Linking Operational Semantics and Algebraic Approach for a Probabilistic Timed Shared-Variable Language

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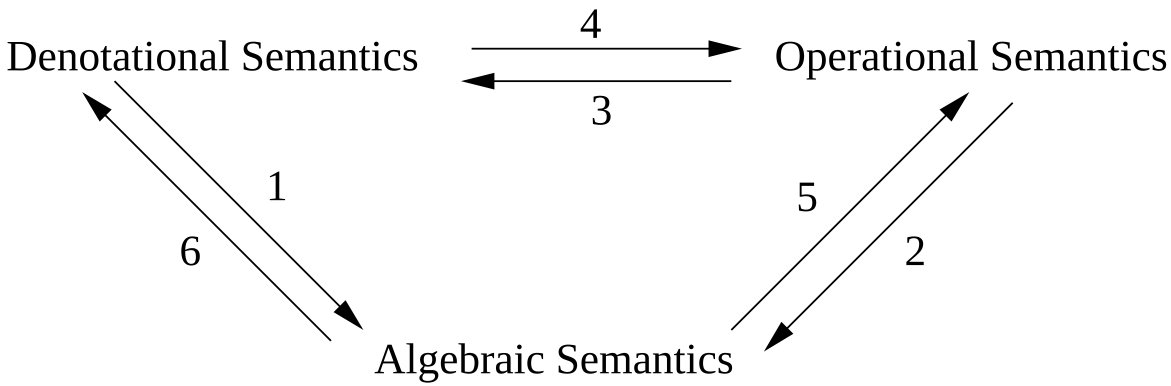
Motivation (1)

- Complex software systems: (a) Probability; (b) Real-time; (c) Shared-Variable Concurrency

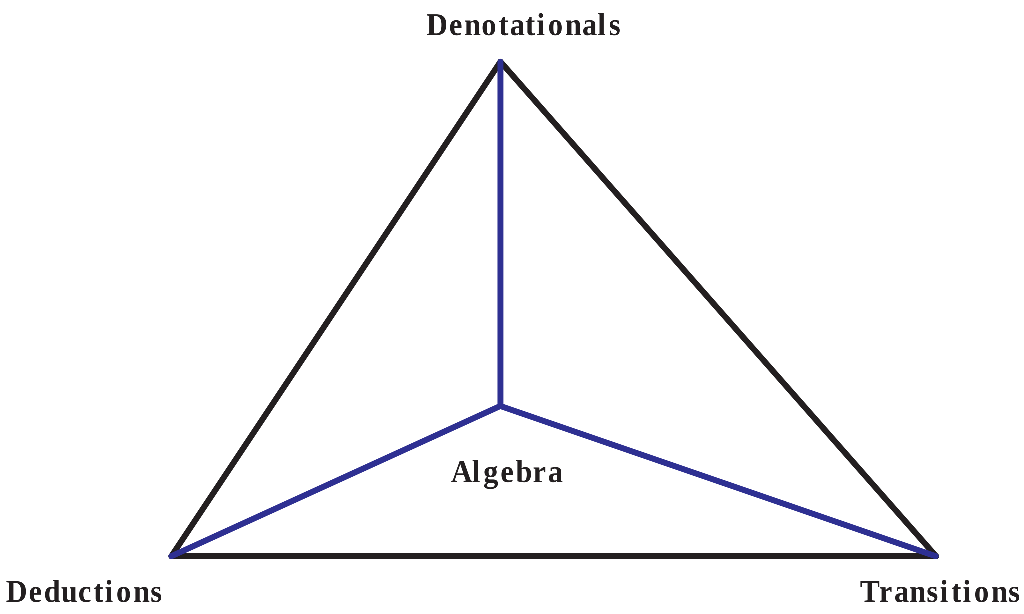
  We have proposed a language PTSC, which integrates these features.

- Semantic Linking:



Related Work

- Recently, Hoare proposed the challenging research for the semantic linking between **algebra**, **denotations**, **transitions** and **deductions** (in Meeting 52 of WG 2.3).



Contribution

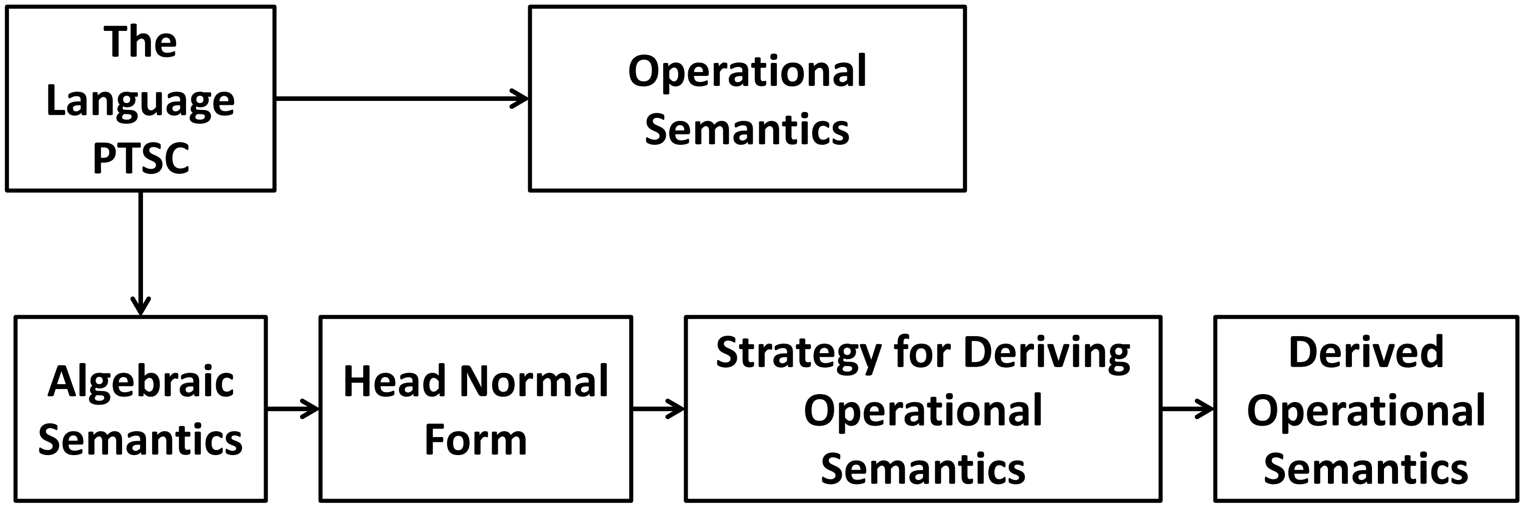
How can we guarantee the consistency between operational semantics and algebraic semantics for PTSC?

