} n dt - \langle n \rangle \right)^2 \right\rangle \propto \frac{\tau_c}{T}$$

- ✓ **In practice, run the code in the "converged" statistically stationary Steady State (SS) and compute the mean solution**
- ✓ **Compute standard deviations too: measure the dispersion of the solution from time step to time step**

Indication on the distance between the *last time step* and *the mean*

Noise-induced terms in the equation

✓ **Key question:**

how much does the mean solution differ from the noise free solution ?

✓ **Equation for the mean density**

$$\nabla \cdot \left(\left(\langle n \rangle \langle u_{\|} \rangle + \Gamma_{\| \eta} \right) \mathbf{b} = \nabla \left(D \nabla_{\perp} \langle n \rangle \right) \right) + \langle n \rangle \langle n_0 \rangle \left[\overline{\sigma \mathrm{v}} \right]_{\eta}$$

✓ **Extra terms induced by noise – *"turbulent fluxes"***

$$\Gamma_{\| \eta} = \langle \delta n \delta u_{\|} \rangle$$

Spurious parallel transport

$$\left[\overline{\sigma \mathrm{v}} \right]_{\eta} = \langle \overline{\sigma \mathrm{v}}(n, T_e) \rangle + \frac{\langle \delta n \; \overline{\sigma \mathrm{v}} \rangle}{\langle n \rangle} + \frac{\langle \delta n_0 \; \overline{\sigma \mathrm{v}} \rangle}{\langle n_0 \rangle} + \frac{\langle \delta n \delta n_0 \; \overline{\sigma \mathrm{v}} \rangle}{\langle n \rangle \langle n_0 \rangle}$$

Executive Summary

- ✓ **The mean solution is the proper solution to the problem**
- ✓ **It can be estimated by time averaging in the SS**
- ✓ **need to run the code for T>>τ_c in that phase = price to pay**
- ✓ **It is the solution of an equation with spurious noise-induced terms, similar to turbulent fluxes**
- ✓ **These terms can be estimated from the SS too:**
- ✓ **Could ultimately lead to a criteria useful to make sure that noise "does not perturb the solution too much".**

