

# Bipartite graphs for computational modeling in systems biology: from KEGG to Petri nets

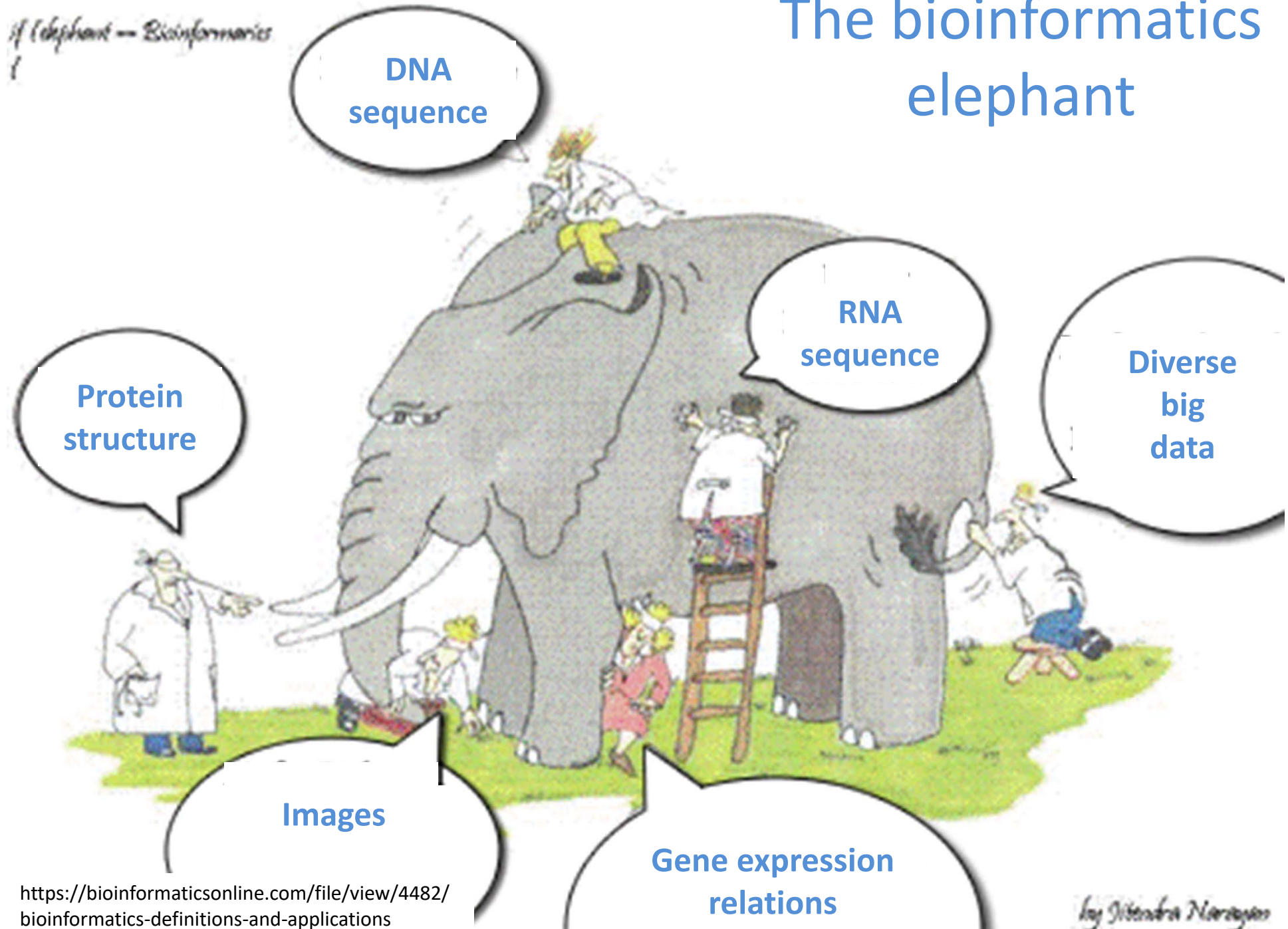
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[www.bioinformatik.uni-frankfurt.de](http://www.bioinformatik.uni-frankfurt.de)

*November, 14<sup>th</sup> 2023*

*3<sup>rd</sup> Edition of the Workshop "Metabolism and mathematical models: Two for a tango"*

# The bioinformatics elephant



<https://bioinformaticsonline.com/file/view/4482/bioinformatics-definitions-and-applications>

*by Sitenra Narayan*

# Challenges of data integration

- ❖ Incomplete data
- ❖ Different time points and different locations in the cell
- ❖ Incomplete data **The quality and quantity of the data determine the modeling approach**
- ❖ Different experiments under varying experimental conditions
- ❖ Different scales: genomics, transcriptomics, proteomics, metabolomics, interactomics, imaging, ...

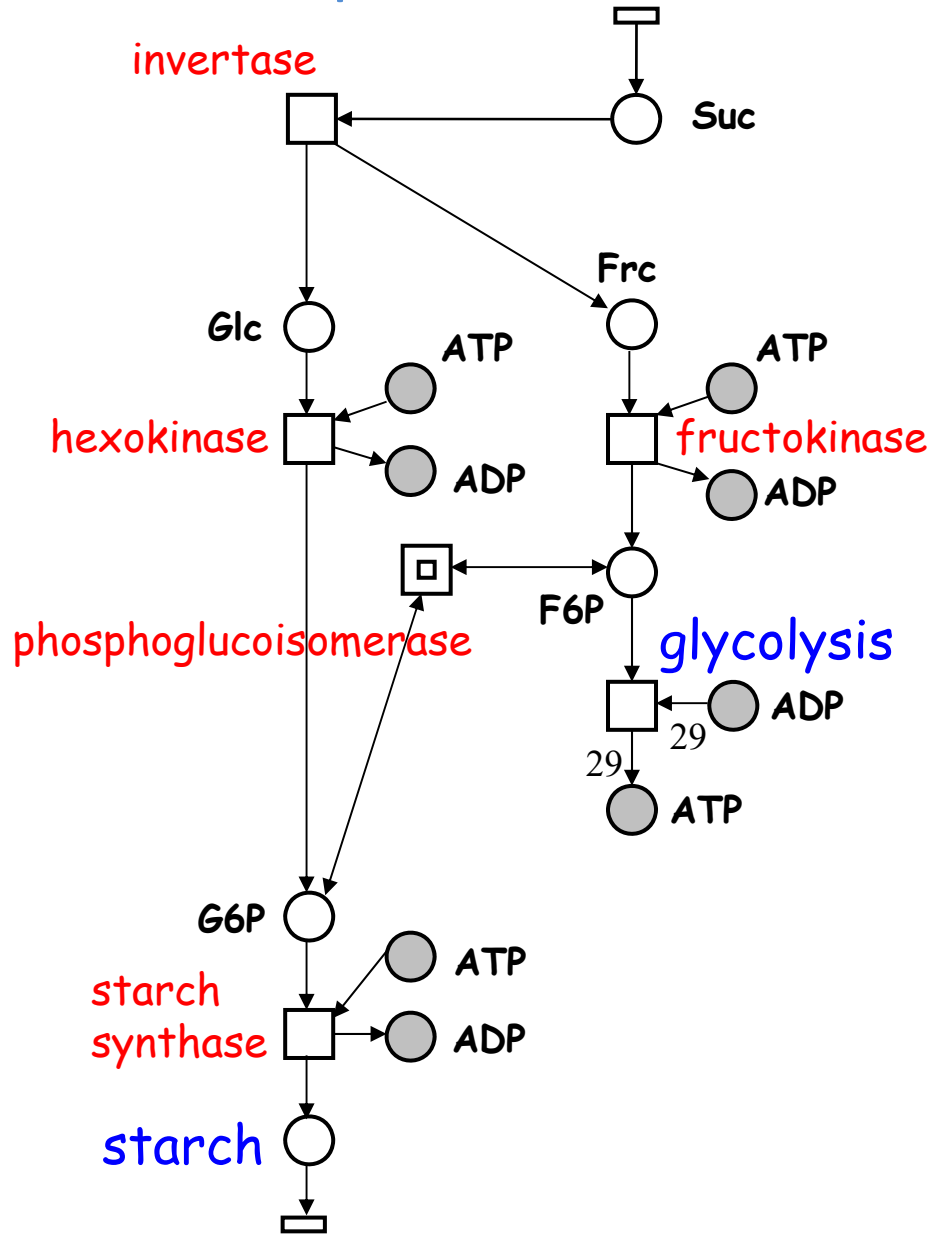
# Hypergraphs and bipartite graphs

A **hypergraph** is a generalization of a graph where edges can join any number of nodes.

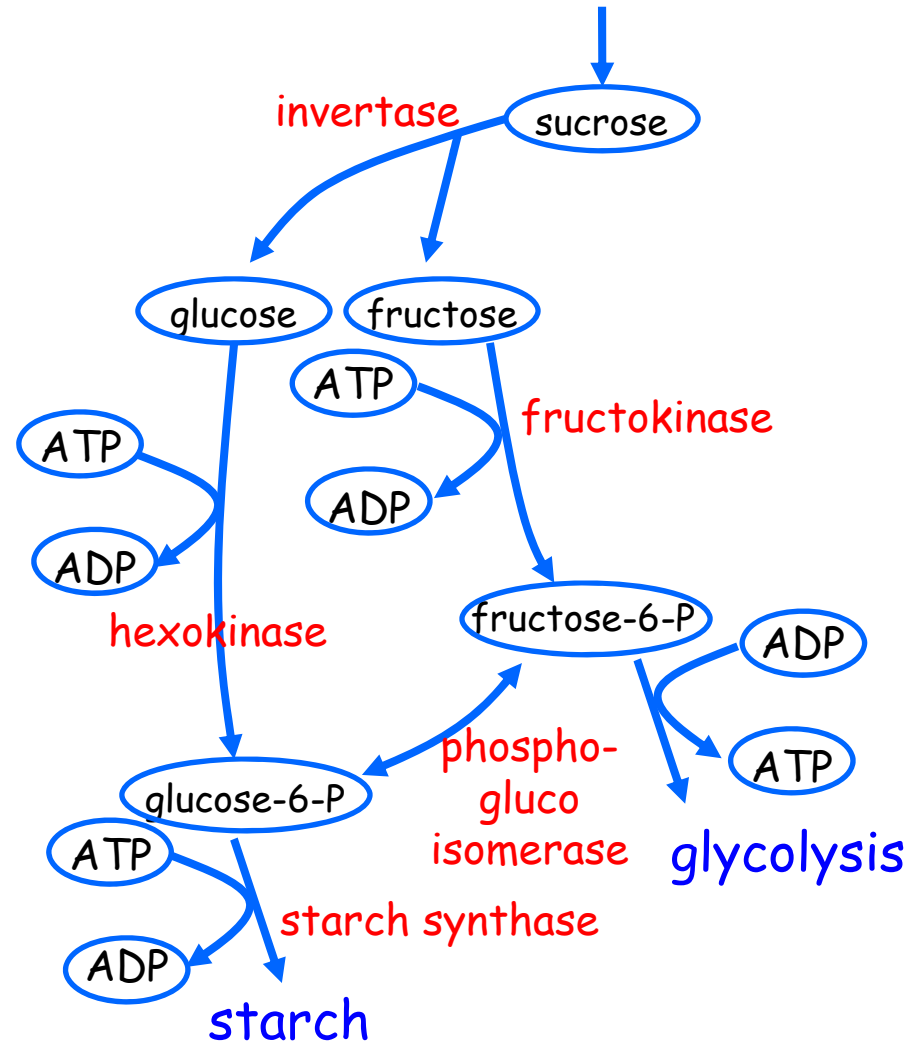
A **bipartite graph** (or **bigraph**) is a graph whose nodes are partitioned into two disjoint sets such that no two nodes within the same set are adjacent.

The image shows a screenshot of a GigaScience article page. The article title is "Bipartite graphs in systems biology and medicine: a survey of methods and applications" by Georgios A. Pavlopoulos, Panagiota I. Kontou, Athanasia Pavlopoulou, Costas Bouyioukos, Evripides Markou, and Pantelis G. Bagos. The page includes the Oxford logo and the GigaScience logo. The article is a review published in GigaScience, 7, 2018, 1-31. The DOI is 10.1093/gigascience/giy014. The advance access publication date is 19 February 2018. The page is annotated with a graph structure. A node labeled '1' is connected to nodes 'a1' and 'a3'. Node '6' is connected to node '1'. A path of nodes is shown: B (blue) - 3 (red) - A (blue) - 1 (red) - E (blue) - 5 (red) - 2 (red). Node '2' is also connected to node 'E'.