(1) **Approach:** Deriving operational semantics from algebraic semantics.

(2) **Methodology:** Theoretically and Mechanically

- Further Algebraic Laws
- Head Normal Form
- Deriving Operational Semantics from Algebraic Semantics
- Mechanizing the three steps

Methodology: A Sketch



Syntax of PTSC (1)

$$P \; ::= \; \textbf{Skip} \mid x := e \mid \textbf{if } b \textbf{ then } P \textbf{ else } P$$
$$\mid \textbf{while } b \textbf{ do } P \mid @b\ P \mid \#n\ P \mid P\ ;\ P$$
$$\mid P \sqcap P \mid P \sqcap_p P \mid P \parallel_p P$$

Five types of guarded choice:

(1) The first type is composed of a set of assignment-guarded components.

$$\|_{i \in I}\{[p_i]\ choice_{j \in J_i}(b_{ij} \& (x_{ij} := e_{ij})\, P_{ij})\}$$

Healthiness conditions:

(a) $\forall i \bullet (\bigvee_{j \in J_i} b_{ij} = true)$ and

$$(\forall j_1, j_2 \in J_i \bullet (j_1 \neq j_2) \Rightarrow ((b_{ij_1} \wedge b_{ij_2}) = false))$$

(b) $\Sigma_{i \in I}\, p_i = 1$

Syntax of PTSC (2)

Five types of guarded choice:

(2) The second type is composed of a set of event-guarded components.

$$\|_{i\in I}\{@b_i\, P_i\}$$

(3) The third type is composed of one time-delay component.

$$\|\{\#1\ R\}$$

(4) The fourth type is the guarded choice composition of the first and second type of guarded choice.

$$\|_{i\in I}\{[p_i]\ choice_{j\in J_i}(b_{ij}\&(x_{ij} := e_{ij})\, P_{ij})\}$$
$$\|\|_{k\in K}\{@b_k\, Q_k\}$$

Syntax of PTSC (3)

Five types of guarded choice:

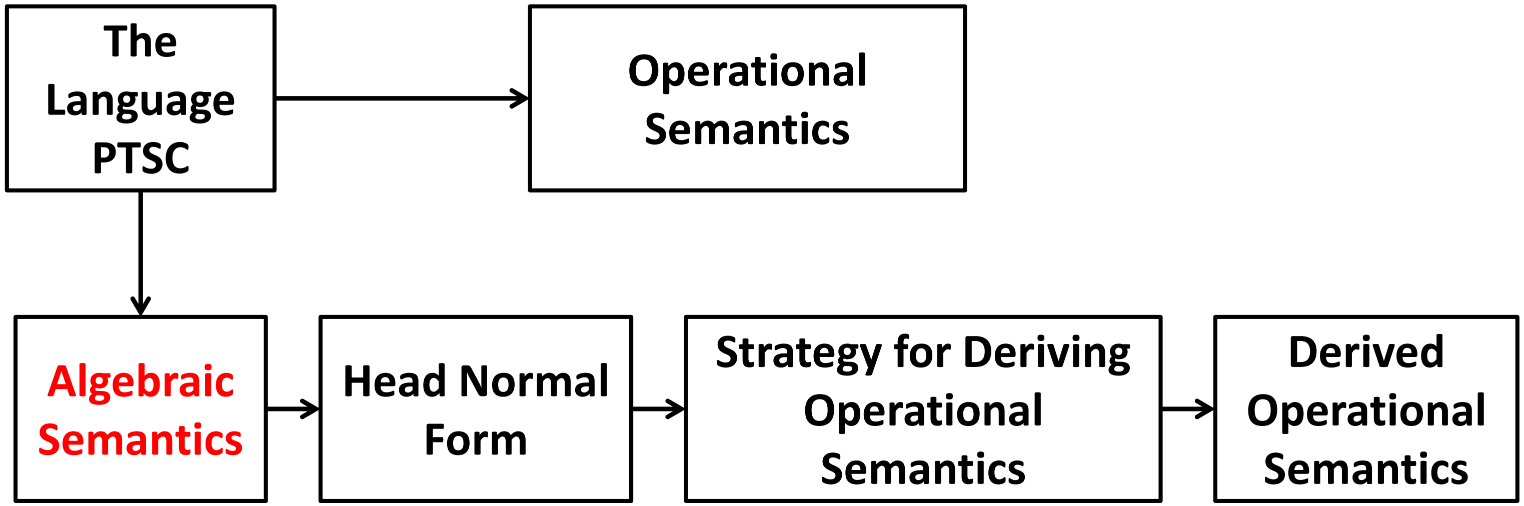
(5) The fifth type is the compound of the second and third type of guarded choice.