Outline

1) General considerations on transport codes convergence

2) Simplified model with synthetic noise

3) Effect of noise on the simulations

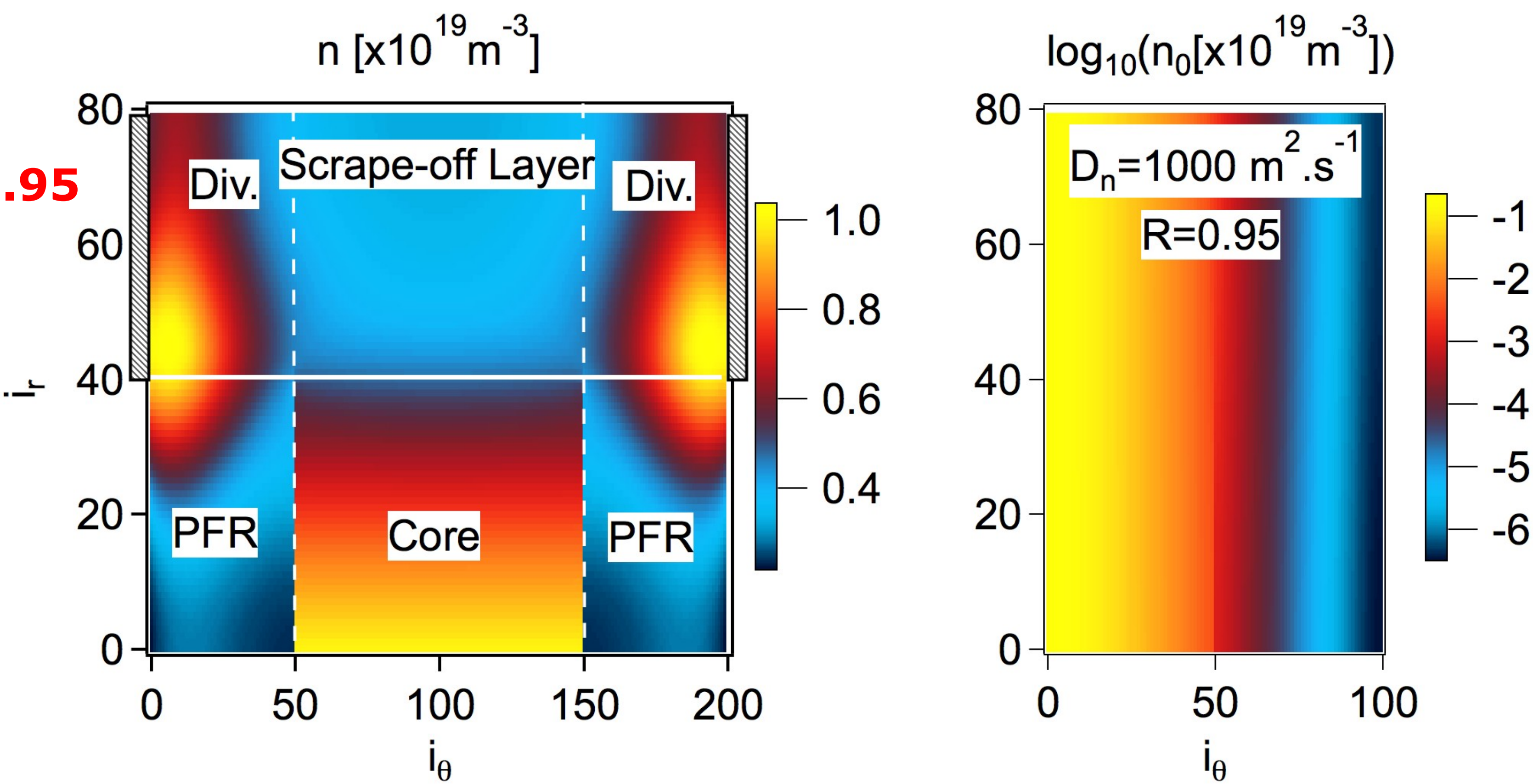
4) Conclusions and Perspectives

Slab case with neutral fluids in SolEdge2D

✓ **Soledge2D neutral fluid model:**

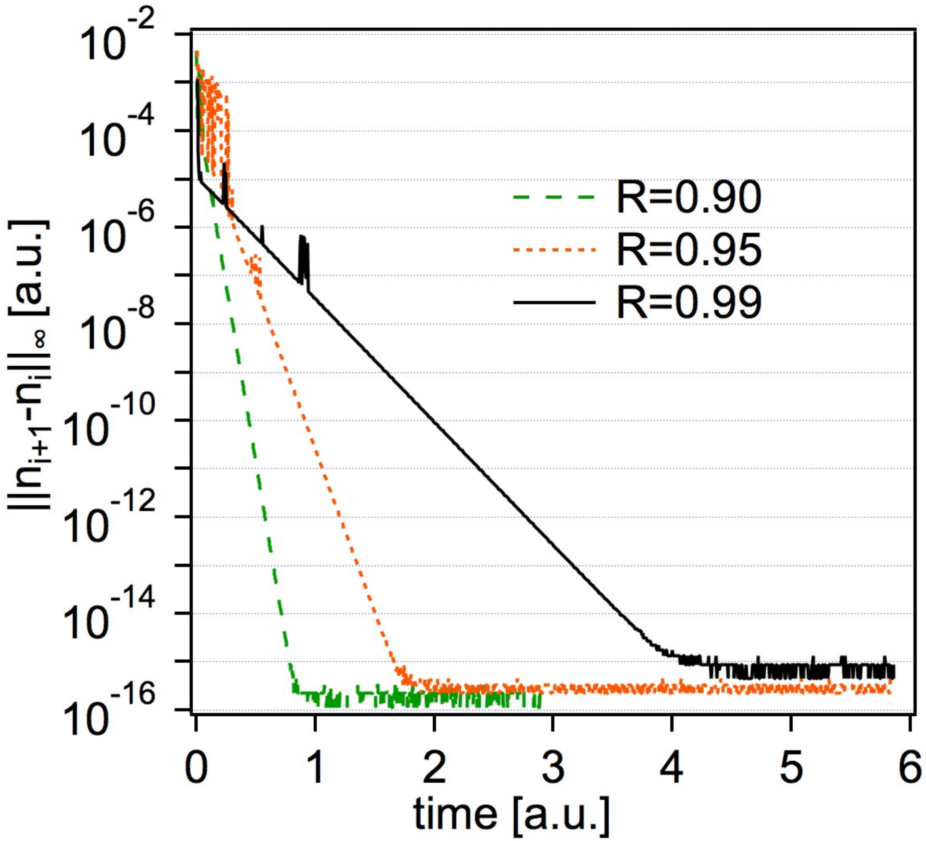
$$\partial_t n_0 - D_N \nabla^2 n_0 = -S_n$$