# Bipartite graph (Petri net) representation

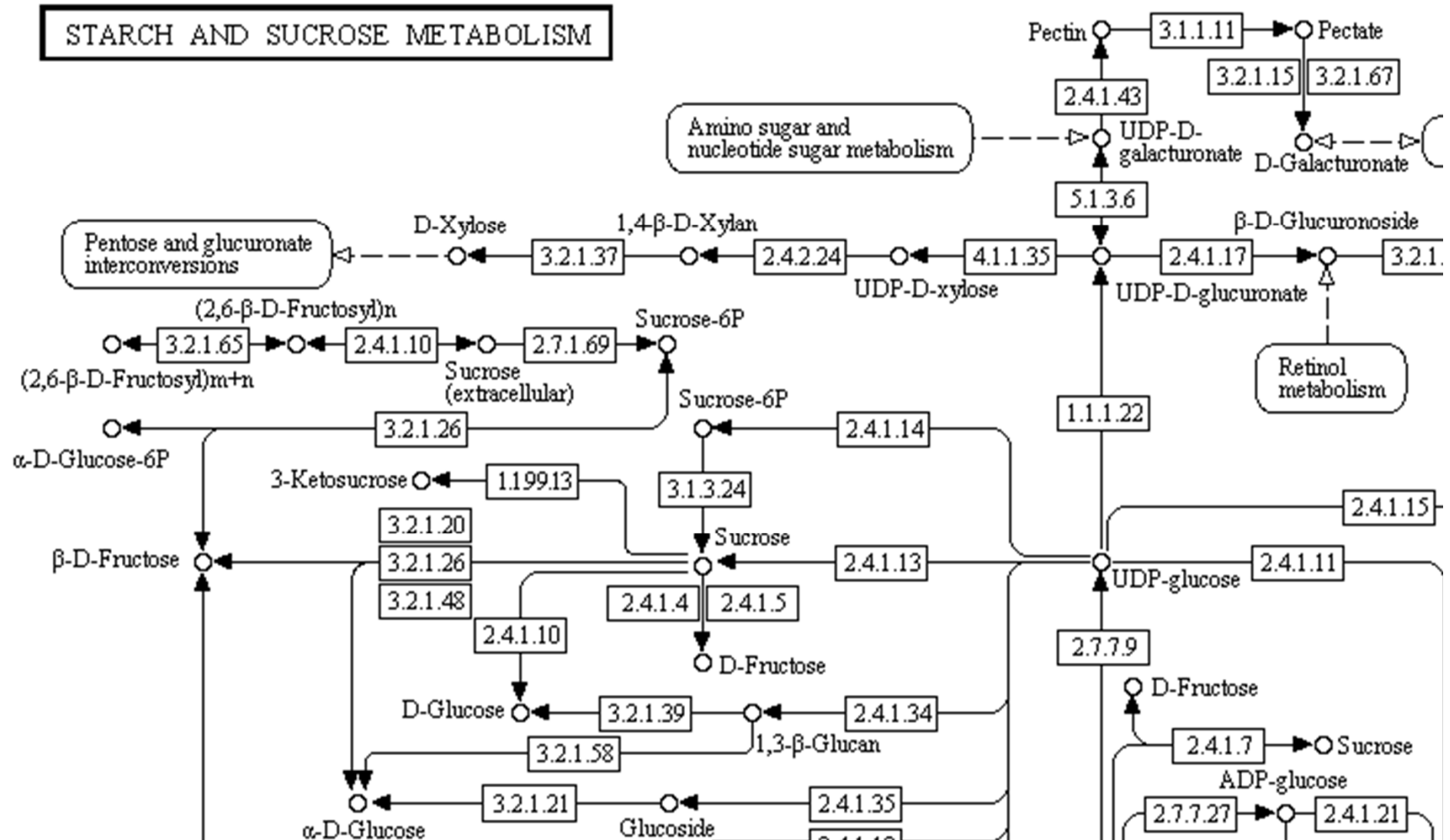


# Hypergraph representation



# Bipartite graph (KEGG) representation

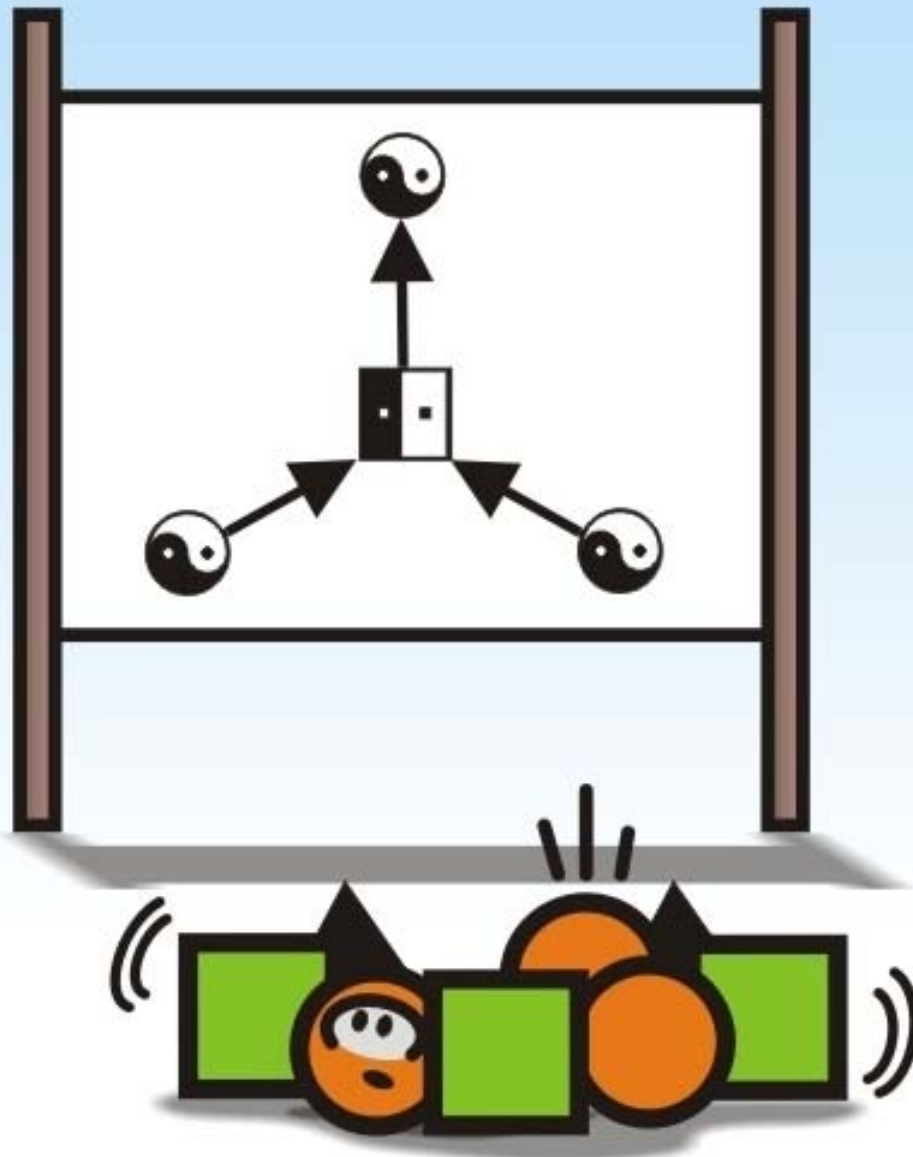
<http://www.genome.jp/kegg/pathway/map/map00500.html>



How and to what extent  
can we analyze and predict  
the system's behavior  
without  
knowing kinetic parameters?

Petri nets

# Petri nets have nothing to do with Petri dishes








Lesson I  
"Petri Nets"



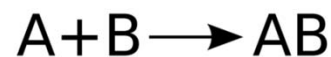
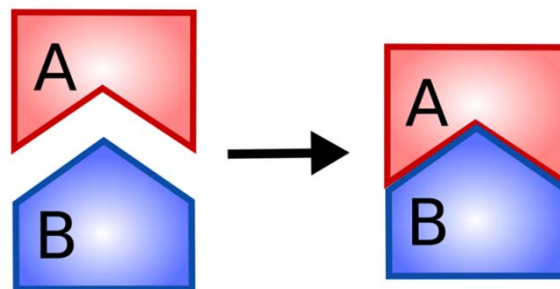
Carl Adam Petri  
(1926 – 2010)



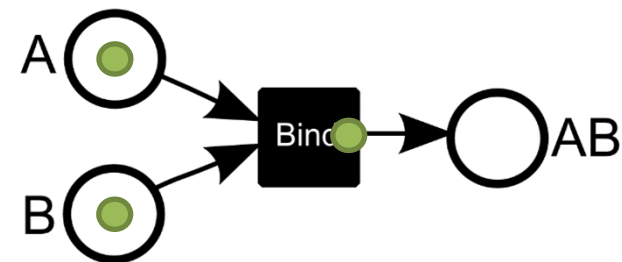
# Petri net definitions

Elements	Symbols	Interpretation
Transitions		Active elements: processes, e.g., reactions
Places		Passive elements: biological species, e.g., compounds, define pre-/post-conditions
Tokens	 	Movable objects: molecules, proteins, enzymes, cofactors, bacteria, or cells, marking as system state
Edges or arcs		Relations, weighted to quantify pre-/post-conditions

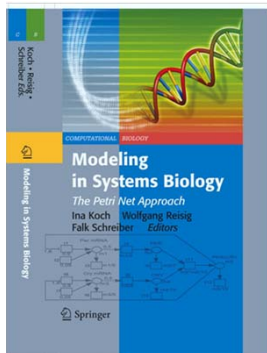
Binding process



Petri net



**Firing rule: untimed (P/T net),**  
timed-discrete, stochastic, continuous



# Outline

## **Part I - Metabolism**

- ❖ Modeling of the central carbon metabolism in potato tubers



Stéphanie Boue

## **Part II – Signal transduction**

- ❖ Modeling of TNFR1-induced signaling pathway as Petri net, including the NF- $\kappa$ B pathway



Leonie Amstein

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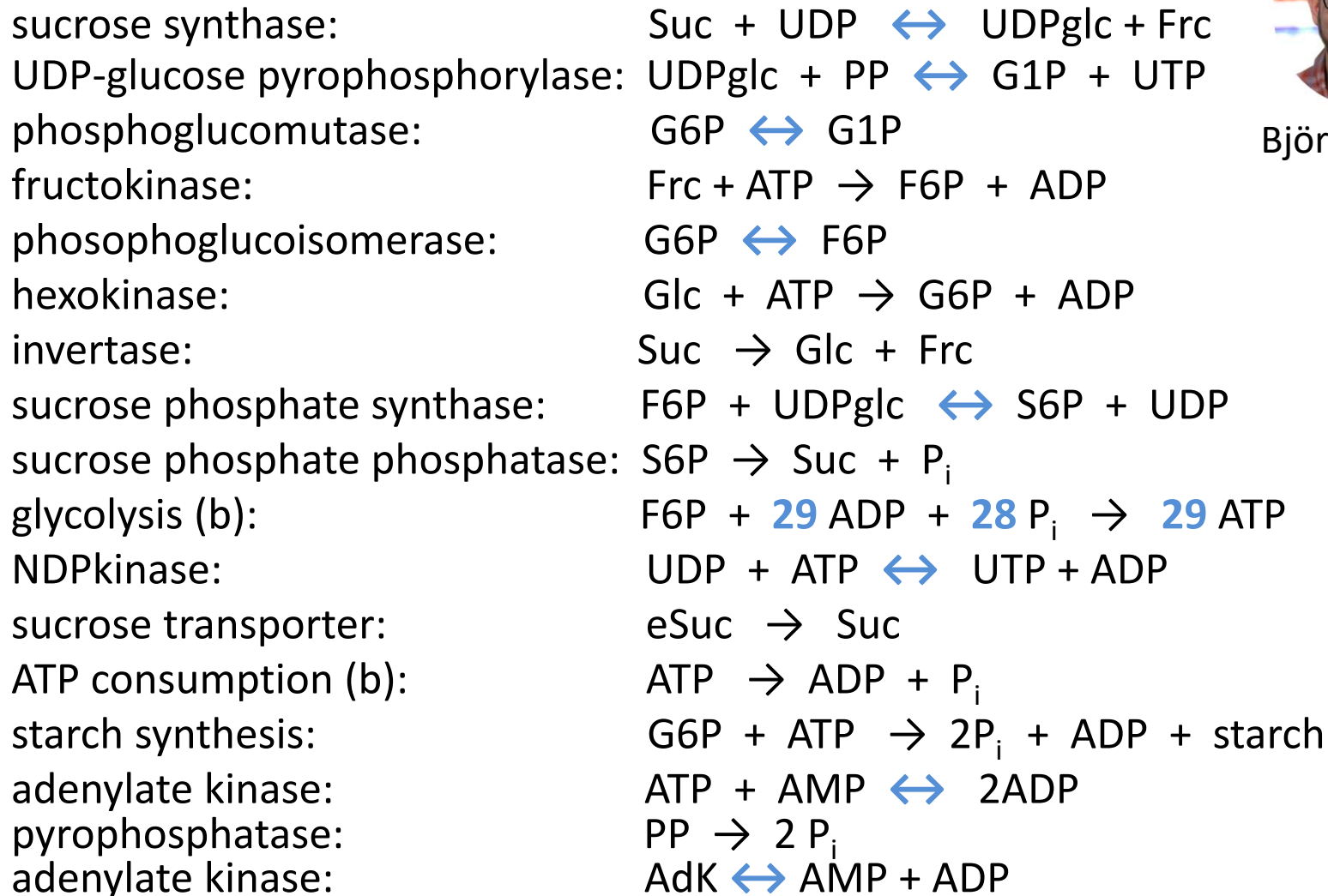