$$\|_{i \in I}\{@b_i\, P_i\}\|\{\#1\ R\}$$

Example: Let

$$P \;=\; \|\{\ [0.7]choice(\ true\&(x := 5)\, P_1\ )\,,$$
$$[0.3]choice(\ (x > 2)\&(x := x)\, P_2,\ \ (x \leq 2)\&(x := x)\, P_3\ )$$
$$\}$$

Algebraic Semantics (1)



Algebraic Semantics (2)

Laws for guarded choice:

(gchoice-1) $[\!]\{C_1, \cdots\cdots, C_n\} \;=\; [\!]\{C_{i_1}, \cdots\cdots, C_{i_n}\}$

where, $C_{i_1}, \cdots, C_{i_n}$ is a permutation of $1, \cdots, n$.

(gchoice-2) $[\!]\{[p]choice\{\mathbf{false}\&(x := e)\,P,\; G_1\},\; G_2\}$

$= \;[\!]\{[p]choice\{G_1\},\; G_2\}$

(gchoice-3) $[\!]\{[p]choice\{b_1\&(x := e)\,P,\; b_2\&(x := e)\,P,\; G_1\},\; G_2\}$

$= \;[\!]\{[p]choice\{(b_1 \vee b_2)\&(x := e)\,P,\; G_1\},\; G_2\}$

(gchoice-6) $[\!]\{[p]choice\{G_1\},\; [q]choice\{G_1\},\; G_2\}$

$= \;[\!]\{[p+q]choice\{G_1\},\; G_2\}$

Algebraic Semantics (3)

(1) $x := e \;=\; \|\{\,[1]choice\{\mathbf{true}\&(x := e)\ \varepsilon\}\,\}$

(2) $\#n \;=\; \|\{\#1\ \#(n-1)\}$, where $n > 1$

(3) **if** b **then** P **else** Q

$= \|\{\,[1]choice\{b\&(x := x)\ P,\ \ \neg b\&(x := x)\ Q\}\,\}$

(4) Assume $P \;=\; \|\{C_1,\ \ldots, C_n\}$, then

$P\ ;\ Q \;=\; \|\{\mathbf{seq}(C_1, Q), \ldots, \mathbf{seq}(C_n, Q)\}.$

Algebraic Semantics (4)

Parallel Expansion Laws

(par-3-1) Let

$$P = \;\|_{i\in I}\{[p_i]\; choice_{j\in J_i}(b_{ij}\&(x_{ij} := e_{ij})\, P_{ij})\}$$

$$Q = \;\|_{k\in K}\{[q_k]\; choice_{l\in L_k}(b_{kl}\&(x_{kl} := e_{kl})\, P_{kl})\}$$

Then

$$P \;\|_r\; Q$$

$$= \;\|_{i\in I}\{[r \times p_i]\; choice_{j\in J_i}(b_{ij}\&(x_{ij} := e_{ij})\, \mathbf{par}(P_{ij}, Q, r)\}$$

$$\|\|_{k\in K}\{[(1-r) \times q_k]\; choice_{l\in L_k}(b_{kl}\&(x_{kl} := e_{kl})\, \mathbf{par}(P, Q_{kl}, r)\}$$

Algebraic Semantics (5)

Parallel Expansion Laws

(par-3-2) Let $P = \|_{i\in I}\{[p_i]\ choice_{j\in J_i}(b_{ij}\&(x_{ij} := e_{ij})\, P_{ij})\}$
and $Q = \|_{k\in K}\{@c_k\ Q_k\}$

Then

$$P \|_r Q \;=\; \|_{i\in I}\{[p_i]\ choice_{j\in J_i}(b_{ij}\&(x_{ij} := e_{ij})\,\mathbf{par}(P_{ij}, Q, r)\}$$
$$\|\|_{k\in K}\{@c_k\ \mathbf{par}(P, Q_k, r)\}$$

(par-3-6) Let $P \;=\; \|_{i\in I}\{@b_i\ P_i\}$ and $Q \;=\; \|_{j\in J}\{@c_j\ Q_j\}$