R=0.95



Convergence of the simulations

- ✓ **No noise: residuals go to machine precision**



- ✓ **Time scale depends on the recycling coefficient R:**

$$\frac{d\mathcal{N}}{dt} = \Gamma_{in} + S - \Gamma_{out}$$

$$S = R\,\Gamma_{out}$$

Confinement time τ_0

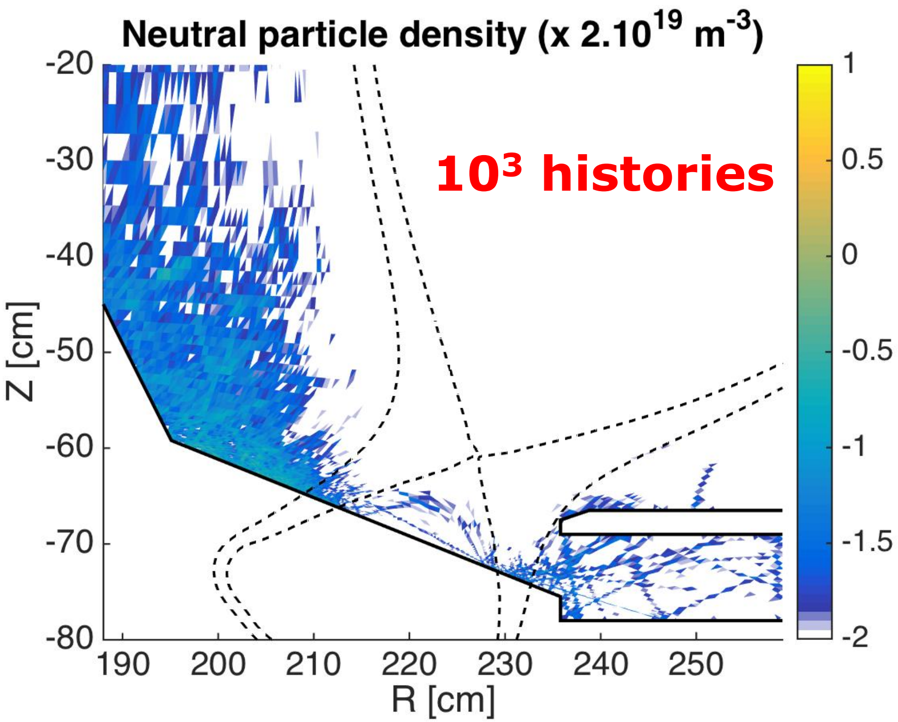
$$\Gamma_{out} = \mathcal{N}/\tau_0$$

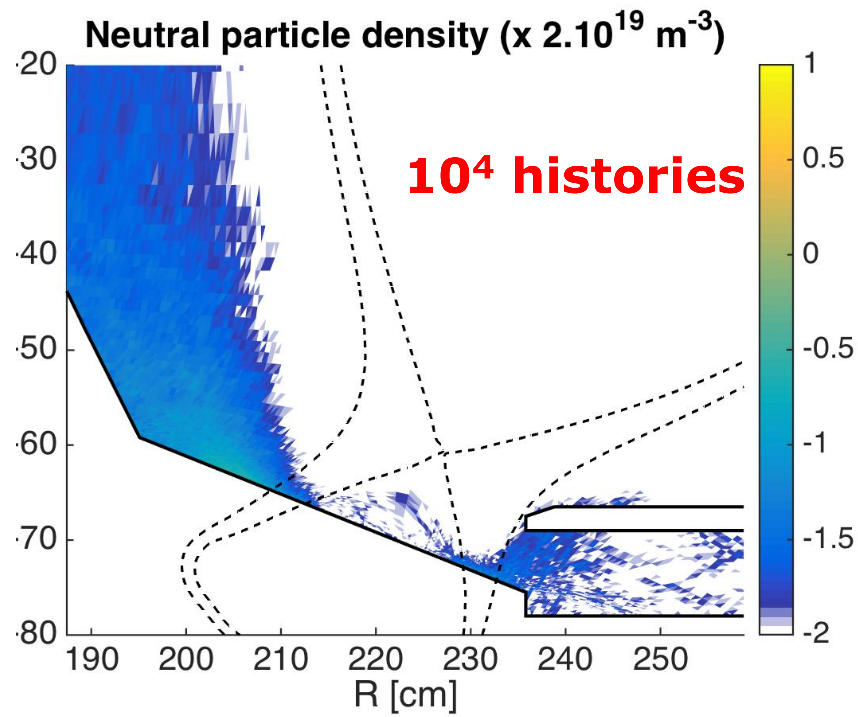
$$\frac{d\mathcal{N}}{dt} = -\frac{\mathcal{N}}{\tau^\star} \text{ , } \tau^\star = \frac{\tau_0(R)}{1-R}$$

Synthetic noise model

Wish list:

- ✓ ***Gaussian*** **at low noise level (C.L.T.)**
- ✓ **Providing** ***positive densities*** **only at high noise levels**
- ✓ **Substantial probability for** ***zero densities***



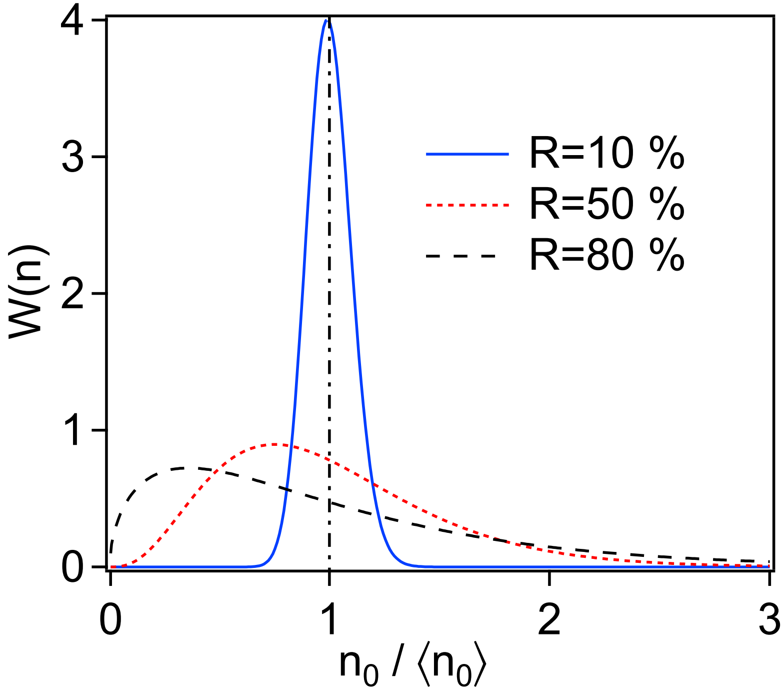


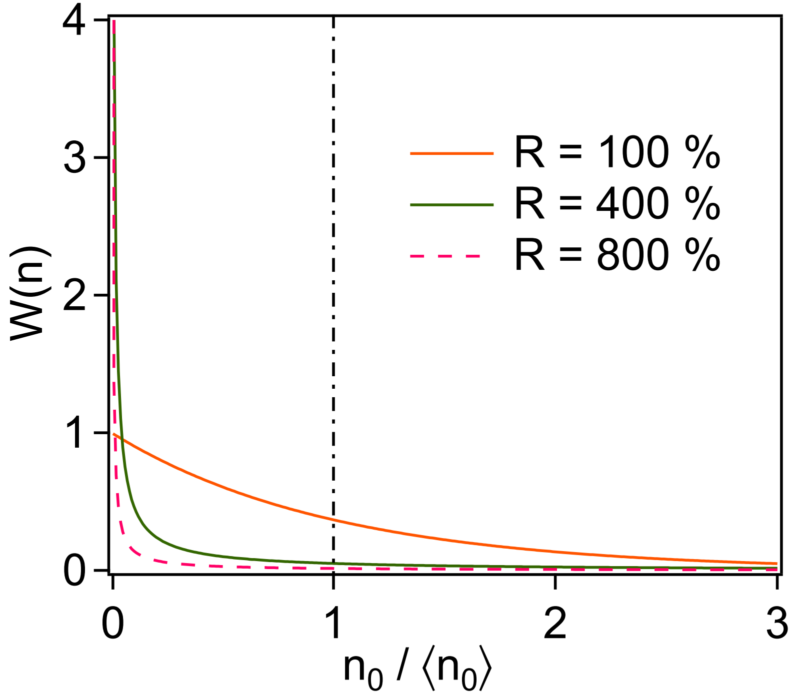
(WEST case, log scale)

A good candidate : the gamma distribution

$$W(n_0) = \frac{1}{\Gamma(\beta)\alpha^\beta} n_0^{\beta-1} \exp\left(-\frac{n_0}{\alpha}\right)$$

