DE LA RECHERCHE À L'INDUSTRIE

Gyrokinetic simulations with GYSELA: Main current issues in physics & numerics

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Basics on Gyrokinetics

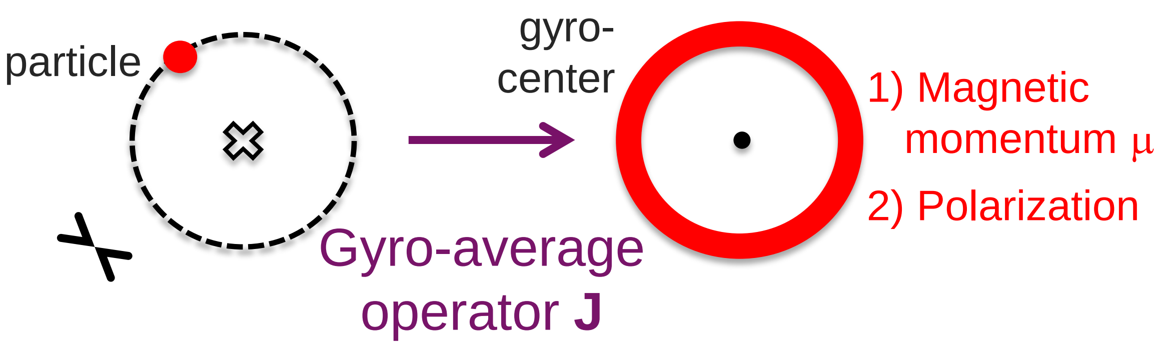
- Fusion plasmas weakly collisional (ITER: $\nu^* \sim n/T^2 \sim 10^{-3} \sim qR/\lambda_{\ell pm}$)

  F can depart from a Maxwellian $\Rightarrow$ kinetic description mandatory

- Gyro-freq. $d\varphi_c/dt \sim 10^8$ >> Turb. freq.$\sim 10^5$ $\rightarrow$ φ_c can be safely "averaged out"

  $\Rightarrow$ phase-space reduction: 6D $F(\mathbf{x},v_{//},\mu,\varphi_c,t)$ $\rightarrow$ 4D+1D $F_G(\mathbf{x}_G,v_{//},\mu,t)$

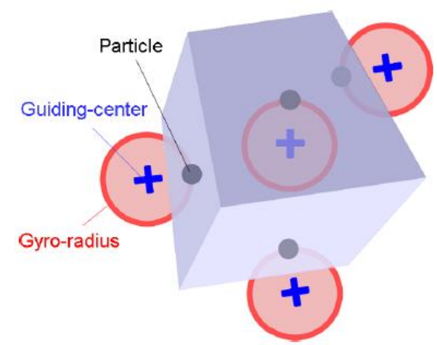
  Adiabatic invariant $\mu = mv_\perp^2/2B$



[Frieman-Chen, 1982, Littlejohn, 1983; Brizard-Hahm, 2007]

- Maxwell's eqs. on F $\Rightarrow$ requires relation $F \leftrightarrow F_G$

  $\Rightarrow$ polarization density: $n = n_G + n_{pol}$

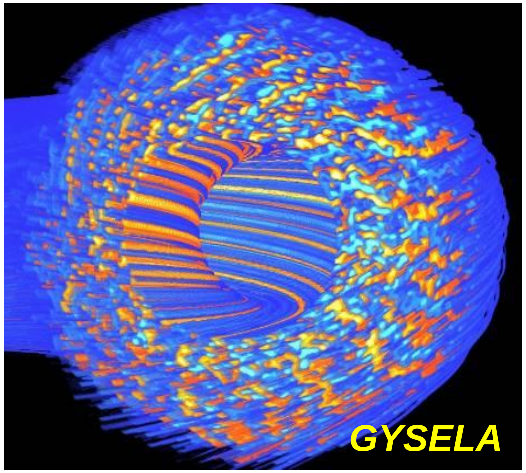


The semi-Lagrangian GYSELA code

[Grandgirard, CPC 2016]

- Self-consistently coupled Gyrokinetic & Quasi-Neutral eqs:

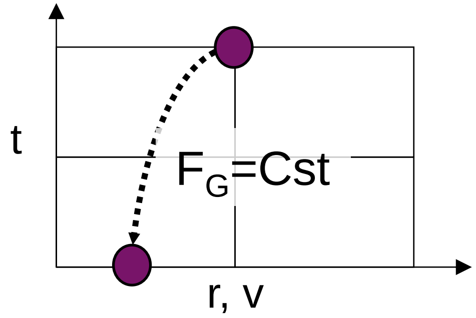
$$\begin{cases} \underbrace{dF_{Gs}/dt}_{\text{4D advection}} = \underbrace{S}_{\text{Source}} + \underbrace{C(F_{Gs})}_{\text{Collisions}} + \underbrace{D_{BC}}_{\text{Boundary Conditions}} \\ L(\phi) = \Sigma_s \int d\mathbf{v}\, J_s.F_{Gs} \end{cases}$$



- Peculiarities:
  - global → boundary conditions
  - full-F → multi-scale physics
  - flux-driven → steady-state on τ_E

- Backward semi-Lagrangian scheme:

  Trajectories (F_G=Cst) followed backwards on fixed grid (weak noise, moderate dissip.)