Then $P \|_r Q = \|_{i\in I}\{@(b_i \wedge \neg c)\ \mathbf{par}(P_i, Q, r)\}$
$$\|\|_{j\in J}\{@(c_j \wedge \neg b)\ \mathbf{par}(P, Q_j, r)\}$$
$$\|\|_{i\in I\wedge j\in J}\{@(b_i \wedge c_j)\ \mathbf{par}(P_i, Q_j, r)\}$$

where, $b \;=\; \vee_{i\in I}\, b_i$ and $c \;=\; \vee_{j\in J}\, c_j$

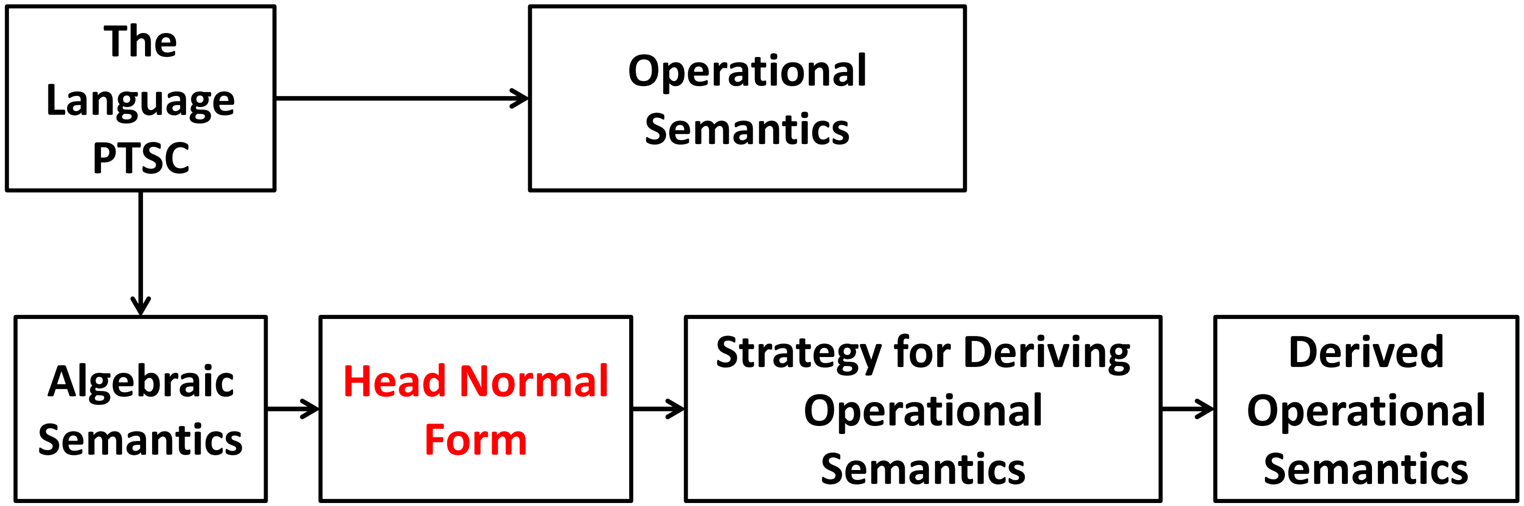
Algebraic Semantics (5)

(1) $$P \sqcap_p Q \;=\; \|\{\, [p]choice\{\mathbf{true}\&(x := x)\, P\},\\ [1-p]choice\{\mathbf{true}\&(x := x)\, Q\}\}$$

(2) Summation: $\bigoplus\{P_1, \cdots, P_n\}$

- $\bigoplus\{P_1, \cdots, P_n\} \;=\; \bigoplus\{P_{i_1}, \cdots, P_{i_n}\}$
- If $P = \bigoplus\{P_1, \cdots, P_n\}$ and $Q = \bigoplus\{Q_1, \cdots, Q_m\}$,
  then $P \sqcap Q \;=\; \bigoplus\{P_1, \cdots, P_n, Q_1, \cdots, Q_m\}$
- If $P = \bigoplus\{P_1, \cdots, P_n\}$,
  then $P\,; Q \;=\; \bigoplus\{(P_1; Q), \cdots, (P_n; Q)\}$

Head Normal Form (1)



Head Normal Form (2)

We assign every program P a normal form called head normal form $\mathcal{HF}(P)$, which can be applied in deriving operational semantics from algebraic semantics.

(1) $\mathcal{HF}(x := e) =_{df} [\![\{\, [1]choice\{\mathbf{true}\&(x := e)\ \varepsilon\}\,\}$

(2) $\mathcal{HF}(@b) =_{df} [\![\{@b\ \varepsilon\}$

(8) If $\mathcal{HF}(P) = \bigoplus_{i\in I} P_i$ and $\mathcal{HF}(Q) = \bigoplus_{j\in J} Q_j$ then $\mathcal{HF}(P \sqcap Q) =_{df} \bigoplus_{i\in I} P_i \bigoplus \bigoplus_{j\in J} Q_j$

(9) If $\mathcal{HF}(P) = \bigoplus_{i\in I} P_i$ and $\mathcal{HF}(Q) = \bigoplus_{j\in J} Q_j$ then $\mathcal{HF}(P \parallel_r Q) =_{df} \bigoplus_{i\in I,\, j\in J} (P_i \parallel_r Q_j)$
For $\mathcal{HF}(P_i \parallel_r Q_j)$, it can be defined as the result of applying the parallel expansion laws for $HF(P_i) \parallel_r HF(Q_j)$.

Animation of Alg. Sem. and HF(P) (1)

Aim: Supporting the mechanical derivation of operational semantics from algebraic semantics.