Leonie Amstein

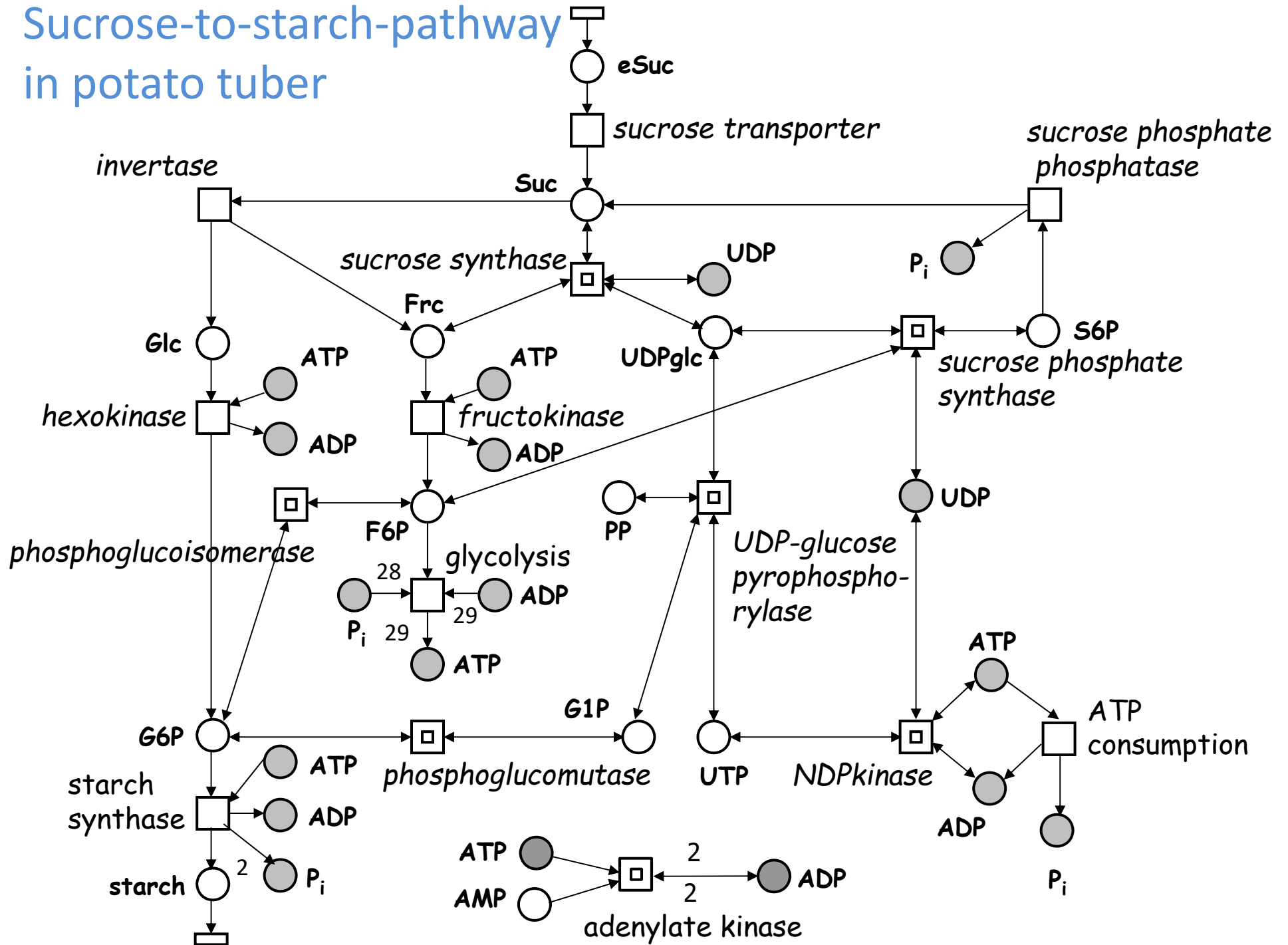
# Sucrose-to-starch-pathway in potato tuber



Björn Junker



# Sucrose-to-starch-pathway in potato tuber

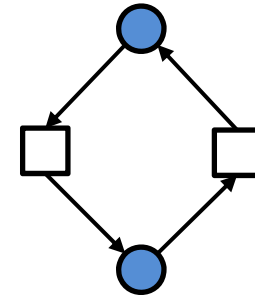


# Invariant analysis at steady state

## Substance conservations

### Place invariants

- ❖ Sets of places whose weighted sum of tokens remains constant
- ❖ Substance conservations



Place invariant:  $C^T x = 0$

Transition invariant:  $C y = 0$

Solutions: **minimal, nonnegative, nontrivial, integer**

Minimal:  $\exists z: \text{supp}(z) \subseteq \text{supp}(u)$  and the largest common divisor of all non-zero entries of  $u$  is 1

## Functional modules

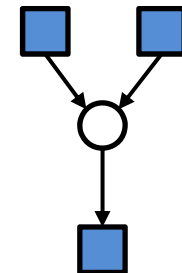
### Transition invariants

- ❖ Multi-sets of transitions whose firing reproduces an arbitrary initial marking

Lautenbach (1973) *GMD Report No. 82*

- ❖ Basic functional modules  
(elementary modes)

Schuster *et al.* (1993) *Second Gauss Symposium*



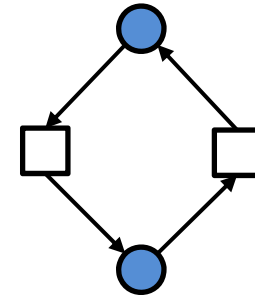
# Invariant analysis for the sucrose-to-starch-pathway

## Substance conservations

### Place invariants

The net is not covered by P-invariants.

1. UDPglc, UTP, UDP
2. ATP, AMP, ADP
3. G6P, F6P, G1P, UTP, ATP(2), ADP, S6P, P<sub>i</sub>, PP(2)



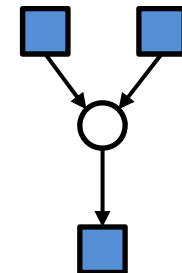
## Functional modules

### Transition invariants

The net is not covered by transition invariants!

- Trivial:
1. SPS, SPS\_rev
  2. UGPase, UGPASE\_rev
  3. SuSy\_SuSy\_rev
  4. PGM, PGM\_rev
  5. NDPkin, NDPkin\_rev
  6. AdK, AdK\_rev
  7. PGI, PGI\_rev

Removing SPS\_rev, AdK, and AdK\_rev, the Petri net becomes covered by transition invariants



# Transition invariant analysis

Invariant number	sucrose cleavage		hexoses go into		ATP cons	ATP used for cycling		
	SuSy	Inv	Glyc	StaSy		Inv SuSy_rev	Inv SPS, SPP	SuSy SPS, SPP
8	x		x	x				x
9	x		x	x				x
10	x		x	x				
11		x	x	x				
12		x	x	x			x	
13		x	x	x				x
14		x	x	x				x
15	x		x	x				
16		x	x					
17		x	x				x	
18		x	x					x
19		x	x					

Koch *et al.* (2005) *Bioinformatics*

Malik-Sheriff *et al.* (2020) *Nucl Acids Res*  
 BioModels ID: MODEL1308080002



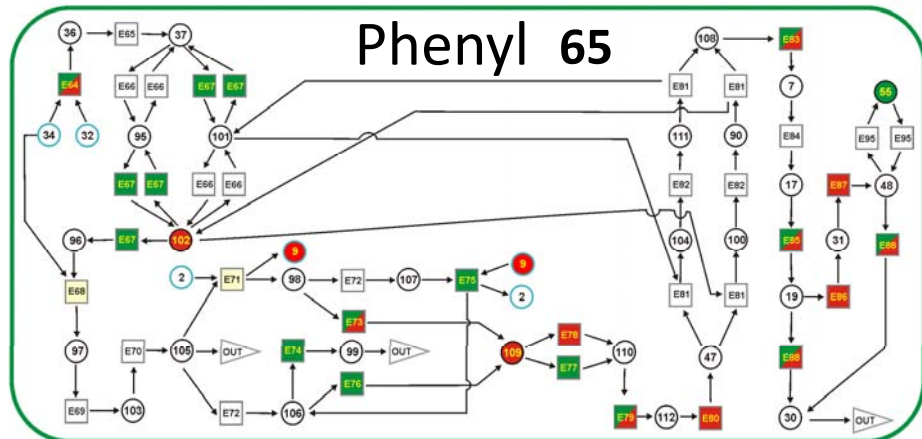
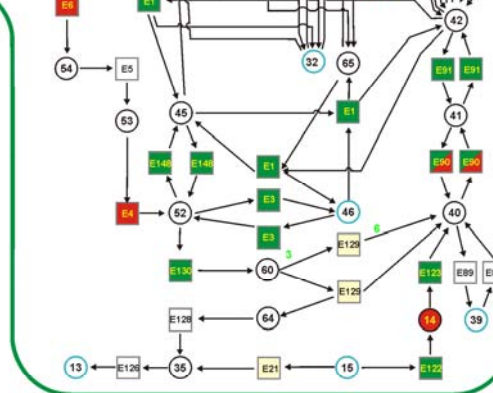
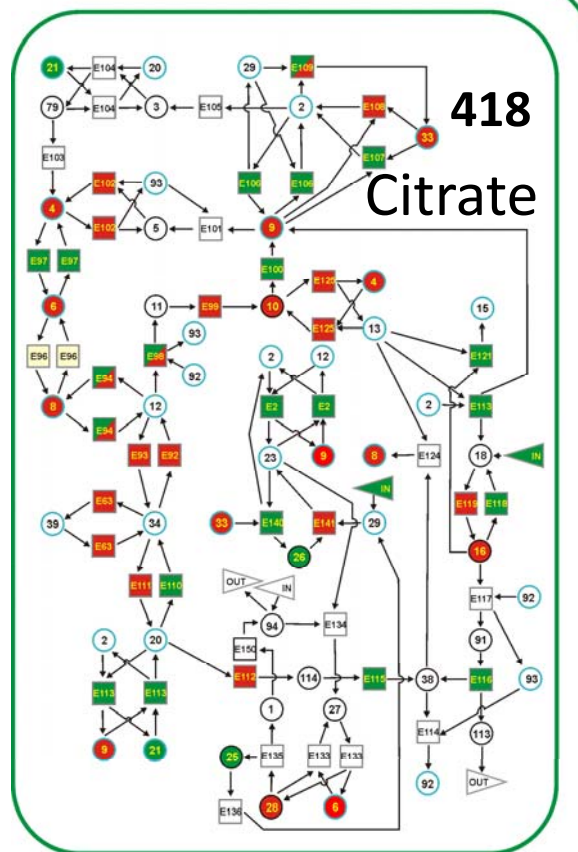
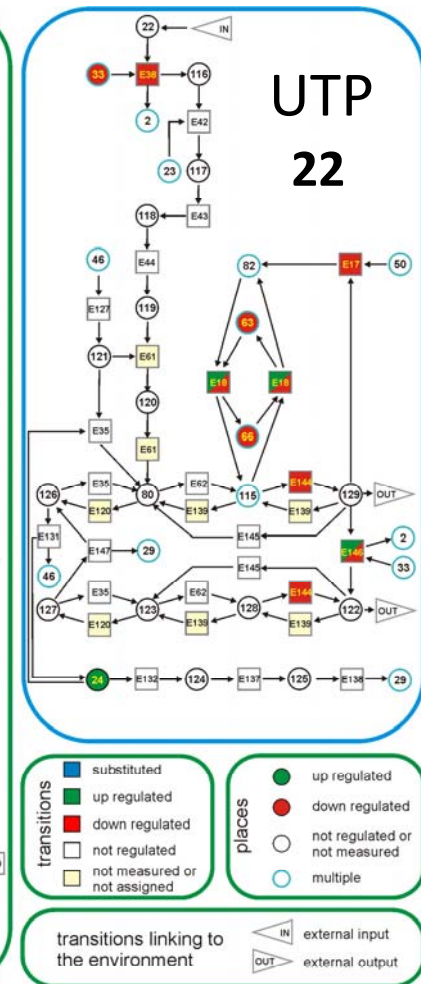
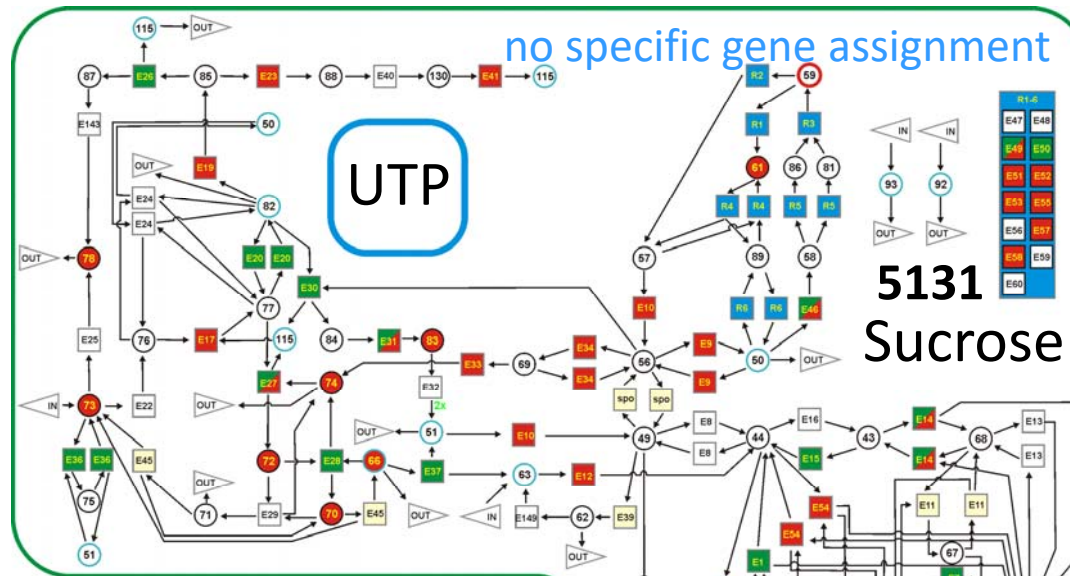
*Arabidopsis thaliana*