Outline

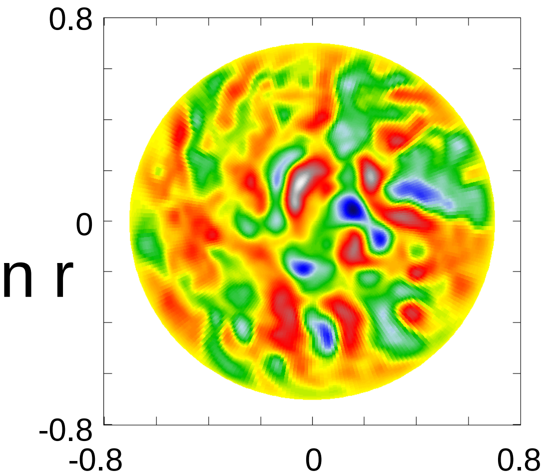
Physics upgrades → numerical challenges → present solutions & issues

- Boundary conditions: core (r=0) & scrape-off layer
- Kinetic electrons: Field aligned method & open issues
- Small scale physics: gyro-average operator & boundary issue
- Neoclassical transport: collision operator & parallelization issue

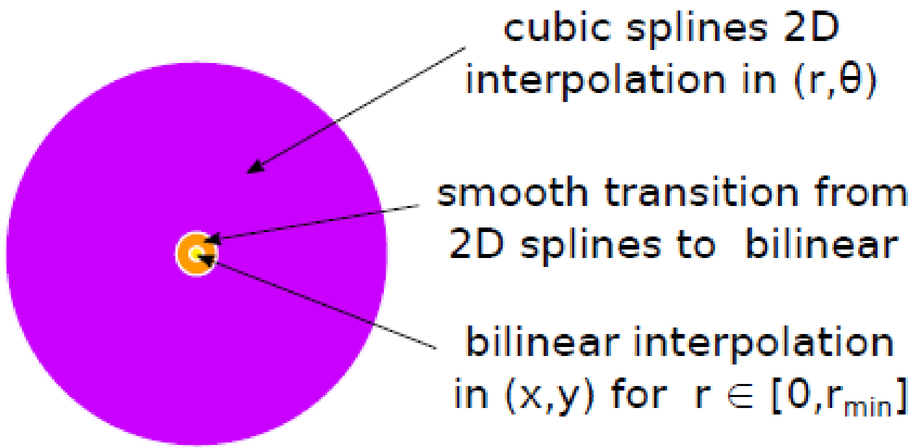
Removing Inner Boundary Condition: r = 0

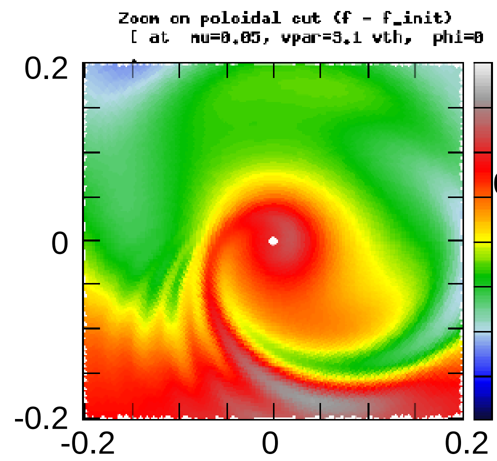
Issue at r=0: divergence of metric (1/r) + too many θ points

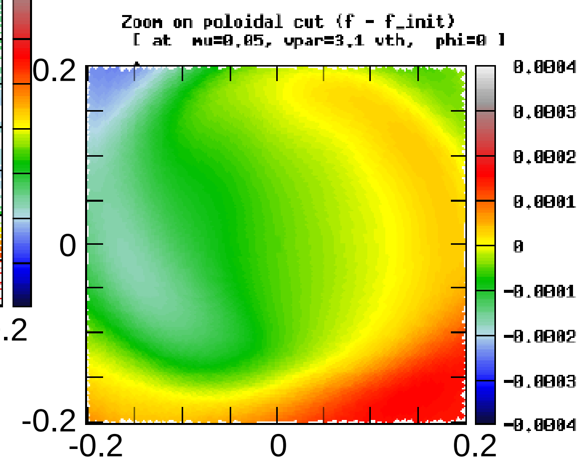
- Previously: $r_{min}>0$ → Dirichlet for ϕ_{mn} / Neumann for ϕ_{00}
- Upgrade:
  - Poisson (trick): $r_{min}=\Delta r/2 \Rightarrow$ no BC required in r
  - Vlasov: bilinear interpolation in $0<r<r_{min}$



[Latu-Mehrenberger, 2016]







Towards Scrape-Off Layer physics r>a

- Coupling core (r/a<1) – SOL (r/a>1) is important: H-mode, impurities & neutrals