Generating Algebraic Laws: $npar(S_1\|_R S_2, T)$

(1) For $S_1\|_R S_2$, where S_1 and S_2 are both of assignment guarded choice.

$$\begin{aligned} npar(S_1 \|_R S_2, T) :- \; & assignGuardChoice(S_1),\ assignGuardChoice(S_2), \\ & assign_2L(S_1 \|_R S_2, T_1),\ assign_2R(S_1 \|_R S_2, T_2), \\ & append(T_1, T_2, T). \end{aligned}$$

(2) For $S_1\|_R S_2$, where S_1 and S_2 are both of event guarded choice.

$$\begin{aligned} npar(S_1 \|_R S_2, T) :- \; & eventGuardChoice(S_1),\ eventGuardChoice(S_2), \\ & event_2L(S_1 \|_R S_2, T_1), event_2R(S_1 \|_R S_2, T_2), \\ & event_2Both(S_1 \|_R S_2, T_3), append(T_1, T_2, T_3, T). \end{aligned}$$

Animation of Alg. Sem. and HF(P) (2)

Generating Head Normal Forms: $hf(P,\ T)$

(1) Assignment:

$hf\ (V = E, \quad [[\ 1\ for\ true\ then\ V = E\ \$\ epsilon\ ]]).$

(5) Nondeterministic processes:

$hf\ (S_1 \sqcap S_2, T)\ :\!-\ summation\ (S_1 \sqcap S_2, T).$

where

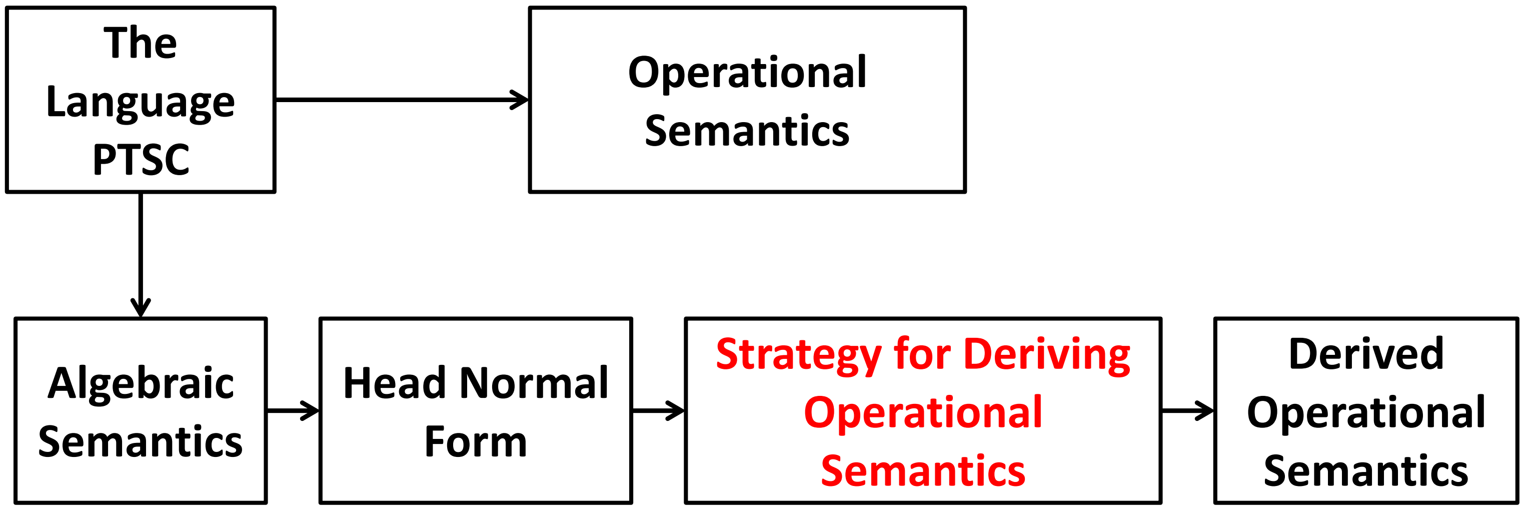
$$summation\ (L_1 \sqcap L_2, T)\ :\!-\ summation\ (L_1, T_1),\ summation\ (L_2, T_2),$$
$$append\ (T_1, T_2, T).$$

$$summation\ (S, [S]).$$

(7)Parallel processes:

$$hf(S_1 \|_R S_2, T)\ :\!-\ summation(S_1, S_1'), summation(S_2, S_2'),$$
$$combination(S_1' \|_R S_2', T).$$

Strategy for Deriving Op. Sem. from Alg. Sem.



Transition Types of Operational Semantics

(1) The execution of an atomic action with certain probability

$$\langle P,\ \sigma\rangle \ \ \xrightarrow{c}_p \ \ \langle P',\ \sigma'\rangle$$

(2) The time delay

$$\langle P,\ \sigma\rangle \ \ \xrightarrow{1} \ \ \langle P',\ \sigma'\rangle$$

(3) The selection of the two components for non-deterministic choice.

$$\langle P,\ \sigma\rangle \ \ \xrightarrow{\tau} \ \ \langle P',\ \sigma\rangle$$

(4) The triggered case of event @ b:

$$\langle P,\ \sigma\rangle \ \ \xrightarrow{v} \ \ \langle P',\ \sigma\rangle$$

Derivation Strategy

Definition (Derivation Strategy)

Let $\mathcal{HF}(P) = \bigoplus_{i \in I} P_i$.

(1) If $|I| > 1$, then $\langle P, \sigma \rangle \xrightarrow{\tau} \langle P_i, \sigma \rangle \ (i \in I)$.