Complete network:  
Sucrose-,  
UTP-,  
Citrate-,  
and  
Phenyl-  
Pathway

# transition  
invariants of  
each subnet

Koch et al. (2017)  
*Frontiers in genetics*

**MODEL1801090001**

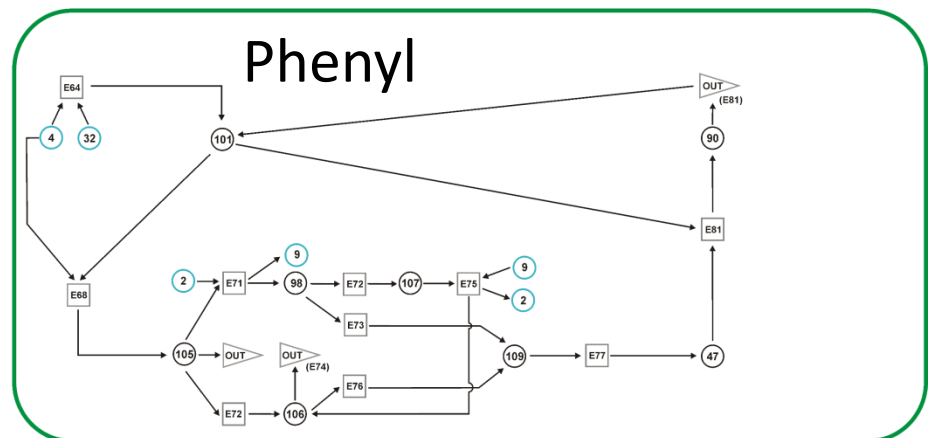
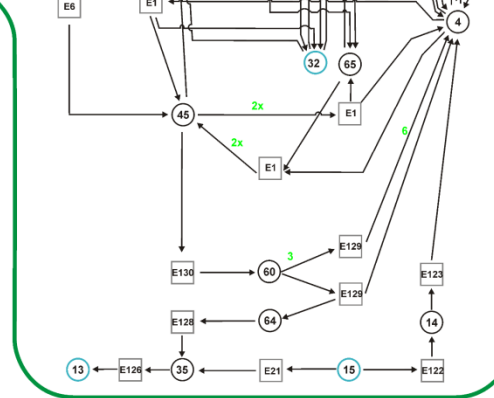
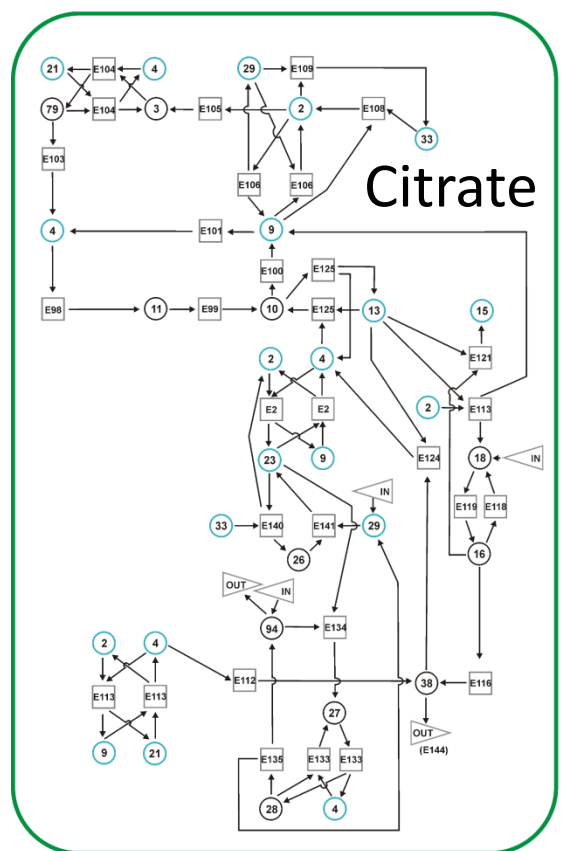
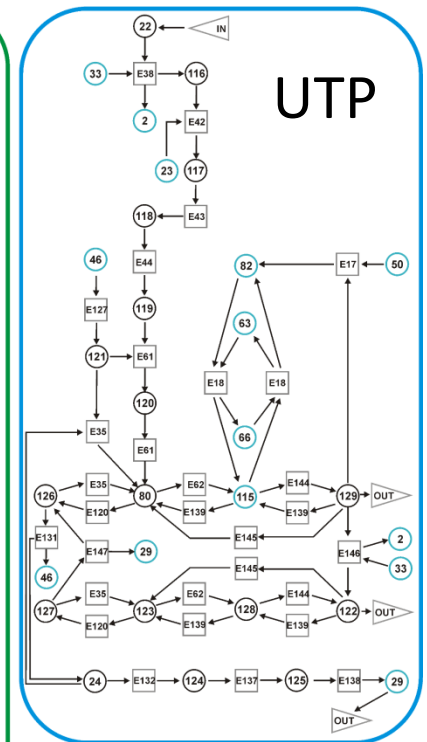
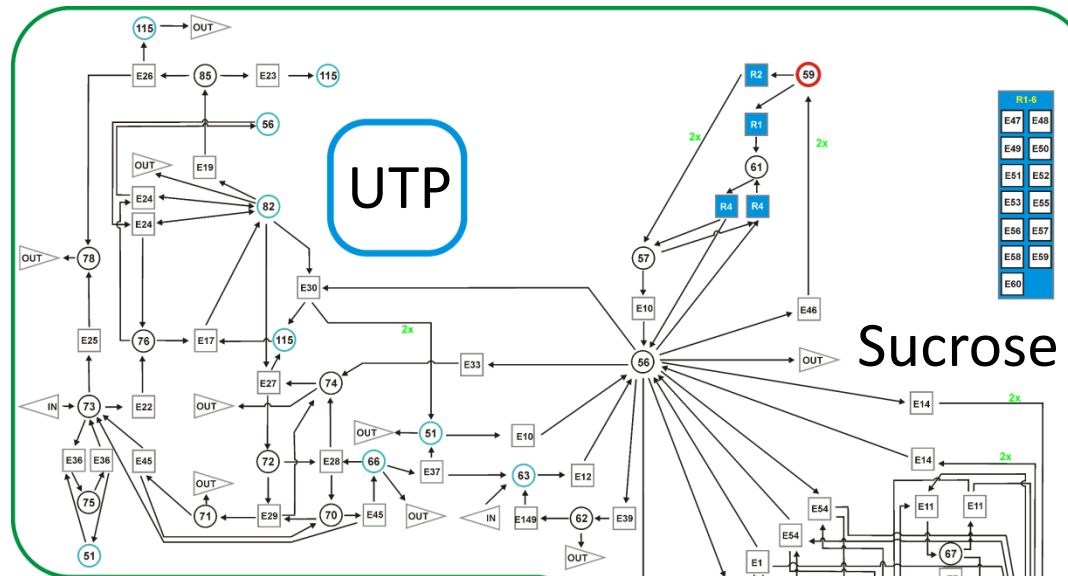


# Reduced network

Net	Normal	Reduced
#pl.	130	61
#tr.	232	123
#ed.	539	309

#transition invariants  
**9775**

Koch et al. (2017)  
*Frontiers in genetics*  
**MODEL1801090001**



transitions

normal

places

normal

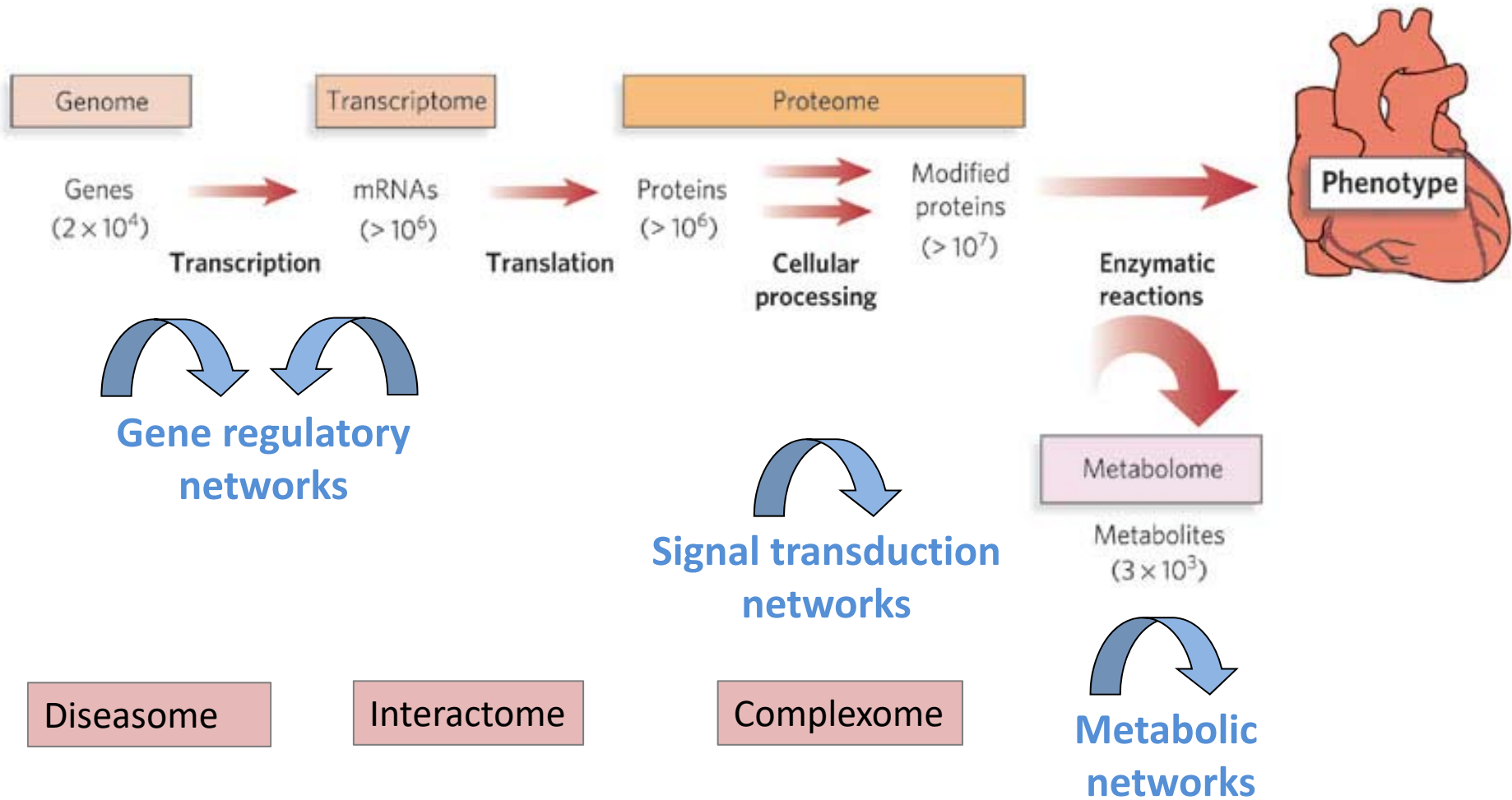
multiple

transitions linking to the environment

external input

external output

# Data of different scales require different modeling methods



# Outline

## Part I - Metabolism

- ❖ Modeling of the central carbon metabolism in potato tubers



Stéphanie Boue

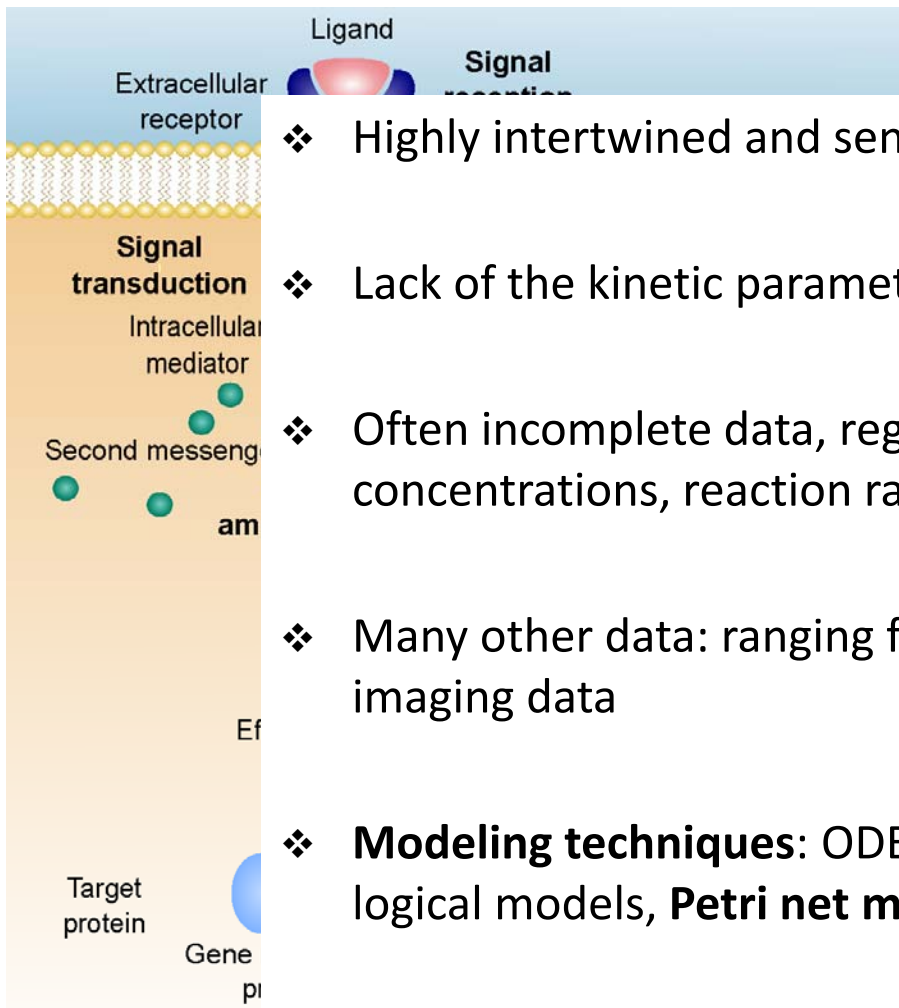
## Part II – Signal transduction

- ❖ Modeling of TNFR1-induced signaling pathway as Petri net, including the NF- $\kappa$ B pathway