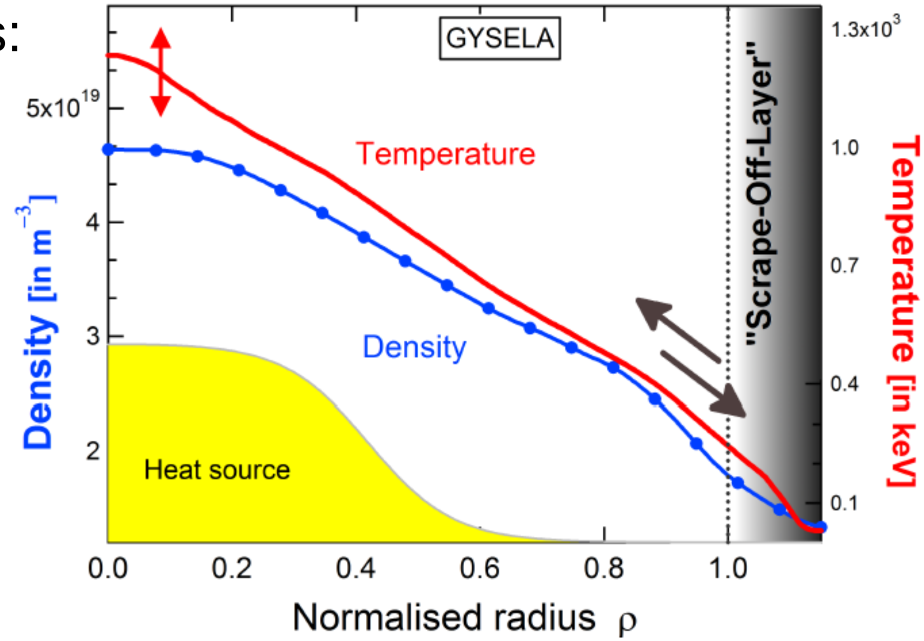
  Critical challenges:
  - close/open magnetic surfaces (periodicity; plasma-surface interaction)
  - relative fluctuation levels
  - particle sources/sinks

- Forced relaxation towards SOL-like profiles:

$$\underbrace{\frac{DF}{Dt} = \mathcal{C}(F) + \mathcal{S}(F)}_{\text{core — edge}} - \underbrace{\nu(F - F_{SOL})}_{\text{"SOL-like"}}$$

  - Smooth transition towards vanishing fluctuations
  - Some evidence of SOL $\rightarrow$ core interplay

[Dif-Pradalier, 2016]



- Possible alternatives: penalization and/or transition towards fluid description?

Kinetic electrons & spurious ω_H modes

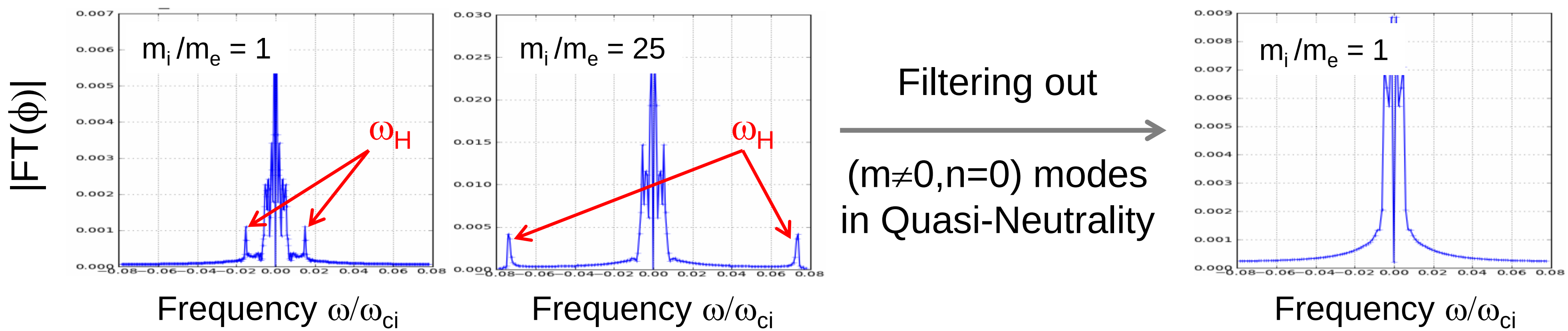
- Kinetic electrons mandatory: particle transport + trapped electron modes
- Electrostatic limit → spurious "ω_H" modes: $\omega_H / \omega_{ci} = (k_{//} / k_\perp)(m_i/m_e)^{1/2}$ [Lee 1987]
  - Correspond to hydro-dynamical limit ($\omega \gg k_{//} v_{th}$) of ITG disp. rel.
  - Also: electrostatic limit ($\beta=0$) of kinetic Alfvén wave

$$\omega_{KAW}^2 = k_\parallel^2 v_A^2 \frac{1+k_\perp^2 \rho_i^2}{1+k_\perp^2 d_e^2} = \frac{k_\parallel^2 \rho_i^2 \;\; \omega_{ci}^2}{k_\perp^2 \rho_i^2 (m_e/m_i) + \beta/2} \left(1+k_\perp^2 \rho_i^2\right)$$ [Scott 1997]