(2) Otherwise,

(a) If $\mathcal{HF}(P) =$

$$\|_{i \in I} \{ [p_i] \ choice_{j \in J_i} (b_{ij} \& (x_{ij} := e_{ij}) \, P_{ij}) \},$$

then $\langle P, \sigma \rangle \xrightarrow{c}_{p_i} \langle P_{ij}, \sigma[e_{ij}/x_{ij}] \rangle$, if $b_{ij}(\sigma)$

(b) If $\mathcal{HF}(P) = \|_{i \in I} \{ @b_i \, P_i \}$,

then $\langle P, \sigma \rangle \xrightarrow{v} \langle P_i, \sigma \rangle$, if $b_i(\sigma)$

$\langle P, \sigma \rangle \xrightarrow{1} \langle P, \sigma \rangle$, if $\bigwedge_{i \in I} \neg b_i(\sigma)$

(c) If $\mathcal{HF}(P) = \|\{\#1\, R\}$,

then $\langle P,\ \sigma\rangle \xrightarrow{1} \langle R,\ \sigma\rangle$.

(d) If $\mathcal{HF}(P) =$

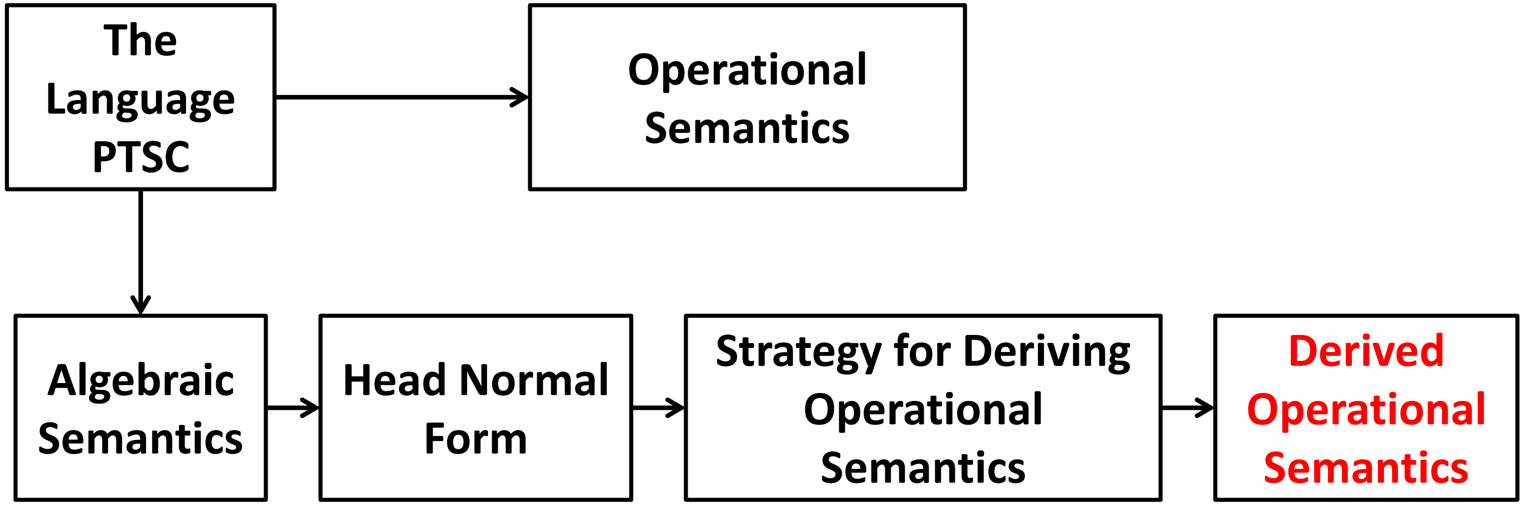
$$\|_{i\in I}\{[p_i]\ choice_{j\in J}(b_{ij}\&(x_{ij} := e_{ij})\, P_{ij})\}$$
$$\|\|_{k\in K}\{@c_k\, Q_k\},$$

then $\cdots\cdots$

(e) If $P = \|_{i\in I}\{@b_i\, P_i\}\|\{\#1\, R\}$,

then $\cdots\cdots$

Deriving Op. Sem. from Alg. Sem.



Deriving Operational Semantics by Proof (1)

Theorem

(1) $\langle x := e,\ \sigma\rangle \xrightarrow{c}_1 \langle \varepsilon,\ \sigma[e/x]\rangle$

(2) $\langle \textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ Q\ ,\ \sigma\rangle \xrightarrow{c}_1 \langle P,\ \sigma\rangle,\quad \text{if } b(\sigma)$

$\langle \textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ Q\ ,\ \sigma\rangle \xrightarrow{c}_1 \langle Q,\ \sigma\rangle,\quad \text{if } \neg b(\sigma)$

(6) $\langle P \sqcap_p Q,\ \sigma\rangle \xrightarrow{c}_p \langle P,\ \sigma\rangle$

$\langle P \sqcap_p Q,\ \sigma\rangle \xrightarrow{c}_{1-p} \langle Q,\ \sigma\rangle$

Deriving Operational Semantics by Proof (2)

Theorem

(1) If $\langle P,\ \sigma\rangle \xrightarrow{\tau} \langle P',\ \sigma'\rangle$, then $\langle P \sqcap Q,\ \sigma\rangle \xrightarrow{\tau} \langle P',\ \sigma'\rangle$

$\langle Q \sqcap P,\ \sigma\rangle \xrightarrow{\tau} \langle P',\ \sigma'\rangle$

(2) If $stable(P)$, then $\langle P \sqcap Q,\ \sigma\rangle \xrightarrow{\tau} \langle P,\ \sigma'\rangle$

$\langle Q \sqcap P,\ \sigma\rangle \xrightarrow{\tau} \langle P,\ \sigma'\rangle$

Theorem

(1) (a) If $\langle P,\ \sigma\rangle \xrightarrow{\tau} \langle P',\ \sigma\rangle$ and $stable(\langle Q, \sigma\rangle)$,

then $\langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{\tau} \langle \mathbf{par}(P', Q, p_1),\ \sigma\rangle$.