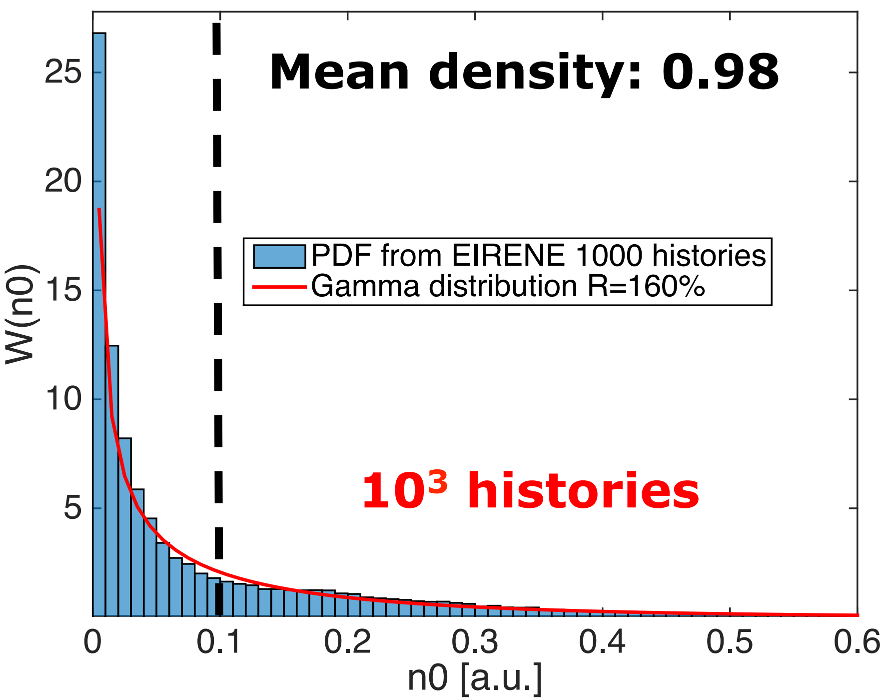
$$\langle n_0 \rangle = \alpha\beta \qquad \sigma_{n_0}^2 = \langle n_0^2 \rangle - \langle n_0 \rangle = \alpha^2\beta \qquad \mathcal{R} = \frac{\sigma_{n_0}}{\langle n_0 \rangle} = \beta^{-1/2}$$

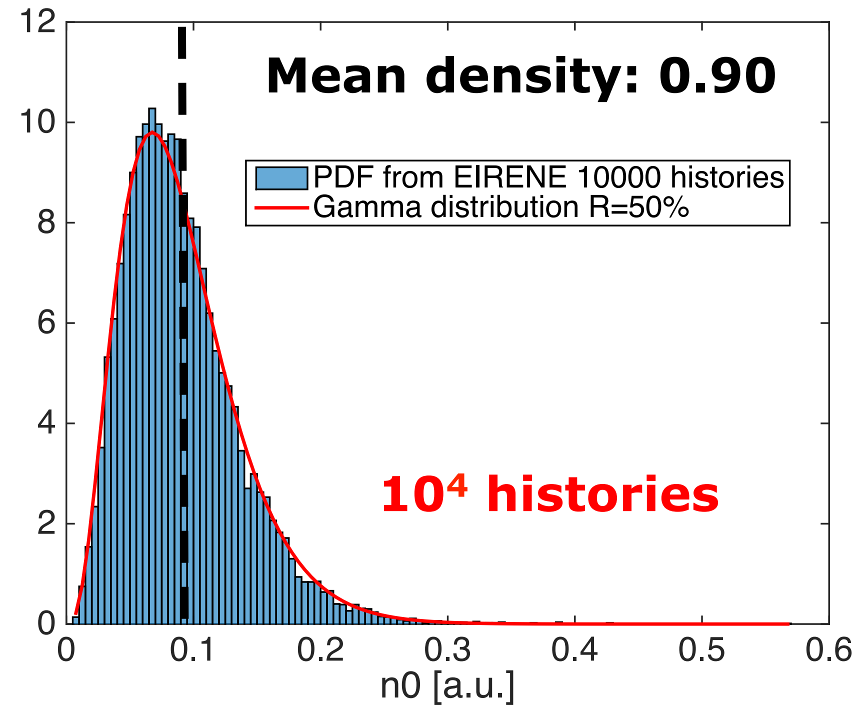




How realistic is the gamma model ?

✓ **PDF of the neutral particle density in the outer divertor leg (WEST simulations), up to 7x10^4 calls to EIRENE in SS**





Too good to be coincidental ? Erlang distribution ?

= *Sum of exponentially (Poisson) distributed events*

Implementation and examples

✓ **Assumption: uniform fluctuation level $\mathcal{R}$**

✓ **Freeze the noise for k iterations : introduce *time correlations***

$$\tau_c = k\Delta t$$

Real life situations :

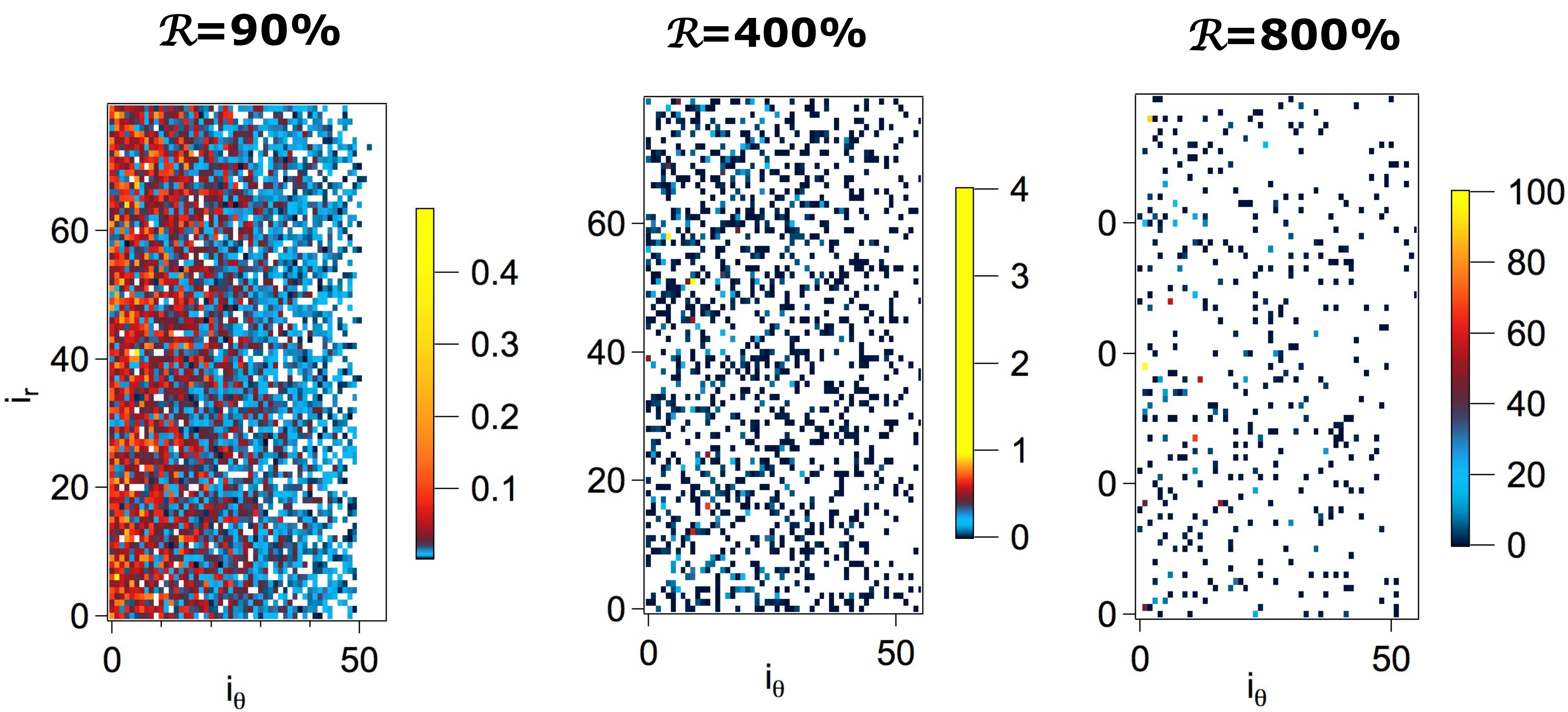
i) Not calling MC solver at each time step ("short cycling")