$\Rightarrow$ Should disappear in electromagnetics (for $\beta > (k_\perp \rho_i)^2 m_e/m_i \sim 2.10^{-5}$)

  - Trick: disappear when filtering out ($m\neq0,n=0$) modes in QN eq. [Idomura 2016]

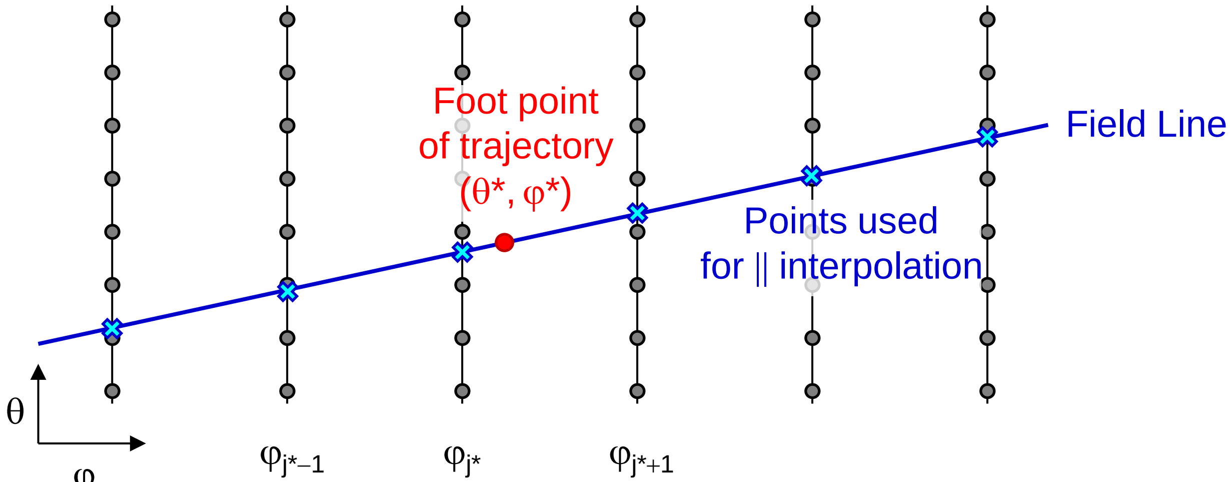
[Ehrlacher 2016]



Kinetic electrons & Field-aligned method

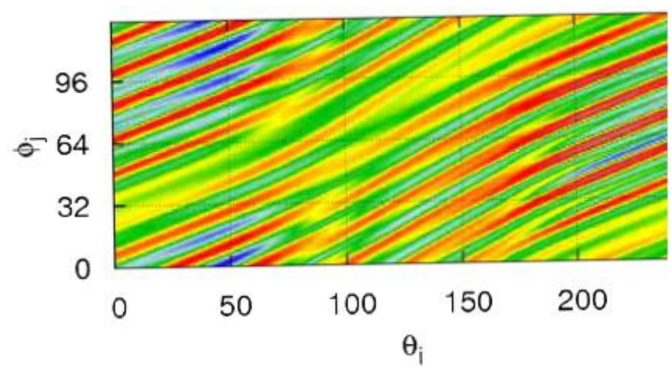
- Kinetic electrons mandatory: particle transport + trapped electron modes
- Numerical issues
  - $v_{the} \sim (m_i/m_e)^{1/2} \times v_{thi} \sim 10^8 m.s^{-1}$ $\Rightarrow$ time step / $(m_i/m_e)^{1/2}$
  - $\rho_e \sim \rho_i/(m_i/m_e)^{1/2} \sim \rho_i/60 \sim 50\mu m$ $\Rightarrow$ nb grid points $\times (m_i/m_e)^{3/2}$
- Reducing numerical cost:
  - filtering passing electrons (adiabatic) $\rightarrow$ artificially large (m_e/m_i) ~OK [Bottino 2016]
  - Using field-aligned method $\rightarrow$ nb grid points $\times(m_i/m_e)$ only

[Ottaviani 2011; Hariri 2013]



[Latu-Mehrenberger, 2016]