$\langle Q \parallel_{p_1} P,\ \sigma\rangle \xrightarrow{\tau} \langle \mathbf{par}(Q, P', p_1),\ \sigma\rangle$.

(b) If $\langle P,\ \sigma\rangle \xrightarrow{\tau} \langle P',\ \sigma,\rangle$ and $\langle Q,\ \sigma\rangle \xrightarrow{\tau} \langle Q',\ \sigma\rangle$,

then $\langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{\tau} \langle \mathbf{par}(P', Q', p_1),\ \sigma\rangle$

(2) (a) If $\langle P,\ \sigma\rangle \xrightarrow{v} \langle P',\ \sigma\rangle$ and $stable(\langle Q,\sigma\rangle)$ and $stableE(\langle Q,\sigma\rangle)$,

$$\text{then } \langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{v} \langle \mathbf{par}(P',Q,p_1),\ \sigma\rangle.$$

$$\langle Q \parallel_{p_1} P,\ \sigma\rangle \xrightarrow{v} \langle \mathbf{par}(Q,P',p_1),\ \sigma\rangle.$$

(b) If $\langle P,\ \sigma\rangle \xrightarrow{v} \langle P',\ \sigma\rangle$ and$\langle Q,\ \sigma\rangle \xrightarrow{v} \langle Q',\ \sigma\rangle$,

$$\text{then } \langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{v} \langle \mathbf{par}(P',Q',p_1),\ \sigma\rangle$$

(3) If $\langle P,\ \sigma\rangle \xrightarrow{c}_{p_2} \langle P',\ \sigma'\rangle$ and $stable(\langle x,\sigma\rangle)$ and $stableE(\langle x,\sigma\rangle)$ $(x = P,\ Q)$,

$$\text{then } \langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{c}_{p_1\times p_2} \langle \mathbf{par}(P',Q,p_1),\ \sigma'\rangle$$

$$\langle Q \parallel_{p_1} P, \sigma\rangle \xrightarrow{c}_{(1-p_1)\times p_2} \langle \mathbf{par}(Q,P',p_1), \sigma'\rangle$$

(4) If $\langle P,\ \sigma\rangle \xrightarrow{1} \langle P',\ \sigma'\rangle$ and $\langle Q,\ \sigma\rangle \xrightarrow{1} \langle Q',\ \sigma'\rangle$ and $stable(\langle x,\sigma\rangle)$ and $stableE(\langle x,\sigma\rangle)$ $(x = P,\ Q)$,

$$\text{then } \langle P \parallel_{p_1} Q,\ \sigma\rangle \xrightarrow{1} \langle \mathbf{par}(P',Q',p_1),\ \sigma'\rangle.$$

Equivalence of Deriv Stra and Tran Syst

Lemma

(1) If transition $\langle P, \alpha \rangle \stackrel{\beta}{\longrightarrow} \langle P', \alpha' \rangle$ exists in the *transition system*, then it also exists in the *derivation strategy*.

(2) If transition $\langle P, \alpha \rangle \stackrel{\beta}{\longrightarrow} \langle P', \alpha' \rangle$ exists in the *derivation strategy*, then it also exists in the *transition system*.

Theorem: Regarding the derived operational semantics for our probabilistic language, the derivation strategy is equivalent to the transition system.

Animation Approaches to Operational Semantics

(1) **Animation of Operational Semantics:**

For the derived operational semantics of PTSC, we can animate this transition system.

(2) **Animation of Derivation Strategy of Operational Semantics:**

With the mechanical approach of algebraic semantics and head normal form, we can animate the execution of a program based on the derivation strategy of operational semantics from algebraic semantics.

3 **Advantages:**

Using the simulated execution of the two animation approaches, the fact of the equivalence between the derivation strategy and the derived operational semantics can be shown through various test examples.

Animation of Operational Semantics for PTSC

Assignment Guarded Component:

(1) assignment guarded choice;

(2) guarded choice composed of assignment and event guarded components

$$\frac{EB\ \$\ (Sigma)\ \wedge\ Sigma\_ = Sigma \otimes (V = E) \wedge\ [S', Sigma]\ /\!\!-\!\![\,'v'] \!\!\rightarrow\ [\_, Sigma]}{[[[Pr\ for\ EB\ then\ (V = E)\ \$\ S]|S'], Sigma] -[\,'c', Pr] \!\!\rightarrow [S, Sigma\_].}$$

$$\frac{EB\ \$\ (Sigma)\ \wedge\ [S', Sigma] -[\,'c', Pr'] \!\!\rightarrow [S_1, Sigma\_]\ \wedge\ [S', Sigma]\ /\!\!-\!\![\,'v'] \!\!\rightarrow\ [\_, Sigma]}{[[[\_Pr\ for\ EB\ then\ (\_V = \_E)\ \$\ \_S]\ |\ S'], Sigma] -[\,'c', Pr'] \!\!\rightarrow [S_1, Sigma\_].}$$

$$\frac{[S', Sigma] -[\,'v'] \!\!\rightarrow [S_1, Sigma]}{[[[\_Pr\ for\ \_EB\ then\ (\_V = \_E)\ \$\ \_S]|S'], Sigma] -[\,'v'] \!\!\rightarrow [S_1, Sigma].}$$

Animation of Gene Oper Sem from Alge Sem (1)

Definition:

(1) If the head normal form of process P can be expressed as a guarded choice, then the transition rules for the process P are the same as the transition rules of its corresponding guarded choice.