ii) Correlated sampling : freezing noise

NB: Soledge2D relies on a mixed implicit/explicit scheme

$$\Delta t \simeq 10^{-8} s$$

Examples of neutral particle density maps



Outline

1) General considerations on transport codes convergence

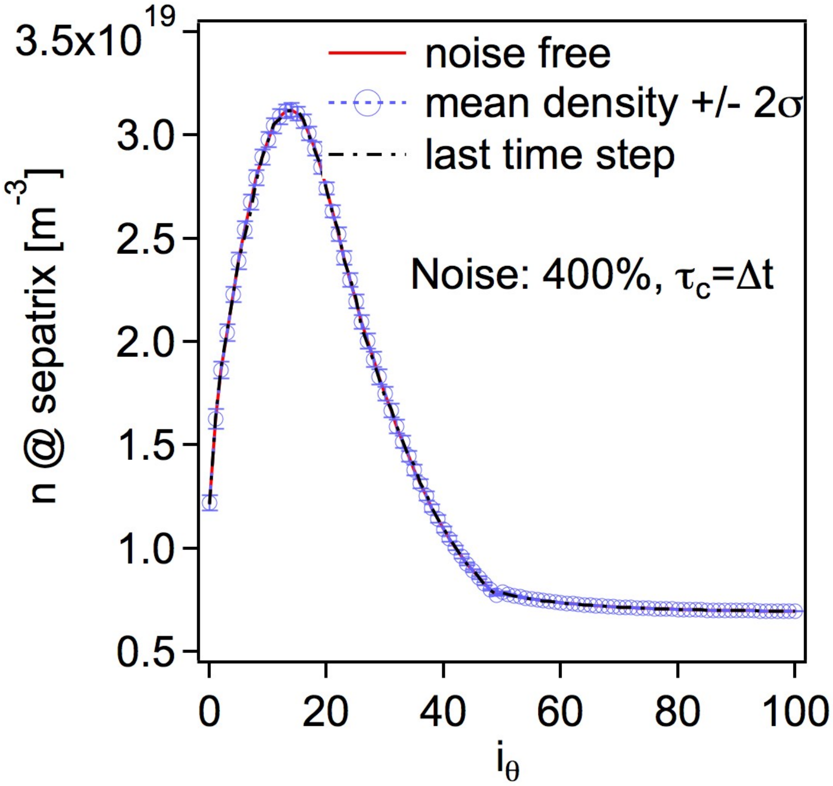
2) Simplified model with synthetic noise

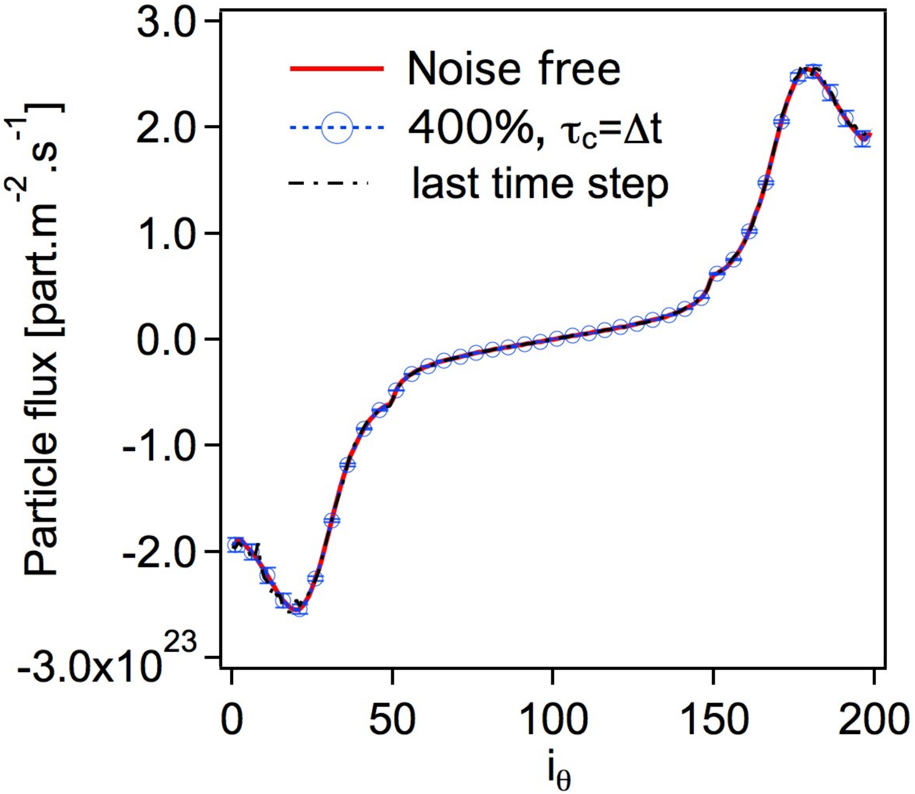
3) **Effect of noise on the simulations**