- Optimized parallelization: memory OK, time+30%



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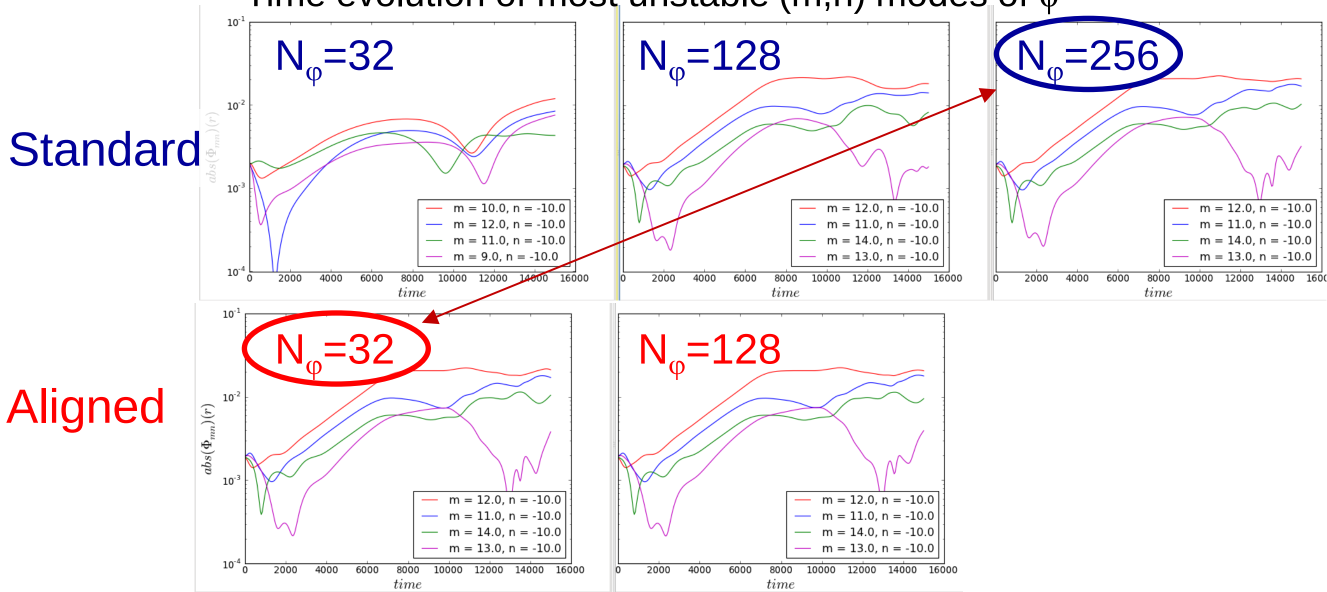
[Ottaviani 2011; Hariri 2013]

Time evolution of most unstable (m,n) modes of ϕ



- Optimized parallelization: memory OK, time+30%
- Less toroidal points: ~ N_φ / 8

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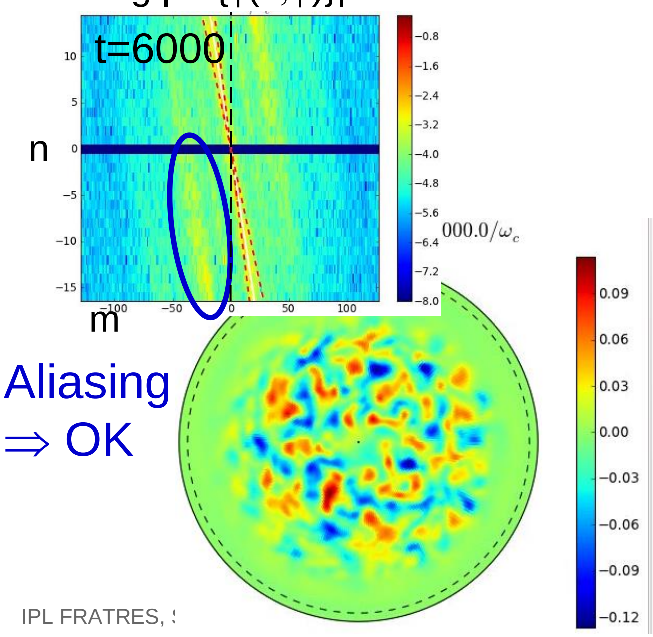
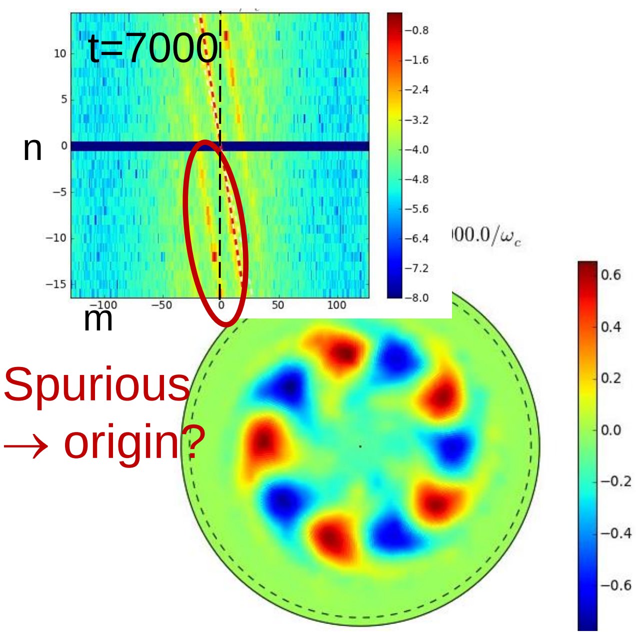
Log |FT{ϕ(θ,φ)}|



Aliasing
⇒ OK



Spurious
→ origin?