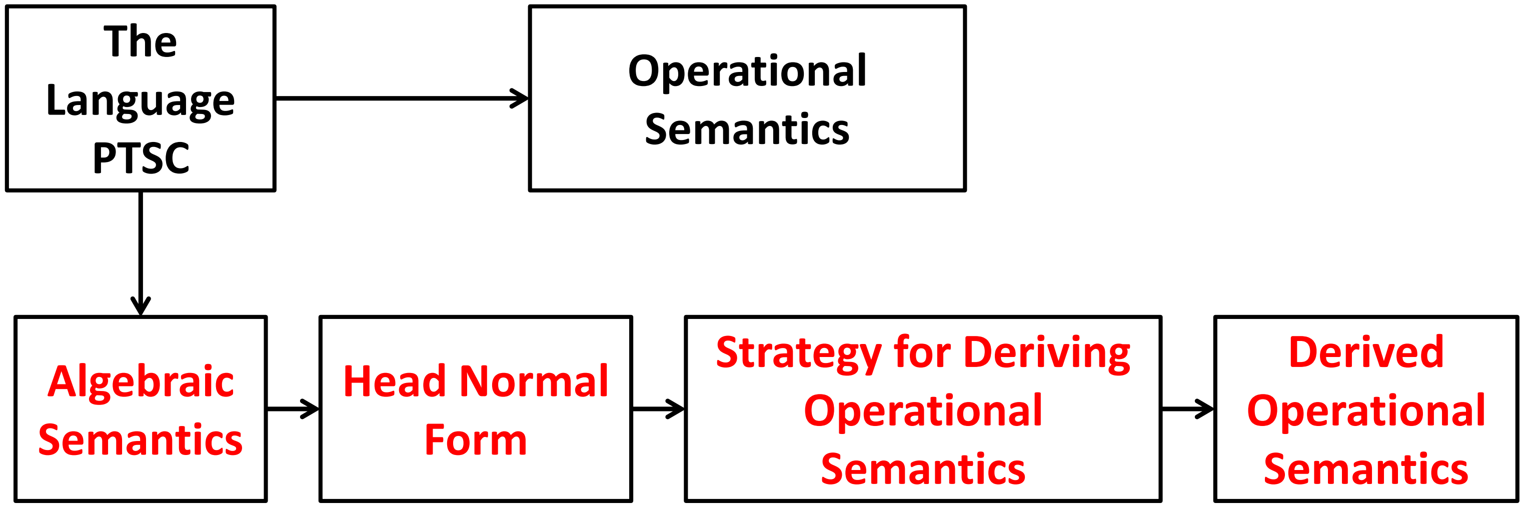
(2) On the other hand, if the head normal form of process P has a structure of summation, then the process P can first do $['tau']$ transitions and reach to all the processes that are initially deterministic.

$$\frac{\sim pgc([X \mid L])}{[[X \mid L], Sigma] \dashv['tau']\mapsto [X, Sigma].}$$

$$\frac{L \sim= [\ ] \wedge [L, Sigma] \dashv['tau']\mapsto [Y, Sigma]}{[[\_ \mid L], Sigma] \dashv['tau']\mapsto [Y, Sigma].}$$

Here, $\sim pgc([X \mid L])$ indicates that the head normal form of $[X \mid L]$ is not in the form of the five types of guarded choice, which means it is a summation.

Conclusion (1)



Conclusions (2)

Theoretical Approach:

- We have provided algebraic laws. A process can be expressed as either a guarded choice, or the summation of a set of processes that are initially deterministic. program.
- We have studied the derivation of the operational semantics for our language from its algebraic semantics. A transition system (i.e., operational semantics) for our language can be derived via the derivation strategy.
- We have investigated the relationship between the derivation strategy and the derived operational semantics (the equivalence).

Conclusions (3)

Mechanical Approach:

- We explored the algebraic laws for $PTSC$ using a mechanical approach. We mainly focused on the mechanical generation of the parallel expansion laws.
- We studied the mechanical generation of the head normal form for a program.
- We implemented the theoretical derivation strategy for deriving the operational semantics from the algebraic semantics. For the derived operational semantics as a whole system, we also investigated its animation.

Current and Future Work

1. **Our language PTSC:**
   We are exploring the denotational semantics for PTSC and doing the link between various semantics for PTSC.

2. Quality Calculus + PTSC:
   Currently we are working on integrating quality calculus with PTSC.

3. **Wireless System and Mobile Ad Hoc Networks:**
   For wireless system and mobile ad hoc networks, we are studying various semantics and their linking theories.

4. **Cyber-Physical Systems:**
   We are doing the algebraic semantics, denotational semantics, etc for Cyber-Physical Systems.