4) Conclusions and Perspectives

The (toy) system is robust to noise

- ✓ **Push the model close to the brink: R=0.99, "detached"**
- ✓ **Start with $\tau_c=\Delta t$, ramping up the noise level up to 400%**

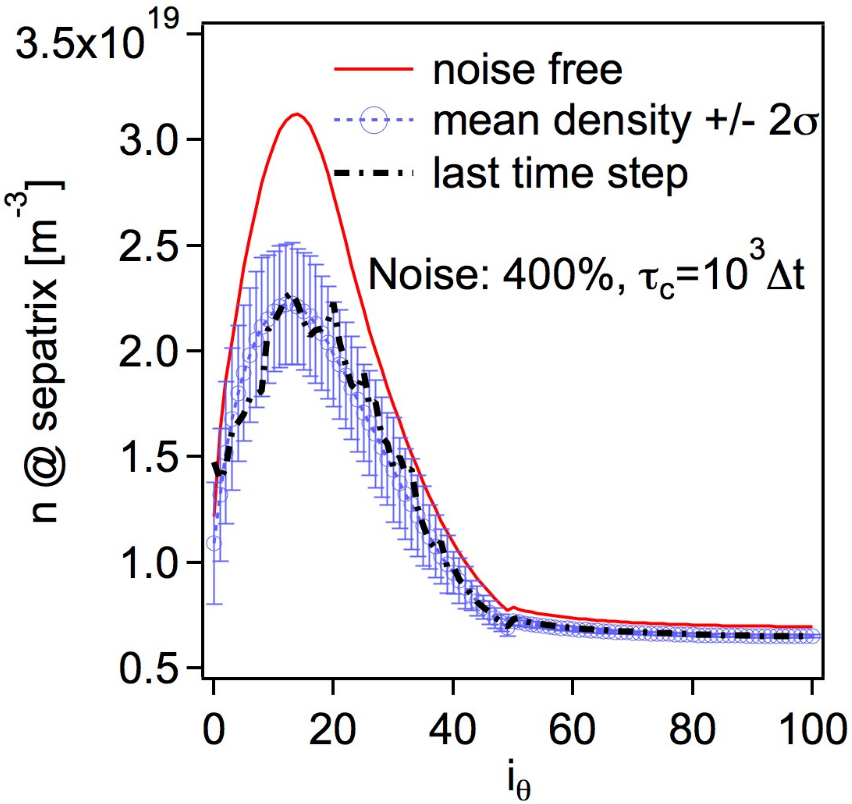


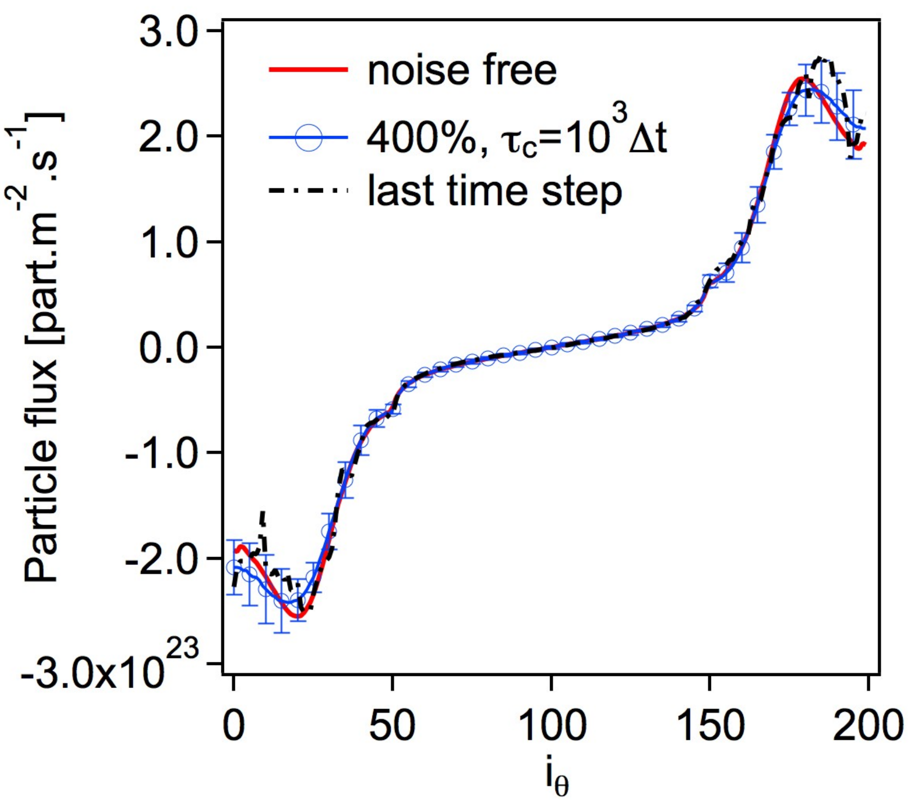


Effects of noise *immaterial* even for such strong amplitudes

Role of the noise correlation time

✓ **Same noise level, but $\tau_c = 10^3\ \Delta t$: things start to go wrong …**