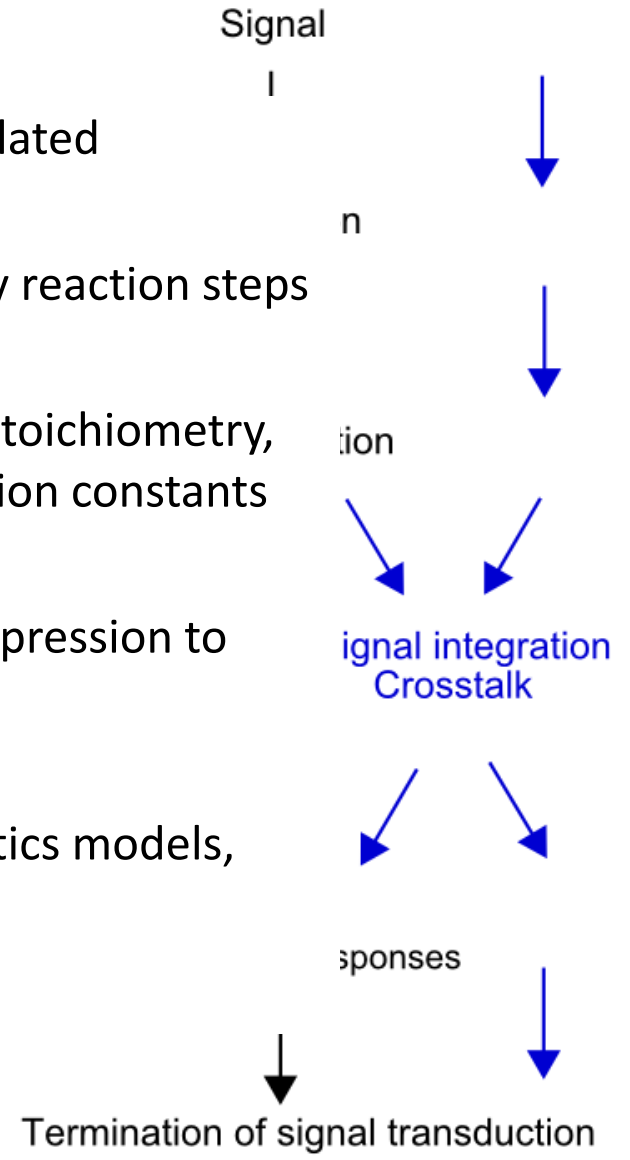
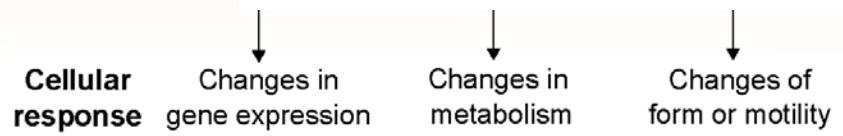


Leonie Amstein

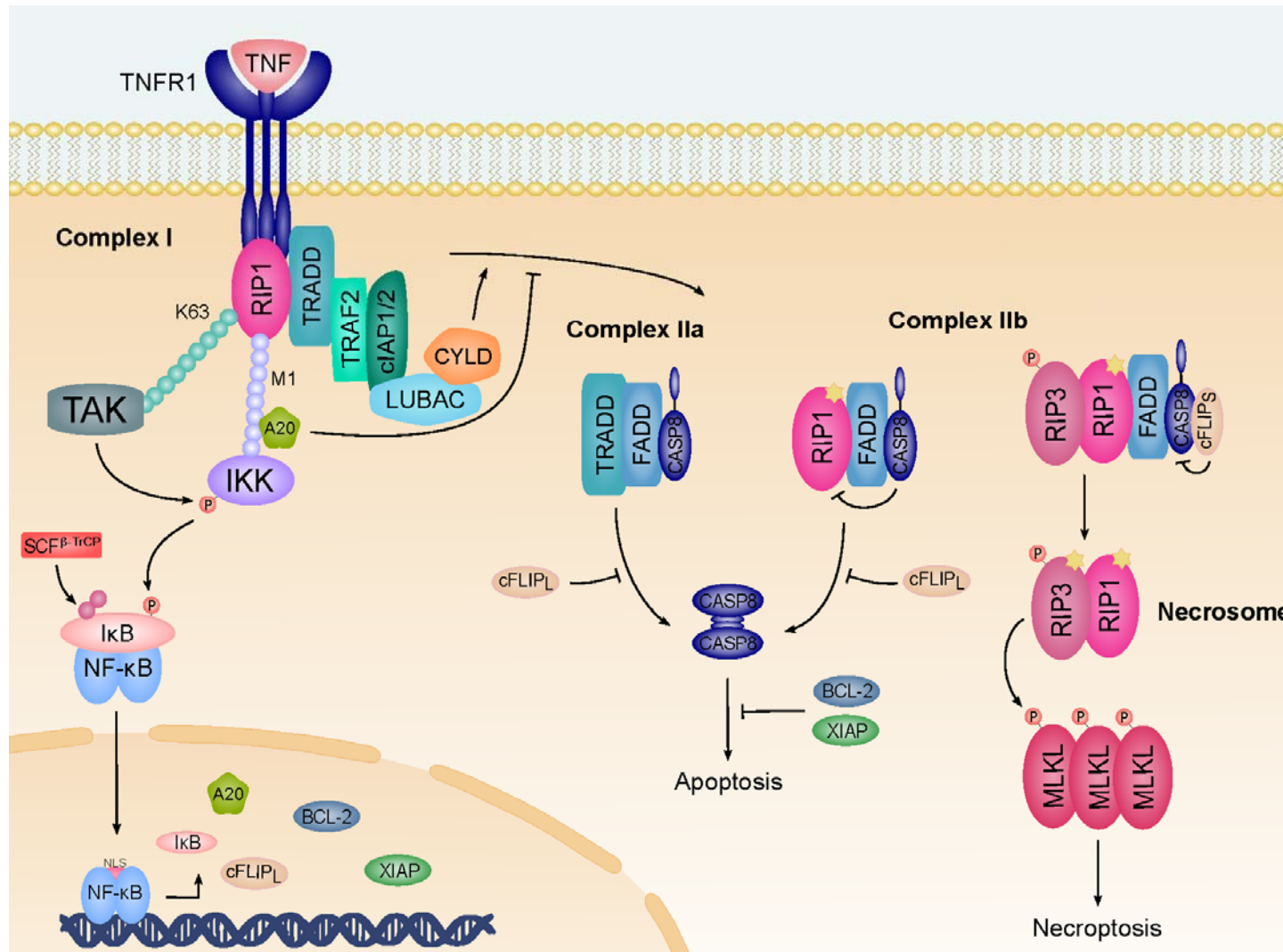
# Functionality in signaling pathways



- ❖ Highly intertwined and sensitively regulated
- ❖ Lack of the kinetic parameters for many reaction steps
- ❖ Often incomplete data, regarding the stoichiometry, concentrations, reaction rates, or reaction constants
- ❖ Many other data: ranging from gene expression to imaging data
- ❖ **Modeling techniques:** ODE-based kinetics models, logical models, **Petri net models**

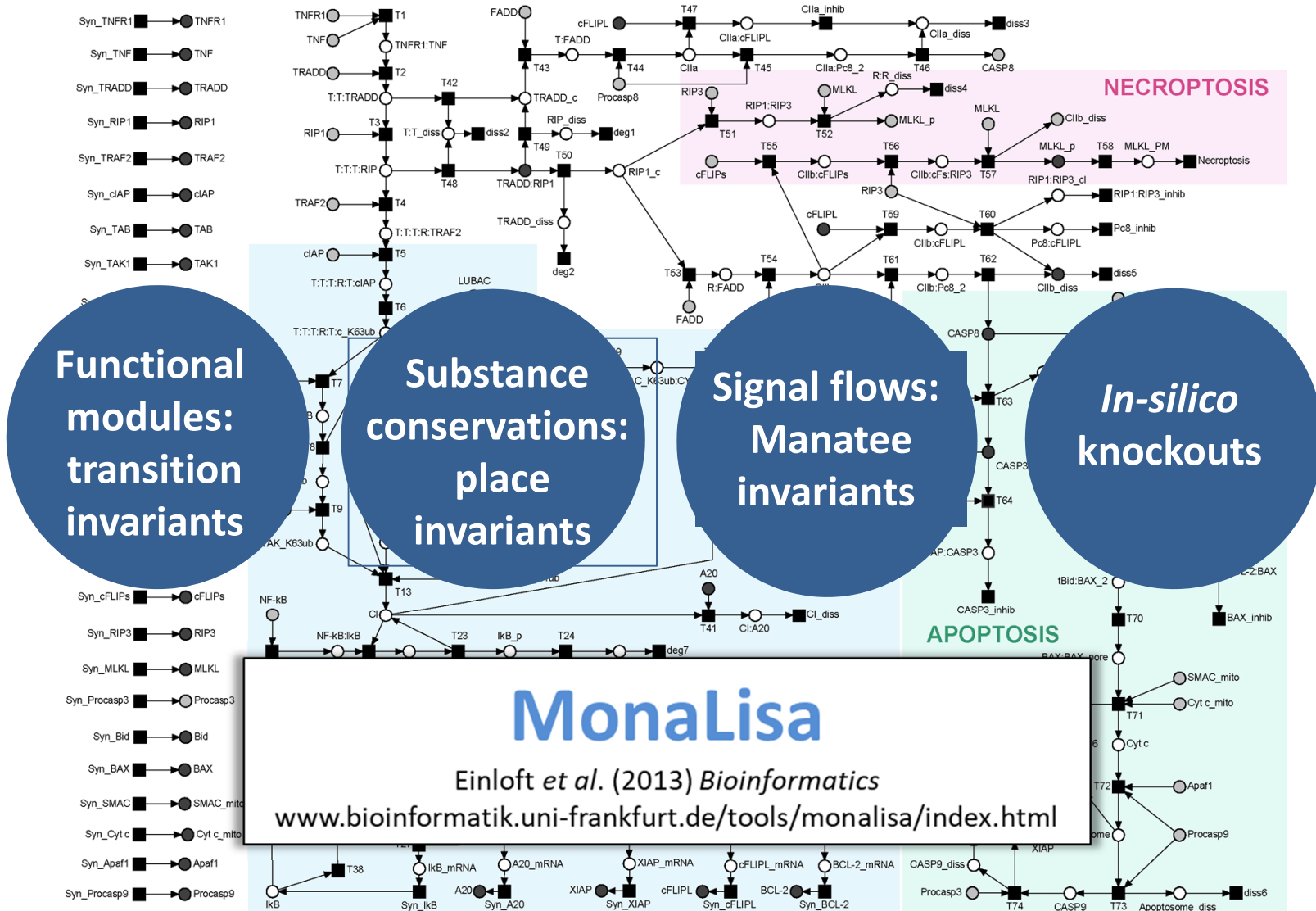


# The TNFR1 signal transduction pathway



Amstein et al. (2017) *BMC Systems Biol* ; Amstein et al. (2022) *PLoS Comp Biol*

# The Petri net analysis



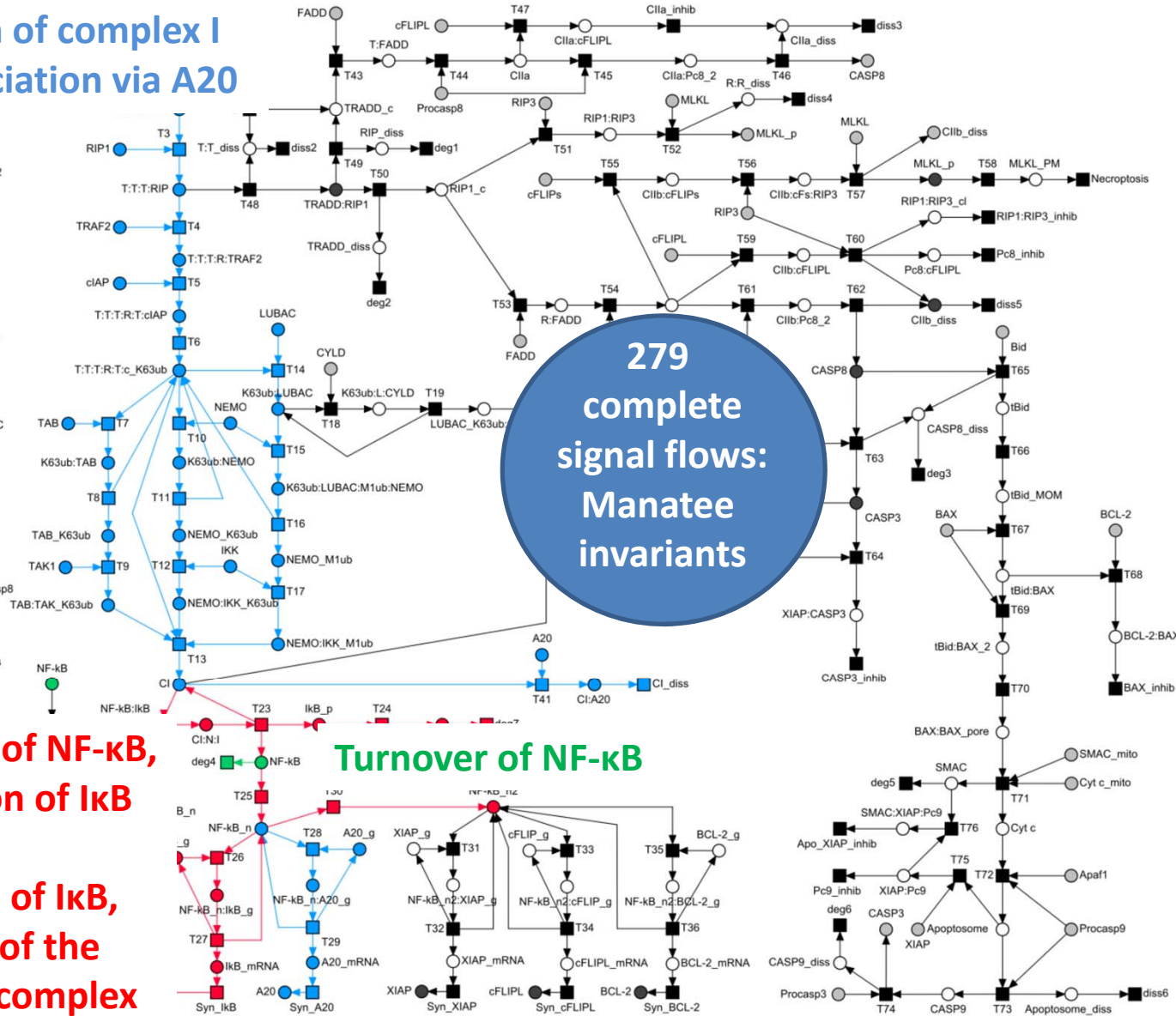
# Manatee invariants predict signaling pathways