- Optimized parallelization: memory OK, time+30%
- Less toroidal points: ~ N_φ / 8
- Spurious modes: under investigation

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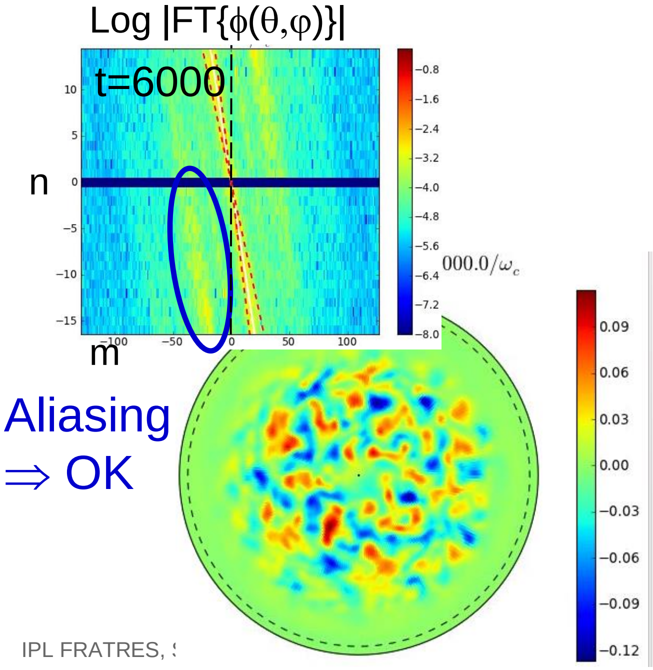
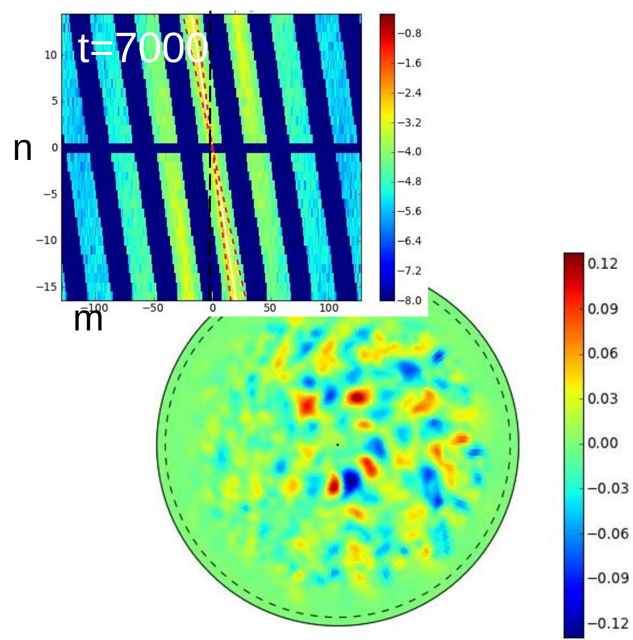




- Optimized parallelization: memory OK, time+30%
- Less toroidal points: ~ N_φ / 8
- Spurious modes: under investigation
  $\Rightarrow$ OK when filtered out

Small scales & gyro-average operator

- Gyro-average → Finite Larmor Radius effects

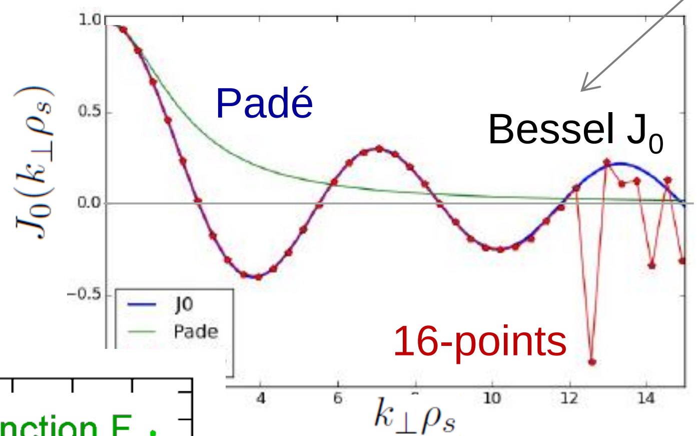
$$\bar{g}(\mathbf{x}_G, v_\perp) = \oint_0^{2\pi} \frac{d\varphi_c}{2\pi} g(\mathbf{x}) = \int_{-\infty}^{+\infty} \frac{d^3\mathbf{k}}{(2\pi)^3} J_0(k_\perp \rho_s) \hat{g}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}_G}$$