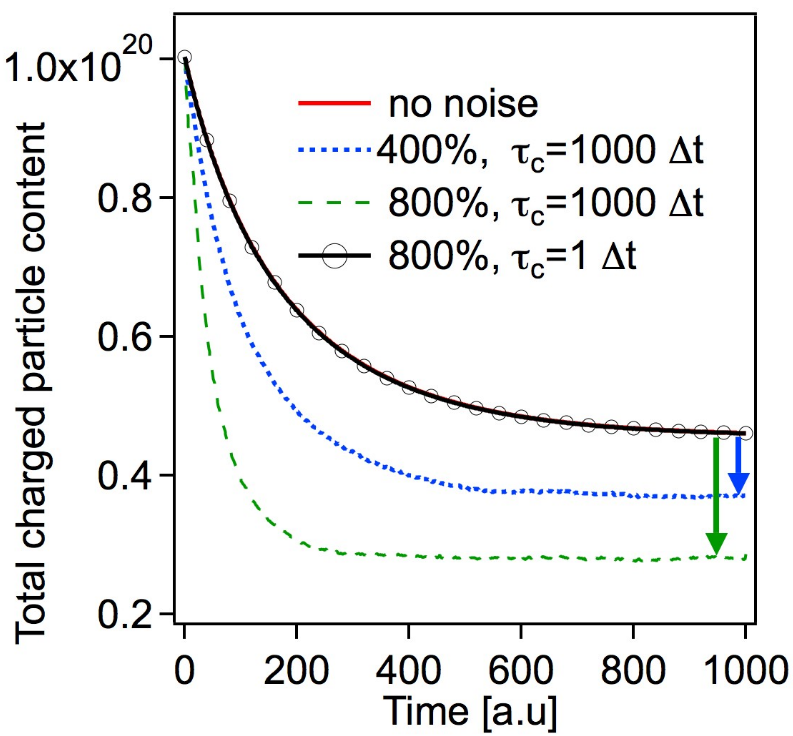


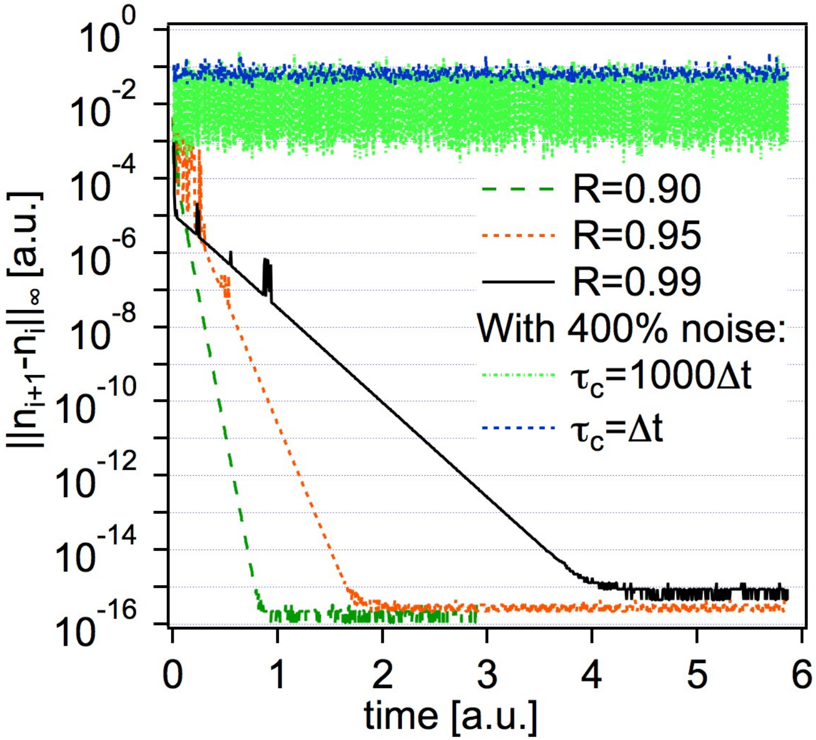
✓ **Noise with short correlation is filtered out by the system**

✓ **Here frozen long enough to build strong gradients**

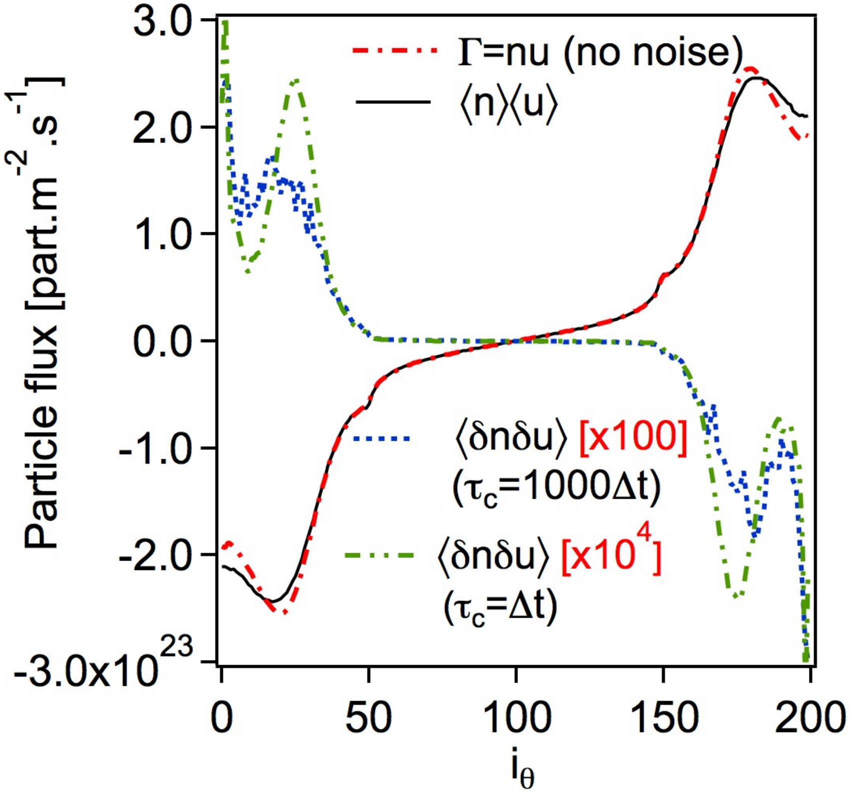
Particle balance, total content and residuals