Amstein *et al.*  
(2017)  
*BMC Systems Biol*

## Formation of complex I and dissociation via A20

- Syn\_RIP1 → RIP1
- Syn\_TRAF2 → TRAF2
- Syn\_cIAP → cIAP
- Syn\_TAB → TAB
- Syn\_TAK1 → TAK1
- Syn\_NEMO → NEMO
- Syn\_IKK → IKK
- Syn\_LUBAC → LUBAC
- Syn\_CYLD → CYLD
- Syn\_NF-κB → NF-κB
- Syn\_FADD → FADD
- Syn\_Procasp8 → Procasp8
- Syn\_SCF → SCF
- Syn\_cFLIPs → cFLIPs
- Syn\_RIP3 → RIP3



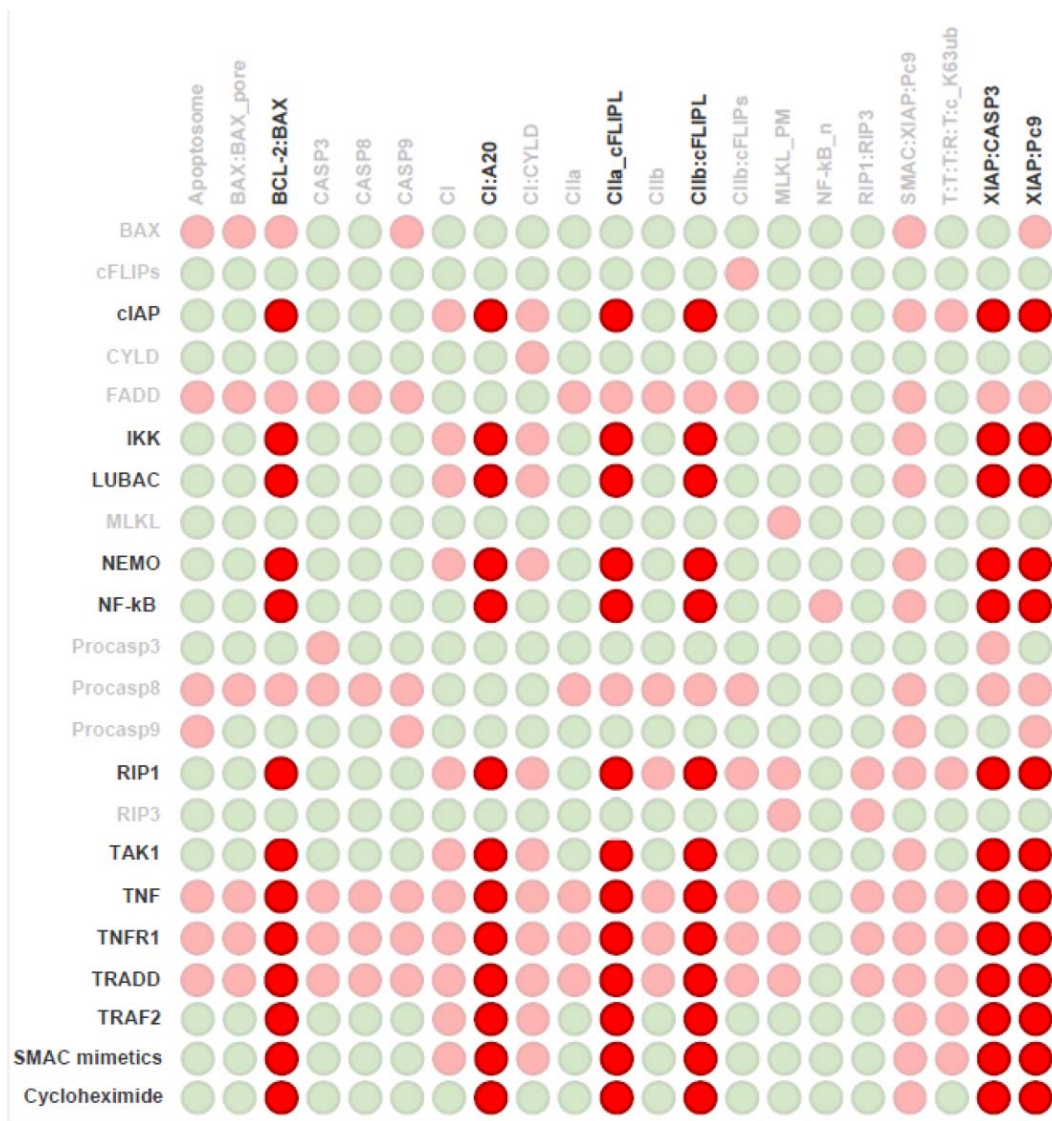
**Activation of NF-κB, degradation of IκB and gene expression of IκB, formation of the inhibitory complex**

**Turnover of NF-κB**

**279 complete signal flows: Manatee invariants**



# In-silico knockouts of the TNFR1 Petri net



- ❖ Knockout matrix based on Manatee invariants
- ❖ 20 *in-silico* knockouts, 2 therapies (SMAC mimetic, cycloheximide), 21 complexes
- ❖ Identification of knockouts that overcome the robust *survival* response
- ❖ Identification of functional patterns



**isiKnock**

Jennifer Hannig  
(former Scheidel)

Christoph Welsch

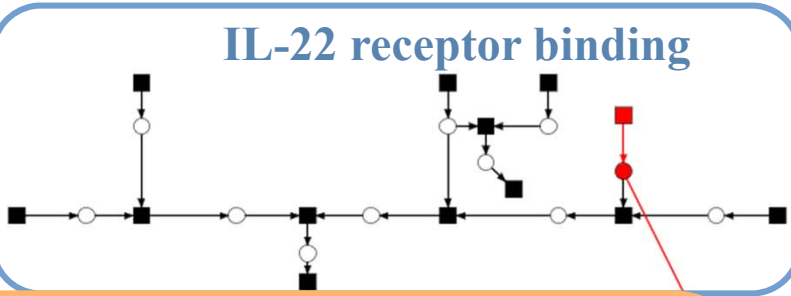


# Ongoing projects

Heiko Mühl



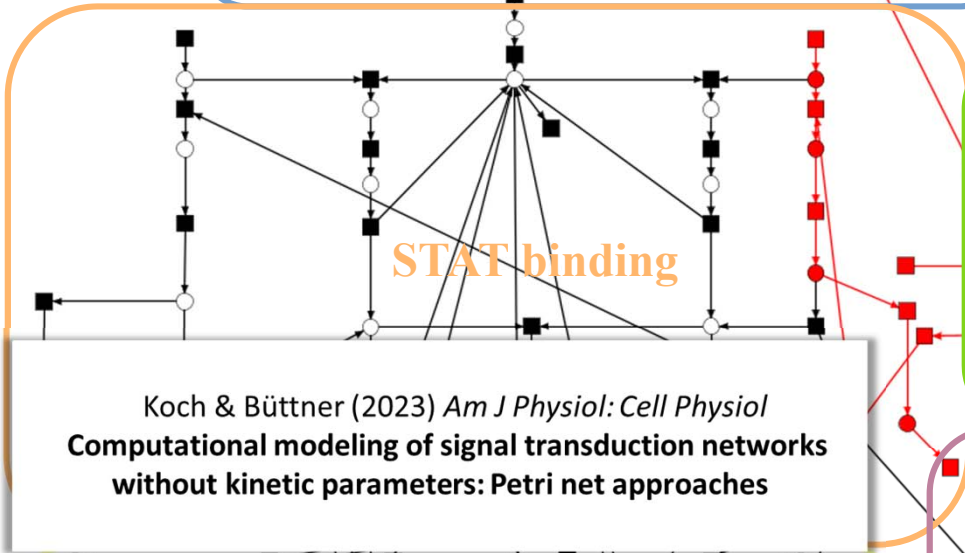
IL-22 receptor binding



Classical Petri net and development of a mass-action reaction model showing the interactions between IL-6 and IL-22 pathways unpublished

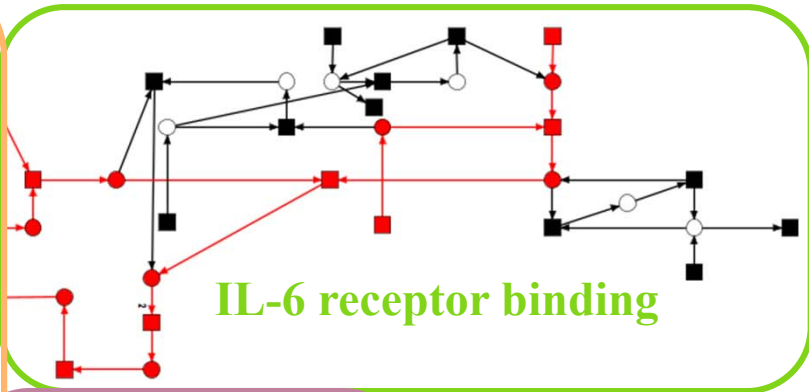


STAT binding

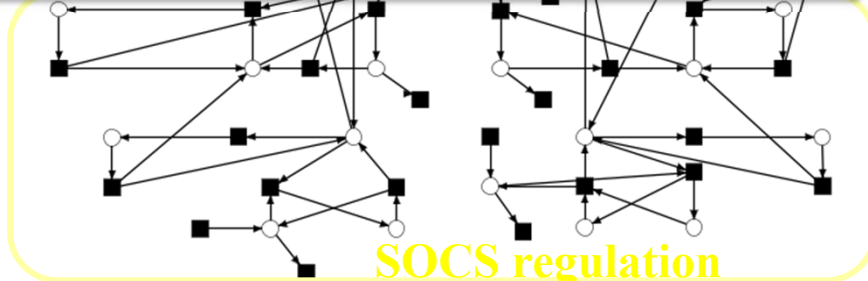


Koch & Büttner (2023) *Am J Physiol: Cell Physiol*  
Computational modeling of signal transduction networks without kinetic parameters: Petri net approaches

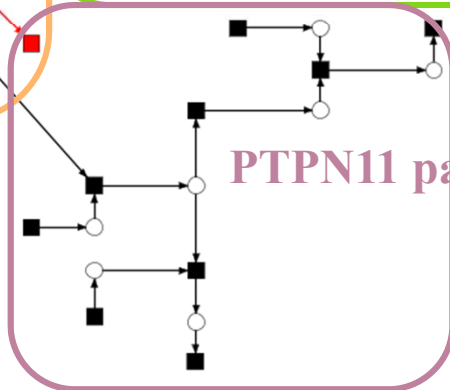
IL-6 receptor binding



SOCS regulation



PTPN11 pathway



Marcus Keßler

# Ongoing projects

Influence of the gut microbiota on the translocation of bacteria and fungi in patients with ACLF  
 Agent-based model on the interaction between antibiotics and microbiota

- ❖ Bacterial movement through gut motility (peristalsis, laminar flow, water absorption)
- ❖ **Bacterial metabolism** (production of SCFA)
- ❖ Bacterial growth in dependence on the pH value



Kristiyana Tsenova

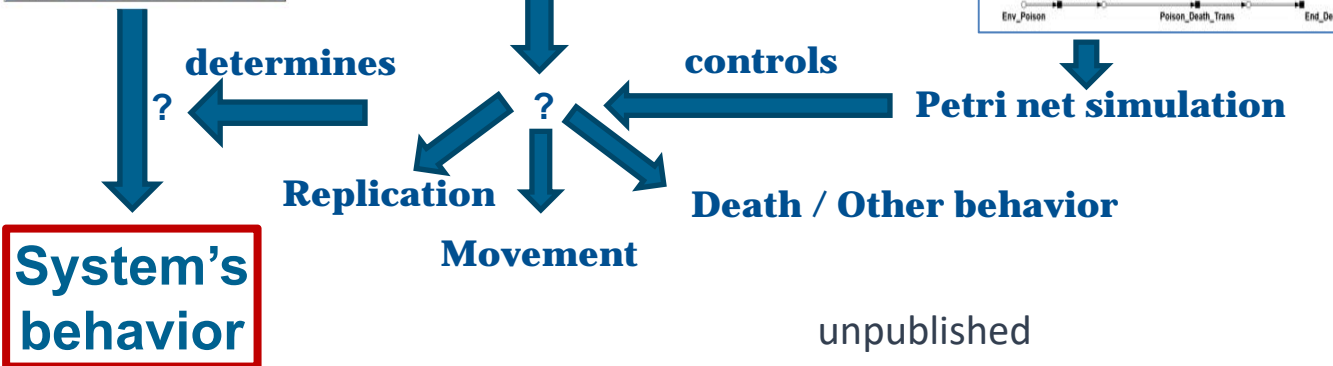
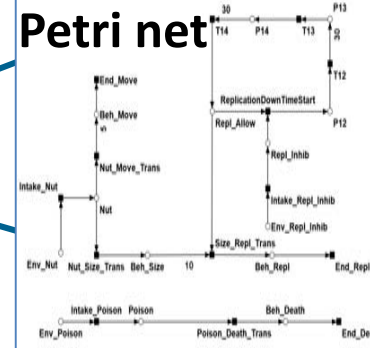
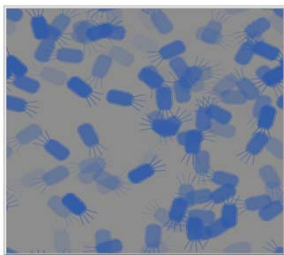