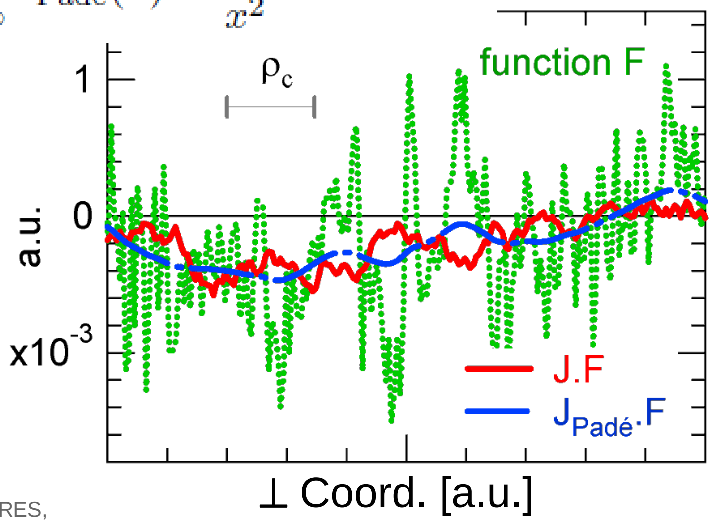
- From **Padé** to **N-point average**

Padé: $1 / [1+(k_\perp \rho_c/2)^2]$

small scales filtered out

$$\begin{cases} \lim\limits_{x\to\infty} J_0(x) = \sqrt{\frac{2}{\pi}} \frac{\cos(x-\pi/4)}{x^{1/2}} \\ \lim\limits_{x\to\infty} J_{\text{Padé}}(x) = \frac{4}{x^2} \end{cases}$$

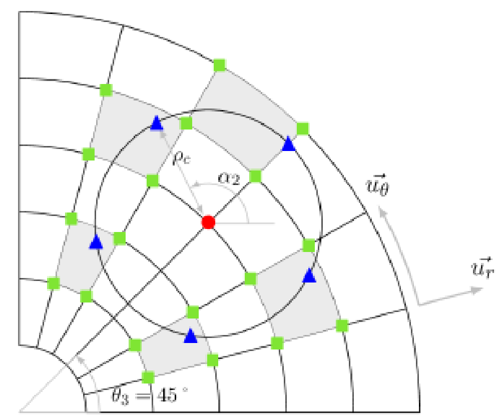




N-point average (Hermite):

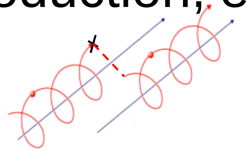
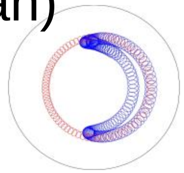
- Convergence reached for N=8 points (ion turb.)
- Issue: boundary condition?

[Steiner, 2015; Rozar, 2015; Bouzat, 2016]



Collisions: physical & numerical issues

- Critical for:
  - flow damping: friction on trapped particles, Zonal Flow damping
  - impurity transport (synergy turbulent-neoclassical) [Estève 2016]
  - momentum & energy exchanges between species

- Constraints: [Hirshman-Sigmar 1977; Helander-Sigmar 2005]
  - Boltzmann H-theorem (entropy production, equil.=Maxwellian)
  - Neoclassical transport = collisions  + trajectories 

- Collisions break down μ-invariance → parallelization issue [Garbet 2009; Dif-Pradalier 2011; Estève 2015]

$$C_a(F_a) = \sum_b \left[\frac{T_b - T_a}{T_a} \frac{m_a v^2}{2T_a} \nu_{E,ab} - \nu_{s,ab}(v) \frac{m_a}{T_a} v_\parallel \left(U_{\parallel d,a} - U_{\parallel ba} \right) \right] F_{M0a} + C_{v,ab}(F_a) + C_{d,ab}(F_a)$$

Energy exchange — Momentum exchange — **v**-motion (radial & deflection)

- Conservation properties OK (on $\tau_{coll.}$):

$$\frac{\Delta n}{n} \simeq 10^{-5} \quad \frac{\Delta p_\parallel}{p_\parallel} \simeq 10^{-5} \quad \frac{\Delta E}{E} \simeq 10^{-4}$$

- Neoclassical results under investigation

[Donnel 2016]

Projection on Laguerre polynomials in $u = \frac{\mu B}{T}$

$$F(\boldsymbol{r}, v_\parallel, u, t) = F_{M0a} \sum_l \alpha_\ell(\boldsymbol{r}, v_\parallel, t) P_\ell(u)$$

$\Rightarrow$ replace differential operators by integrals

Collisions: physical & numerical issues

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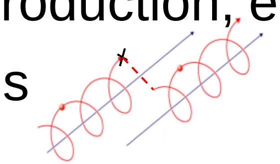
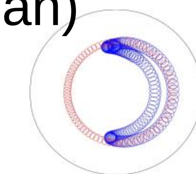
  - Boltzmann H-theorem (entropy production, equil.=Maxwellian)
  - Neoclassical transport = collisions + trajectories