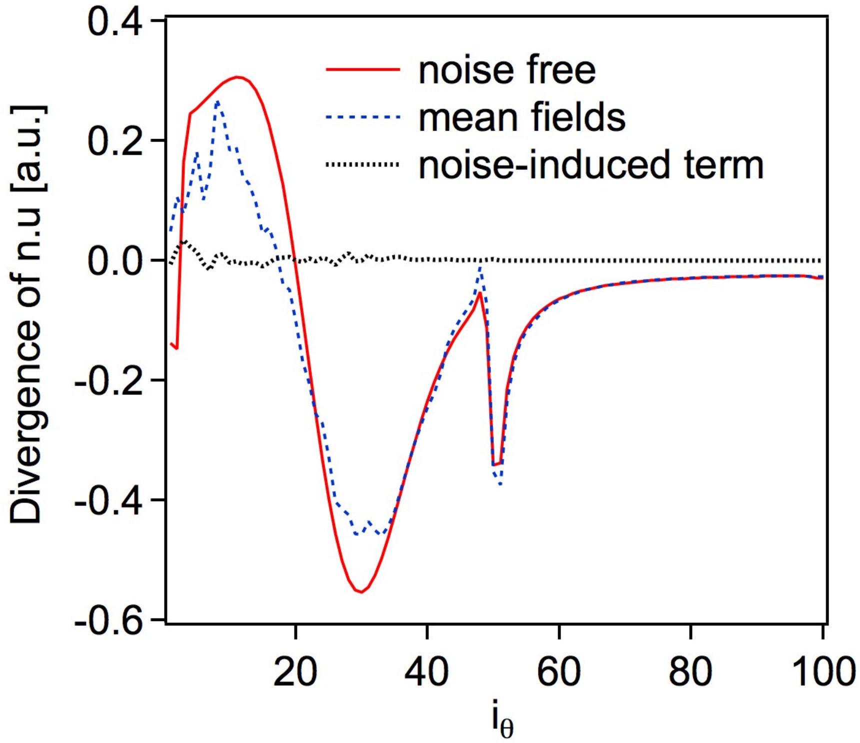
- ✓ **Mean particle balance** $(\langle\Gamma_{in}\rangle + \langle S\rangle - \langle\Gamma_{out}\rangle)/\langle\Gamma_{in}\rangle = 4\times 10^{-3}$
- ✓ **Total content: τ_0 goes down with noise**
- ✓ **Residuals reflect non-stationarity, and "frustrated" relaxation**





First analysis of noise-induced terms





Another possible culprit :

numerical diffusion in the advection scheme ?

Outline

1) General considerations on transport codes convergence

2) Simplified model with synthetic noise

3) Effect of noise on the simulations

4) Conclusions and Perspectives

Conclusions perspectives

- ✓ **The « converged » SS allows one to define a proper stationary solution *and* evaluate its distance to the noise free solution**

- ✓ **First results with our synthetic noise model suggest that there is no strong bias unless relative fluctuation level is very large**

  AND the noise is strongly time correlated

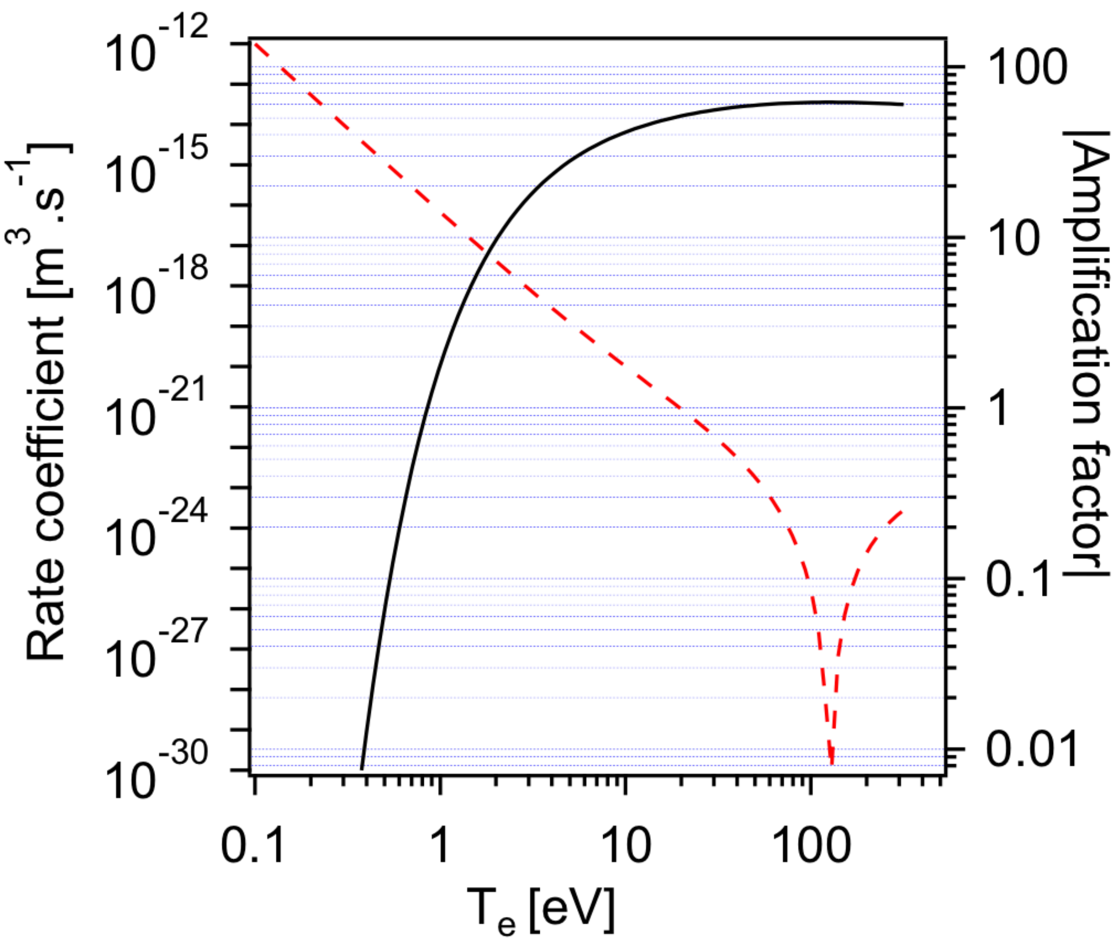
  (see how this transfers *quantitatively* to real life cases …)

- ✓ **With this procedure the KUL group has been able to converge SOLPS ITER simulations ~ 50x faster than what was previously done (Baelmans et al., PSI 2016)**

- ✓ **Effects of numerical scheme need to be assessed too.**

- ✓ **More work needed to see whether practical criteria ruling out strong biases from noise can be established**

Noise amplification by non-linearities



Details on the sampling procedure

✓ **After evolution of neutral fluid model $n_0(r,t_{i+1})$**

At each point r_j in space:

sample with mean $n_0(r_j,t_{i+1})$ and s.d. R $n_0(r_j,t_{i+1})$

calculate S_n, S_m and $S_{Ee,i}$ including noise