Jörg Ackermann



Marius Kirchner

PetriAgents

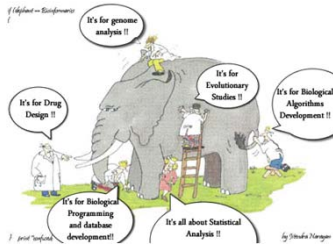


unpublished

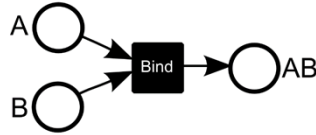


# Summary

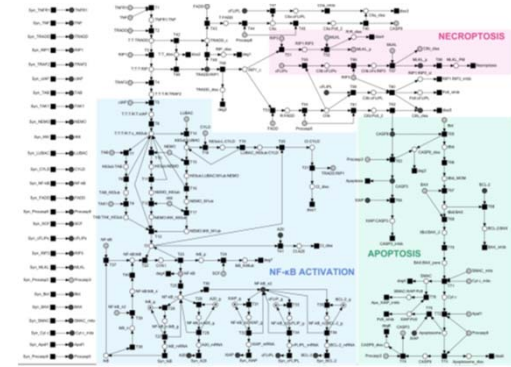
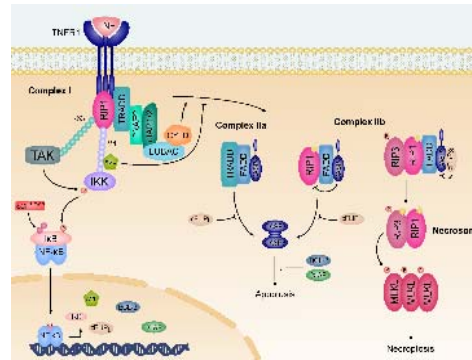
## Challenges of data integration



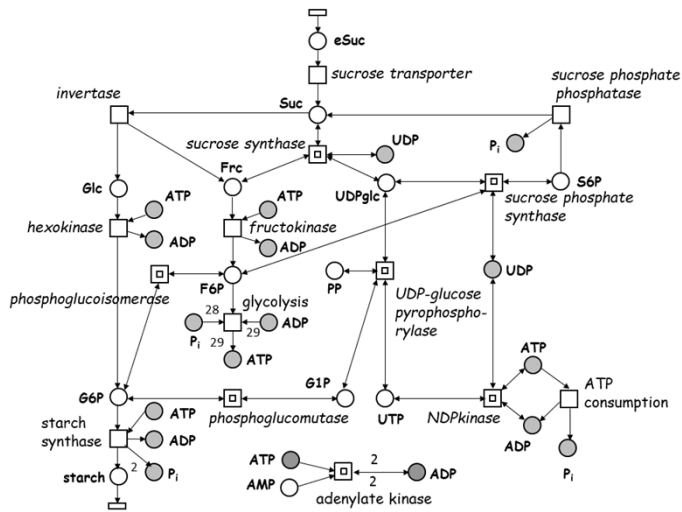
## Petri nets



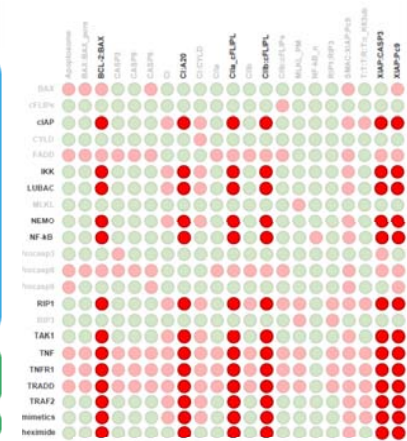
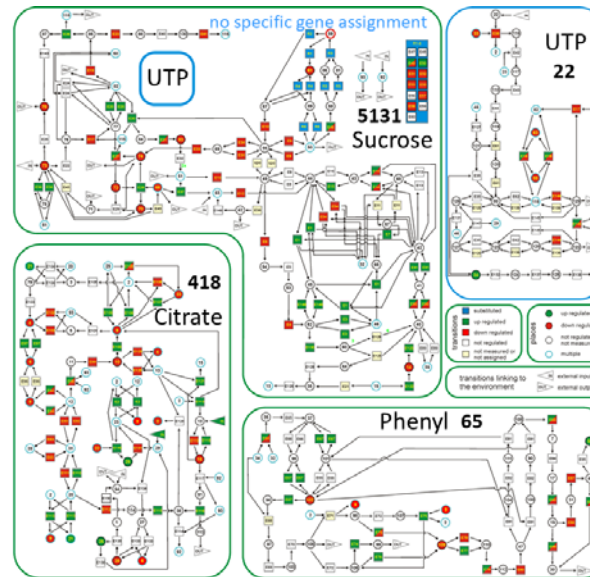
## Petri net model of the TNFR1-mediated signaling pathways



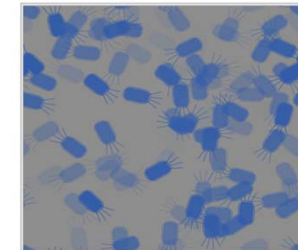
## Petri net model of the carbon metabolism in potato tubers and in *Arabidopsis thaliana*



## Knockout analyses



## PetriAgents



# Take-home messages

- ❖ The quality and quantity of the data determine the choice of the modeling method -> **Check your data** carefully
- ❖ The system's behavior can be predicted **without knowing the kinetic parameters**, using, e.g., Petri nets -> invariant and knockout analyses
- ❖ **Place invariants** describe **substance conservations**
- ❖ **Transition invariants** represent **functional modules**
- ❖ **Manatee invariants** are **linear combinations** of transition invariants -> **complete signaling pathways**
- ❖ **The bottleneck** is the availability of data of sufficient quality and quantity, especially for machine learning applications
- ❖ The exploration of **all possible system states** is still a challenge

There are many databases related to metabolism,  
and there are for a given organism,  
many metabolic networks that have been  
reconstructed that provide different  
and not always compatible information.  
How the community could address these issues?

- ❖ Standardization
- ❖ Computational verification methods
- ❖ Database improvement by automatic curation,  
checking for contradictions
- ❖ **Validation, validation, validation!**

# Acknowledgments

## University Jena

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Prof. Dr. Martin-Leo Hansmann

Prof. Dr. Christoph Welsch

Prof. Dr. Heiko Mühl

Prof. Dr. Maria Vehreschild

## Ecole Normale

## Supérieure Paris

Prof. Dr. Denis Thieffry

## University of Evry

## Val d'Essonne, Évry

Prof. Dr. Anna Niarakis



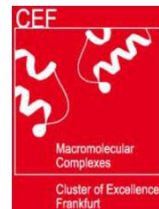
Federal Ministry  
of Education  
and Research

**DFG** Deutsche  
Forschungsgemeinschaft



En<sup>o</sup>ABLE

Ein Clusterprojekt des Landes Hessen



GOETHE  
UNIVERSITÄT  
FRANKFURT AM MAIN



**LOEWE**

Exzellente Forschung für  
Hessens Zukunft

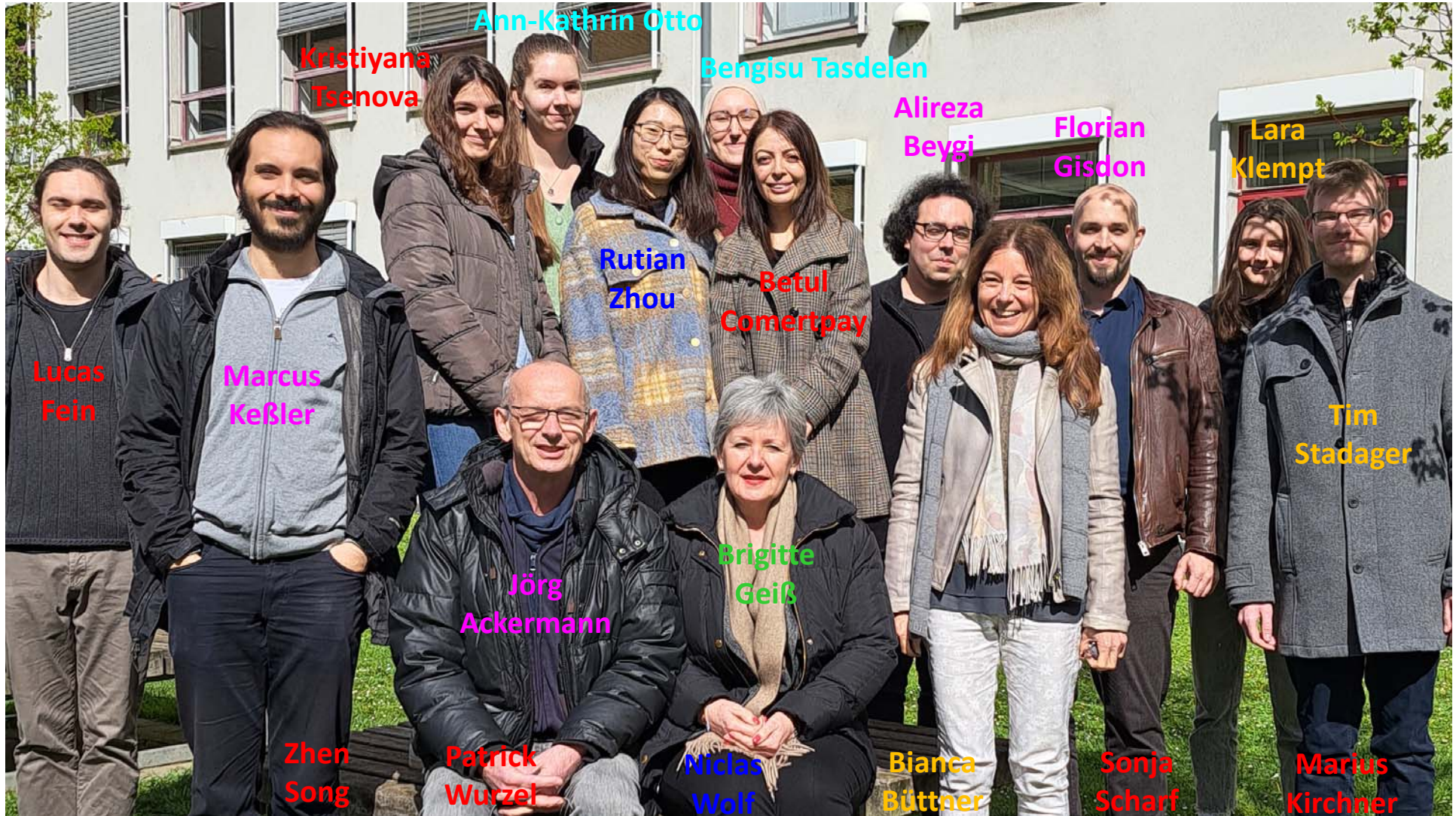
HESSEN



Mol  BI  
Molecular Bioinformatics

April 2023

Mol  BI



Administration  
Bachelor student  
Master student  
Ph.D. student  
Postdoc, Alumni





*Thank you  
for  
your attention!*

# System's invariants

Transition Place	$r_1$	$r_2$	$r_{3f}$	$r_{3b}$
C	-2	-1	-1	+1
O <sub>2</sub>	-1	-1	0	0
CO	+2	0	+2	-2
CO <sub>2</sub>	0	+1	-1	+1
init	0	0	0	0

place (P-) invariant:  $C^T x = 0$

transition (T-) invariant:  $C y = 0$

0: steady-state constraint

Search for **minimal nonnegative, nontrivial integer** solutions

Minimal:  $\nexists z: \text{supp}(z) \subseteq \text{supp}(u)$  and the largest common divisor of all non-zero entries of  $u$  is 1

## P-invariants

$$\begin{aligned}
 -2x_1 - 1x_2 + 2x_3 &= 0 \\
 -1x_1 - 1x_2 + 1x_4 &= 0 \\
 -1x_1 + 2x_3 - 1x_4 &= 0 \\
 +1x_1 - 2x_3 + 1x_4 &= 0 \\
 +3x_1 + 2x_2 - 1x_5 &= 0 \\
 -2x_3 - 1x_4 + 1x_5 &= 0
 \end{aligned}$$

## T-invariants

$$\begin{aligned}
 -2y_1 - 1y_2 - 1y_3 + 1y_4 + 3y_5 &= 0 \\
 -1y_1 - 1y_2 + 2y_5 &= 0 \\
 +2y_1 + 2y_3 - 2y_4 - 2y_6 &= 0 \\
 +1y_2 - 1y_3 + 1y_4 - 1y_6 &= 0 \\
 -1y_5 + 1y_6 &= 0
 \end{aligned}$$