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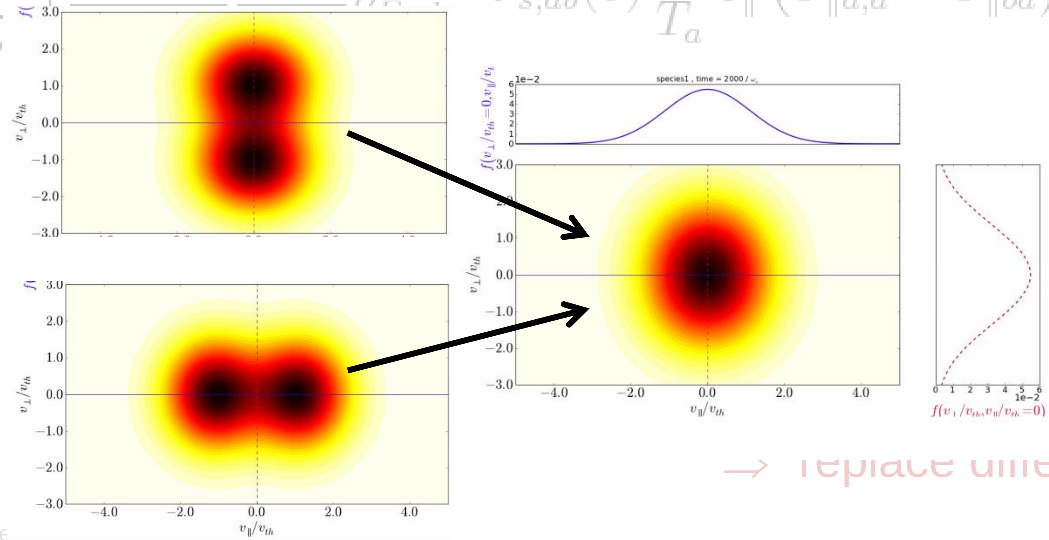
v-motion (radial & deflection)

H-theorem OK (in $v_{//}$ & $v_\perp$):



...aguerre polynomials in $u = \frac{\mu B}{T}$

$$F_{M0a} \sum_l \alpha_\ell(r, v_\parallel, t) P_\ell(u)$$

$\Rightarrow$ replace differential operators by integrals

Summary - conclusions

Intricate upgrades of **physics** & **numerical methods** / **parallelization**

- "Boundary Conditions": towards a model for the SOL → gyro-fluid?

- Fully kinetic electrons (trapped & passing) still out of reach on present HPC

  Electromagnetics cure electrostatic artifacts

  Trapped kinetic + heavy electrons

- Multi-scale physics requires accurate gyro-average operator

- Collisions mandatory BUT: complex (linearized) operator + parallelization issue!

Back-up slides

Collisions

[Donnel 2016]

$$C_a\left(F_a\right) = \sum_b \left\{\left[\frac{T_b - T_a}{T_a}\frac{m_a v^2}{2T_a}\nu_{E,ab} - \nu_{s,ab}(v)\frac{m_a}{T_a}v_\parallel\left(U_{\parallel d,a} - U_{\parallel ba}\right)\right] F_{M0a}\right. + C_{v,ab}(F_a) + C_{d,ab}(F_a)$$

Energy exchange — Momentum exchange — **v**-motion (radial & deflection)

$$\begin{cases} C_{v,ab}\left(F_a\right) = \dfrac{1}{2B_\parallel^\star v_\perp}\dfrac{\partial}{\partial v_\perp}\left[B_\parallel^\star F_{M0a}\nu_{v,ab}v_\perp^2\left(v_\perp\dfrac{\partial g_a}{\partial v_\perp} + v_\parallel\dfrac{\partial g_a}{\partial v_\parallel}\right)\right] \\ \qquad + \dfrac{1}{2B_\parallel^\star}\dfrac{\partial}{\partial v_\parallel}\left[B_\parallel^\star F_{M0a}\nu_{v,ab}v_\parallel\left(v_\perp\dfrac{\partial g_a}{\partial v_\perp} + v_\parallel\dfrac{\partial g_a}{\partial v_\parallel}\right)\right] \\ C_{d,ab}\left(F_a\right) = \dfrac{1}{2B_\parallel^\star v_\perp}\dfrac{\partial}{\partial v_\perp}\left[B_\parallel^\star F_{M0a}\nu_{d,ab}v_\perp v_\parallel\left(v_\parallel\dfrac{\partial g_a}{\partial v_\perp} - v_\perp\dfrac{\partial g_a}{\partial v_\parallel}\right)\right] \\ \qquad + \dfrac{1}{2B_\parallel^\star}\dfrac{\partial}{\partial v_\parallel}\left[B_\parallel^\star F_{M0a}\nu_{d,ab}v_\perp\left(-v_\parallel\dfrac{\partial g_a}{\partial v_\perp} + v_\perp\dfrac{\partial g_a}{\partial v_\parallel}\right)\right] \end{cases}$$

$$g_{ab} = f_a - \frac{m_a v_\parallel U_{\parallel d,a}}{T_a} - \frac{m_a v^2}{2T_a}q_{ba}$$

$$q_{ab} = T_b\frac{\left\langle \nu_{E,ab}\frac{m_a v^2}{2}f_a\right\rangle_a}{\left\langle \nu_{E,ab}\left(\frac{m_a v^2}{2}\right)^2\right\rangle_a}$$

$$\left\langle \dots \right\rangle_a = \int d^3\boldsymbol{v}\,\frac{F_{M0a}}{n_a}\dots$$

- These terms are treated by projection on Laguerre polynomials, using a Crank-Nicolson scheme
- All the other terms are treated via finite differences with an explicit scheme