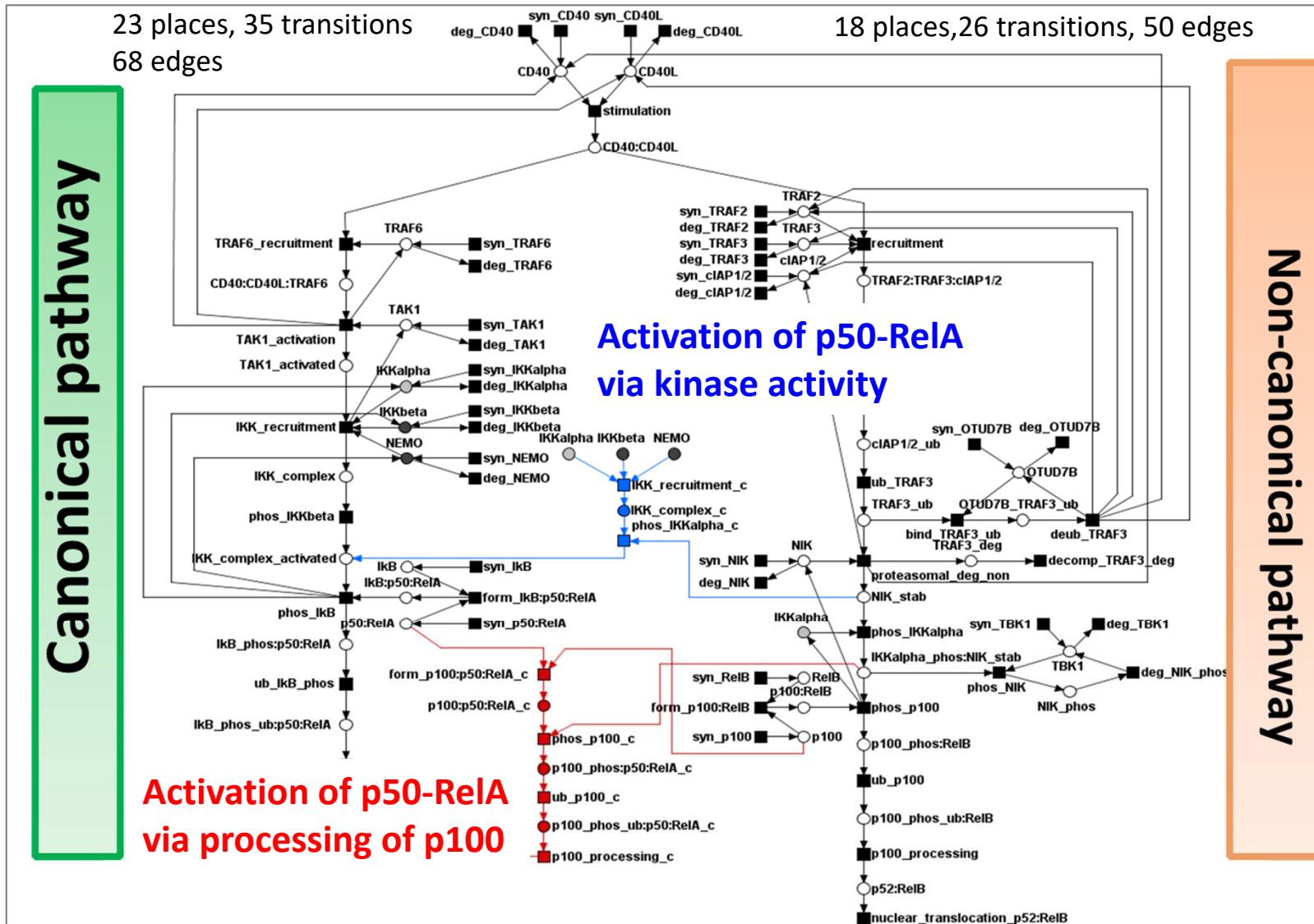
Parikh vector: vector of firing frequencies

# The canonical and non-canonical NF- $\kappa$ B pathways and their crosstalk: a comparative study based on Petri nets

Trares et al. (2022) *Biosystems*; BioModels ID: MODEL2207210001/2/3



Kira Trares



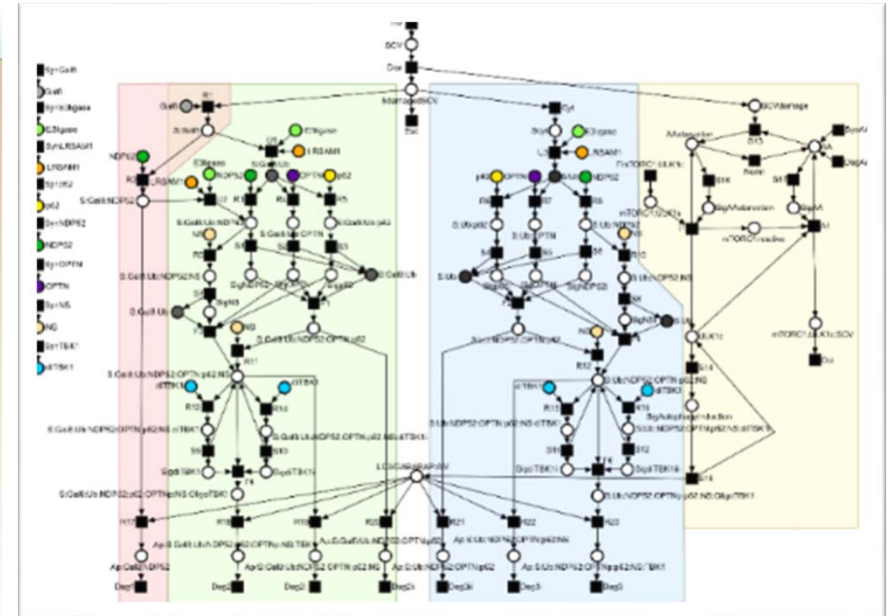
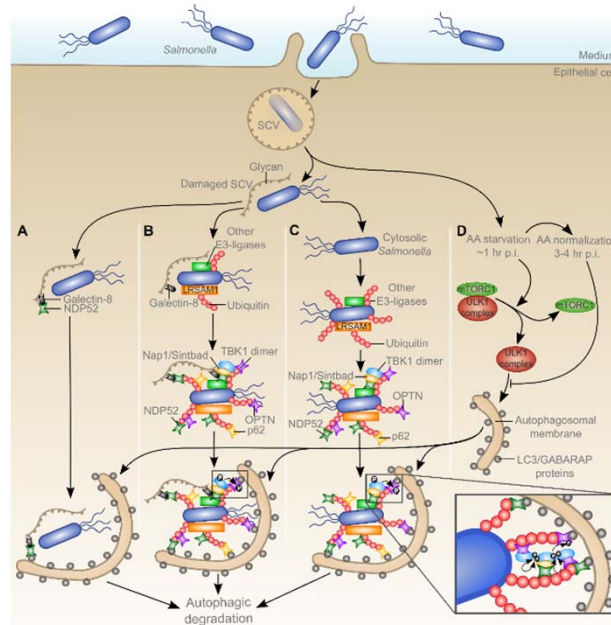
# Xenophagy in epithelial HeLa cells after *Salmonella* Typhimurium infection



Jennifer Hannig



Ivan Dikic



- ❖ Classical Petri net
- ❖ Stochastic Petri net

Hannig *et al.* (2018) *Bioinformatics*

Scheidel *et al.* (2016) *PLoS Computational Biology*

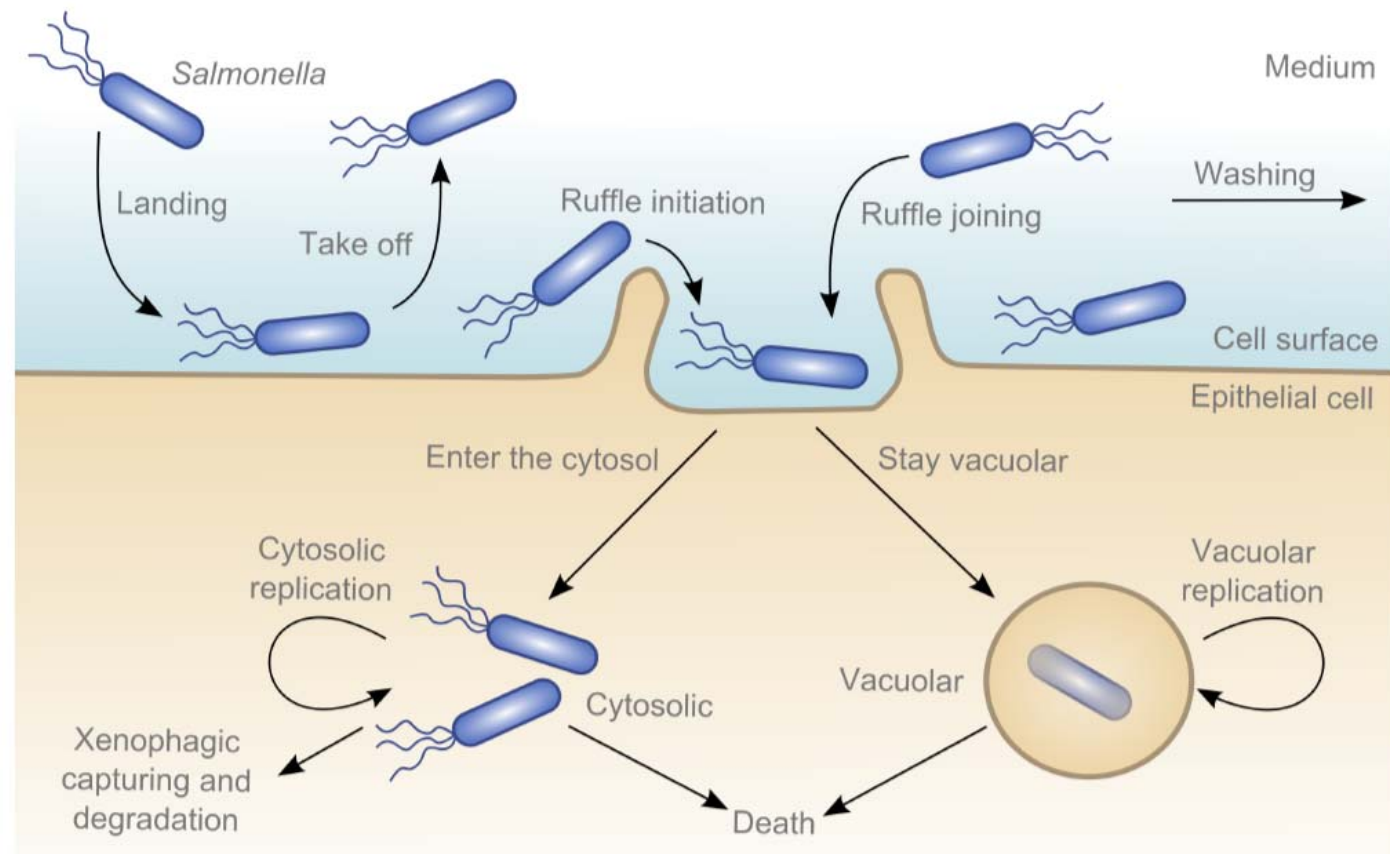
# Agent-based model of *Salmonella* movement on the cell surface



Nasrin Alikhani  
Chamgordani



Jennifer Hannig

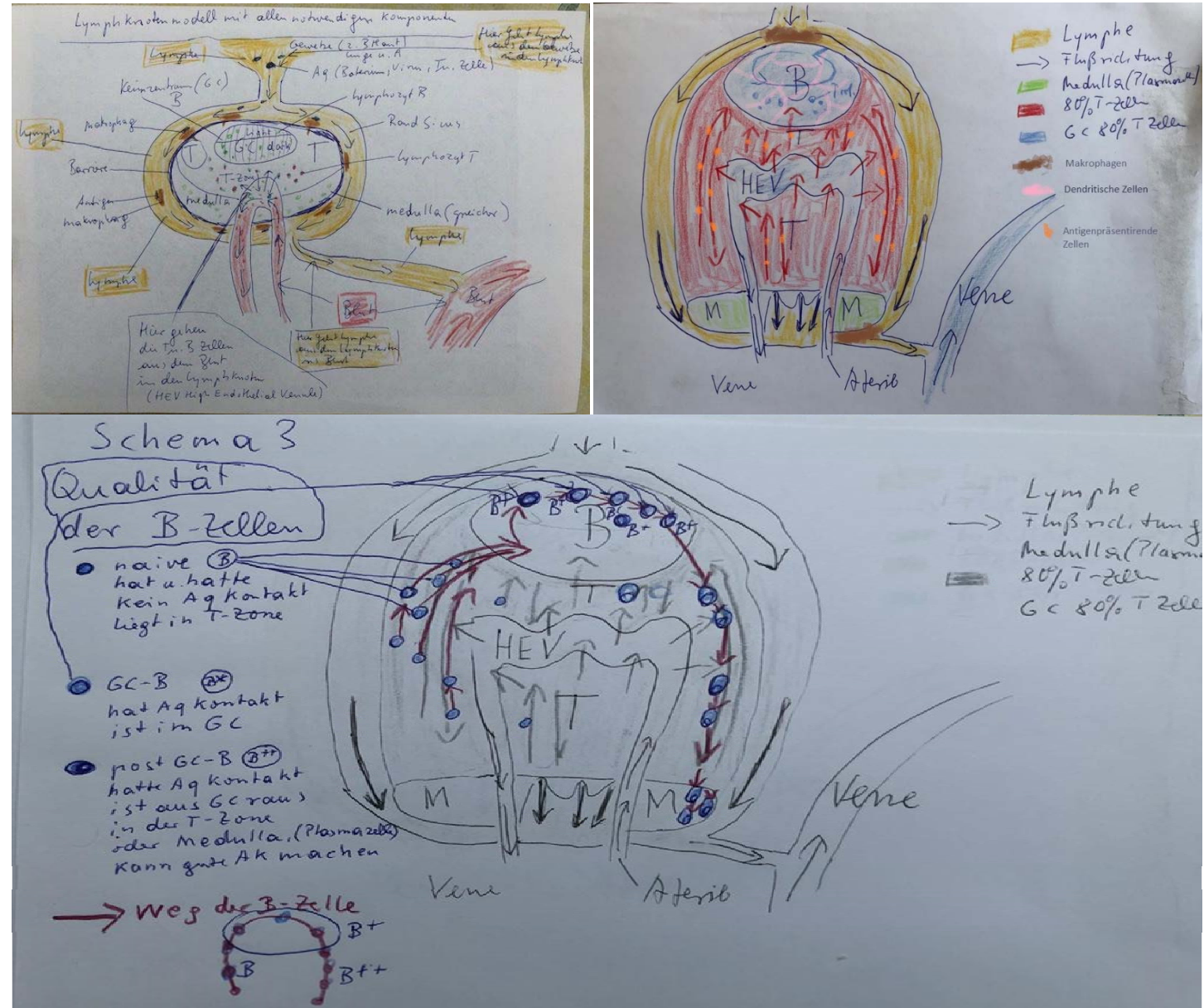


unpublished

# A Petri net model of the human lymph node



Martin-Leo Hansmann



unpublished